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# A fusion approach for managing multi-granularity linguistic term sets in decision making

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**Abstract**

The aim of this paper is to present a fusion approach of multi-granularity linguistic information for managing information assessed in different linguistic term sets (multi-granularity linguistic term sets) together with its application in a decision making problem with multiple information sources, assuming that the linguistic performance values given to the alternatives by the different sources are represented in linguistic term sets with different granularity and/or semantic. In this context, a decision process based on two steps is proposed with a view to obtaining the solution set of alternatives. First, the fusion of the multi-granularity linguistic performance values is carried out in order to obtain collective performance evaluations. In this step, on the one hand, the multi-granularity linguistic information is made uniform using a linguistic term set as the uniform representation base, the *basic linguistic term set*. On the other hand, the collective performance evaluations of the alternatives are obtained by means of an aggregation operator, being fuzzy sets on the *basic linguistic term set*. Second, the choice of the best alternative(s) from the collective performance evaluations is performed. To do that, a fuzzy preference relation is computed from the collective performance evaluations using a ranking method of pairs of fuzzy sets in the setting of Possibility Theory, applied to fuzzy sets on the *basic linguistic term set*. Then, a choice degree may be applied on the preference relation in order to rank the alternatives. 

*Keywords:* Decision-making; Linguistic modelling; Linguistic variables; Multi-granularity linguistic term sets; Fuzzy preference relation

## 1. Introduction

Many aspects of day-to-day activities are evaluated by means of imprecise and fuzzy qualitative values. As was pointed out in [6] this may arise for different reasons. There are some situations in which information may be unquantifiable due to its nature, and thus, it can be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car, terms like "good", "fair", "poor" can be used). In other cases, precise quantitative information cannot be stated because either it is unavailable or the cost for its computation is too high and an "approximate value" can be tolerated (e.g., when evaluating the speed of a car, linguistic terms like "fast", "very fast", "slow" can be used instead of numeric values).

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The use of Fuzzy Sets Theory has given very good results for modelling qualitative information [41]. It is a technique that handles fuzziness and represents qualitative aspects as linguistic labels by means of "linguistic variables", that is, variables whose values are not numbers but words or sentences in a natural or artificial language. The linguistic approach is used in different fields, such as for example, "information retrieval" [2], "clinical diagnosis" [8, 34], "marketing" [40], "risk in software development" [25], "technology transfer strategy selection" [5], "education" [24], "decision making" [10, 16, 33, 37], etc.

In any linguistic approach, an important parameter to determine is the "granularity of uncertainty", i.e., the cardinality of the linguistic term set used to express the information. According to the uncertainty degree that an expert qualifying a phenomenon has on it, the linguistic term set chosen to provide his knowledge will have more or less terms. When different experts have different uncertainty degrees on the phenomenon, then several linguistic term sets with a different granularity of uncertainty are necessary.

In this paper, we deal with the management of multi-granularity linguistic term sets applied to decision making problems with numerous information sources. The information sources may be "experts" [22] or "criteria" [13] or "purposes" [7, 23], but we do not distinguish among them, and interpret the decision process with multiple information sources. Therefore, we consider decision making problems where a set of alternatives must be analyzed according to different sources in order to select the best one(s), being considered as linguistic term sets with different granularity and/or semantics, according to the uncertainty degree of each source, in order to provide the performance evaluations.

We present a fusion approach of multi-granularity linguistic information which allows us to solve the decision process. Following the classical decision scheme (aggregation and exploitation) [32], but considering our particular decision context, we propose the following two steps for developing the decision process:

1. *Fusion of multi-granularity linguistic information.* A collective linguistic performance profile is obtained by means of the fusion of multi-granularity linguistic performance profiles provided by the different information sources. The fusion scheme presented is carried out in two phases:
   (a) *Making the information uniform.* The performance values expressed using multi-granularity linguistic term sets are converted into a specific linguistic domain, which is a *basic linguistic term set* (BLTS), chosen so as not to impose useless precision to the original evaluations and in order to allow an appropriate discrimination of the initial performance values. Each linguistic performance value is defined as a fuzzy set on the BLTS, i.e., the semantic associated to the values of the initial linguistic term sets is obtained by means of fuzzy sets defined in the new specific linguistic domain.
   (b) *Computing the collective performance values.* For each alternative, a collective performance value is obtained by means of the aggregation of the aforementioned fuzzy sets on the BLTS that represents the individual performance values assigned to the alternative according to each information source. Therefore, each collective performance value is a new fuzzy set on the specific linguistic domain, the BLTS.
2. *Choosing the best alternative(s).* A fuzzy preference relation is computed from the collective performance values (their associated fuzzy sets) using a ranking method of pairs of fuzzy sets in the setting of Possibility Theory, applied to the fuzzy sets on the BLTS. Then, a choice degree is used to reach a solution set of alternatives.

In order to do so, this paper is set out as follows. Section 2 briefly explains the linguistic approach in decision making. Section 3 presents a decision process with the fusion approach for managing multi-granularity linguistic term sets. Section 4 develops an example, and finally, some conclusions are pointed out.

## 2. The linguistic approach in decision making

Usually, in a quantitative setting, the information is expressed by means of numerical values, however, when we work with vague or imprecise knowledge, the information could not be estimated with an exact

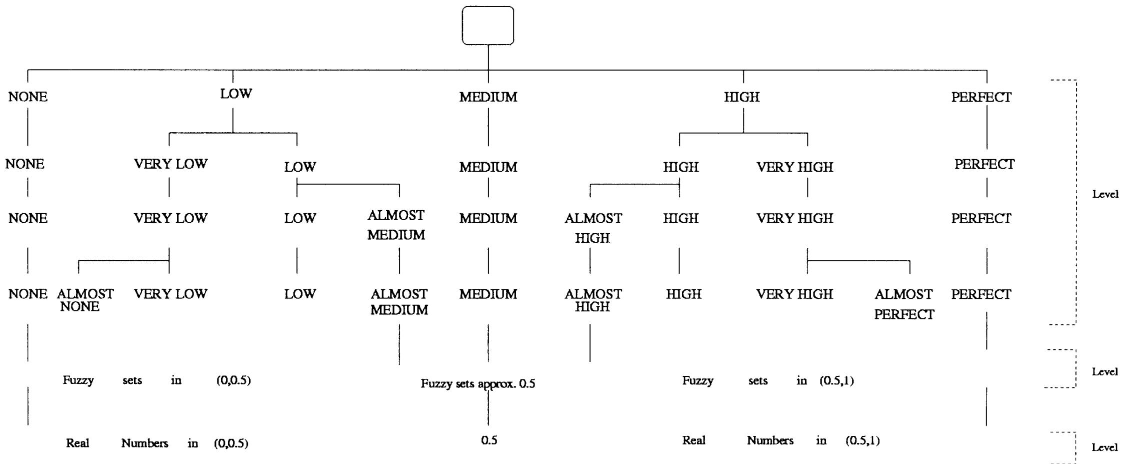


Fig. 1. Hierarchy of expression domains.

numerical value. Then, a more realistic approach may be to use linguistic assessments instead of numerical values, that is, we suppose that the variables which participate in the problem are assessed by means of linguistic terms [41]. This approach is appropriate for a lot of problems, since it allows a representation of the information in a more direct and adequate form if we are unable to express it with precision. In decision making there are a great number of approaches using linguistic information, such as, Multiattribute Decision Making [33], Multiobjective Decision Making [35], Possibility Evaluation Based on Ordinal Utility [20], Multicriteria Multiperson Decision Making [4, 38], Multicriteria Decision Making [5, 15, 26], Linguistic Evidence Based Decision Model [10], Group Decision Making [16, 17, 25], Consensus in Group Decision Making [3, 18, 19], etc.

A linguistic variable differs from a numerical one in that its values are not numbers, but words or sentences in a natural or artificial language. Since words, in general, are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a means of approximate characterization of phenomena, which are too complex, or too ill-defined to be amenable to their description in conventional quantitative terms.

**Definition 1** (Zadeh [41]). A linguistic variable is characterized by a quintuple $(H, T(H), U, G, M)$ in which $H$ is the name of the variable; $T(H)$ (or simply $T$) denotes the term set of $H$, i.e., the set of names of linguistic values of $H$, with each value being a fuzzy variable denoted generically by $X$ and ranging across a universe of discourse $U$ which is associated with the base variable $u$; $G$ is a syntactic rule (which usually takes the form of a grammar) for generating the names of values of $H$; and $M$ is a semantic rule for associating its meaning with each $H$, $M(X)$, which is a fuzzy subset of $U$.

Usually, depending on the problem domain, an appropriate linguistic term set is chosen and used to describe the vague or imprecise knowledge. The number of elements in the term set will determine the granularity of the uncertainty, that is, the level of distinction among different countings of uncertainty. In [1] the use of term sets with an odd cardinal was studied, representing the midterm by an assessment of "approximately 0.5", with the rest of the terms being placed symmetrically around it and the limit of granularity being 11 or not more than 13.

Fig. 1 shows an example of a complete hierarchical structure of expression domains. Level 1 provides the domains with linguistic granularity, formed by the linguistic term sets, level 2 shows a finer granularity than

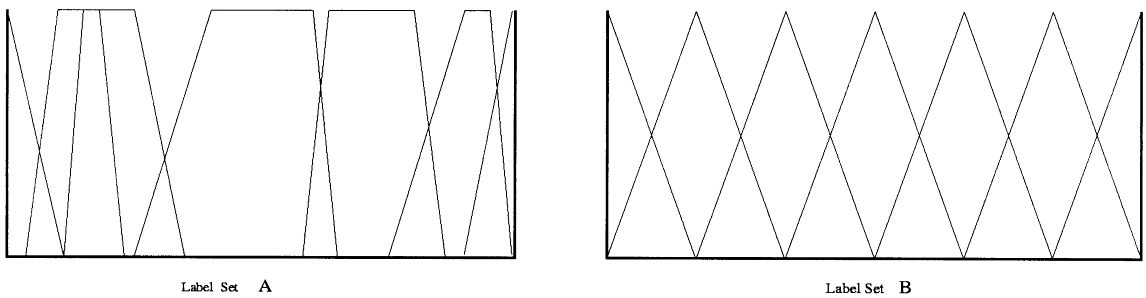


Fig. 2. Linguistic term sets.

level 1, i.e., it uses fuzzy sets as the expression domain, and finally, level 3 presents the finest granularity, the numerical values in $[0,1]$.

The semantic of the elements in a linguistic term set is given by fuzzy numbers defined in the $[0,1]$ interval, which are described by their membership functions. We know that the linguistic assessments are just approximate ones given by the individuals, therefore, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible or unnecessary to obtain more accurate values. This representation is achieved by the 4-tuple $(x_0,x_1,x_2,x_3)$, $x_1$ and $x_2$ indicate the interval in which the membership function value is 1, and $x_0$ and $x_3$ are the left and right limits of the definition domain of trapezoidal membership function. A particular case of this type of representation are the linguistic assesments whose membership functions are triangular, i.e., $x_1=x_2$, then we represent this type of membership functions by a 3-tuple $(x_0,x_1,x_2)$, where $x_1$ is the point where the membership is 1 and $x_0$ and $x_2$ are the left and right limits of the definition domain of the triangular membership function.

Formally speaking, it seems difficult to accept that all individuals should agree on the same membership function associated to linguistic terms, and therefore, there are no universality distribution concepts. We can find some situations where term sets exist with a similar syntax and different semantic are used to evaluate them. For instance, let us look at the following term sets (see Fig. 2):

| Label set A | | | Label set B | | |
|---|---|---|---|---|---|
| *N* | *None* | (0, 0, 0, 0.1) | *N* | *None* | (0, 0, 0.16) |
| *VL* | *Very Low* | (0.04, 0.1, 0.18, 0.23) | *VL* | *Very Low* | (0, 0.16, 0.33) |
| *L* | *Low* | (0.1, 0.15, 0.25, 0.35) | *L* | *Low* | (0.16, 0.33, 0.49) |
| *M* | *Medium* | (0.25, 0.4, 0.6, 0.65) | *M* | *Medium* | (0.33, 0.5, 0.67) |
| *H* | *High* | (0.58, 0.63, 0.8, 0.86) | *H* | *High* | (0.49, 0.67, 0.84) |
| *VH* | *Very High* | (0.75, 0.90, 0.95, 0.99) | *VH* | *Very High* | (0.67, 0.84, 1) |
| *P* | *Perfect* | (0.9, 1, 1, 1) | *P* | *Perfect* | (0.84, 1, 1). |

The first priority ought to be to establish what kind of term set to use. Let $S=\{s_i\}$, $i\in H=\{0,\dots,T\}$, be a finite and totally ordered term set in $[0,1]$ in the usual sense [1, 9]. Any label, $s_i$, represents a possible value for a linguistic variable, that is, a vague property or constraint in $[0,1]$. We consider a finite term set, $S$, as in [1] with its semantic given by linear trapezoidal/triangular membership functions. Moreover, it must have the following characteristics:

(1) The set is ordered: $s_i \geqslant s_j$ if $i \geqslant j$.
(2) There is the negation operator: $\mathrm{Neg}(s_i)=s_j$ such that $j=T-i$.
(3) There is the maximization operator: $\mathrm{Max}(s_i,s_j)=s_i$ if $s_i \geqslant s_j$.
(4) There is the minimization operator: $\mathrm{Min}(s_i,s_j)=s_i$ if $s_i \leqslant s_j$.

## 3. Managing multi-granularity linguistic term sets in decision making

The decision-making problem considered is defined as follows. Let $X=\{x_1,x_2,\dots,x_n\}$ $(n\geqslant 2)$ be a finite set of alternatives to be qualified according to a finite set of information sources $P=\{p_1,p_2,\dots,p_m\}$ $(m\geqslant 2)$. Each source $p_j$ provides a linguistic performance value $p^{ij}$ for each alternative $x_i$. Given that we deal with multi-granularity linguistic term sets in decision-making problems, we assume that each $p_j$ may use a different linguistic term set $S_j$ to express the performance values. The linguistic term sets $\{S_j,\ \forall j\}$ may have a different granularity and/or semantics. Therefore, for each $p_j$, the performance profile of the alternatives is defined as a linguistic fuzzy choice subset defined over $X$ and assessed linguistically on $S_j$:

$$p_j \rightarrow (p^{1j},\dots,p^{nj})\ \ p^{ij}\in S_j \quad S_j=\{s_0^j,\dots,s_{k_j}^j\}\ \ j\in\{1,\dots,m\},$$

where $k_j+1$ is the cardinality of $S_j$. In this framework, the objective of a decision process is the identification of the alternatives which are judged the best according to the evaluations provided by the sources.

As was mentioned earlier, the decision process proposed is developed in two steps:

1. Fusion of multi-granularity linguistic information.
2. Choosing the best alternatives.

In the following subsections, we analyze each step in greater detail. But, before this, we should point out some previous considerations:

- We assume a representation of performance profiles based on linguistic fuzzy choice subsets, but if some sources prefer to use linguistic preference relations [16], then we may easily obtain their respective linguistic fuzzy choice subsets applying some of the linguistic choice functions proposed in [14], and so on using another linguistic preference structure. In this sense, there is no lack of generality on the representation approach.
- We assume that all sources assess the alternatives on the same scale (specifically the unit interval [0,1]), i.e., with term sets covering the same range of that scale and the only difference being the granularity. The problem of managing information when different scales are used is not addressed here.

### 3.1. Fusion of multi-granularity linguistic information

It consists of obtaining a collective performance profile on the alternatives according to the individual performance profiles (assessed in multi-granularity linguistic term sets). So, we present a fusion tool of multi-granularity linguistic information which, as was mentioned earlier, is performed in two phases:

1. Making the information uniform.
2. Computing the collective performance values.

They are analyzed in the following subsections.

#### 3.1.1. Making the information uniform

With a view to managing the information we must make it uniform, i.e., the multi-granularity linguistic information provided by all the sources must be transformed (under a transformation function) into a unified linguistic term set, as we mentioned, called BLTS, and denoted as $S_T$.

Before defining a transformation function to this BLTS, $S_T$, we have to decide how to choose $S_T$. We think that $S_T$ must be a linguistic term set which allows us to maintain the uncertainty degrees associated to each purpose and the ability of discrimination to express the performance values. With this goal in mind, we look for a BLTS with the maximum granularity. We consider two possibilities:

- When there is only one term set with the maximum granularity, then, it is chosen as $S_T$.
- If we have two or more linguistic term sets with maximum granularity, then $S_T$ is chosen depending on the semantics of these linguistic term sets, finding two possible situations to establish $S_T$:

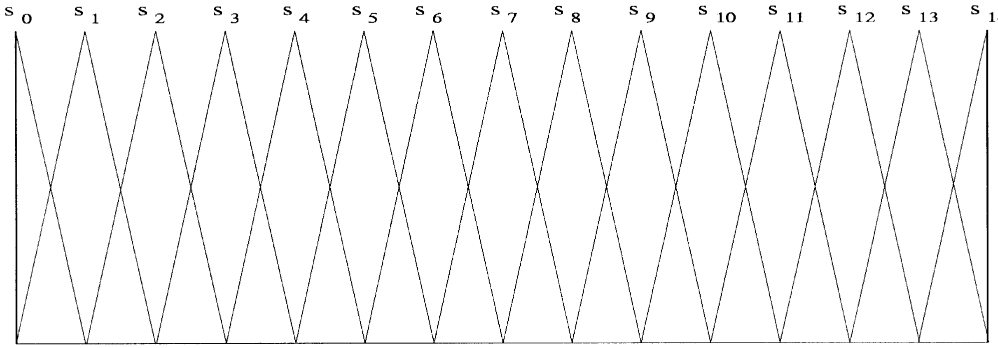


Fig. 3. Term set with 15 terms.

1. If all the linguistic term sets have the same semantics, then $S_T$ is any of them.
2. There are some linguistic term sets with different semantics. Then, $S_T$ is a basic linguistic term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [27]). We define a BLTS with 15 terms and the following semantics (see Fig. 3):

| | | | |
|---|---|---|---|
| $s_0$ | $(0, 0, 0.07)$ | $s_1$ | $(0, 0.07, 0.15)$ |
| $s_2$ | $(0.07, 0.15, 0.22)$ | $s_3$ | $(0.15, 0.22, 0.29)$ |
| $s_4$ | $(0.22, 0.29, 0.36)$ | $s_5$ | $(0.29, 0.36, 0.43)$ |
| $s_6$ | $(0.36, 0.43, 0.5)$ | $s_7$ | $(0.43, 0.5, 0.58)$ |
| $s_8$ | $(0.5, 0.58, 0.65)$ | $s_9$ | $(0.58, 0.65, 0.72)$ |
| $s_{10}$ | $(0.65, 0.72, 0.79)$ | $s_{11}$ | $(0.72, 0.79, 0.86)$ |
| $s_{12}$ | $(0.79, 0.86, 0.93)$ | $s_{13}$ | $(0.86, 0.93, 1)$ |
| $s_{14}$ | $(0.93, 1, 1)$. | | |

**Remark 1.** We should point out that the justification on this choice is based under the idea that the semantic is a parameter used by the conversion process, and thus, it has effect on the final result. We decide to use a symmetrical term set with a granularity bigger than the number of terms that an expert is able to discriminate (11 or 13, see [27]).

We define a transformation function, which represents each linguistic performance value as a fuzzy set defined in the BLTS, $S_T$, as follows.

**Definition 2.** Let $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ be two linguistic term sets, such that, $g \geqslant p$. Then, a multi-granularity transformation function, $\tau_{AS_T}$ is defined as

$$\tau_{AS_T} : A \to F(S_T),$$

$$\tau_{AS_T}(l_i) = \{(c_k, \alpha_k^i) \ / k \in \{0, \ldots, g\}\}, \quad \forall l_i \in A,$$

$$\alpha_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{c_k}(y)\},$$

where $F(S_T)$ is the set of fuzzy sets defined in $S_T$, and $\mu_{l_i}(y)$ and $\mu_{c_k}(y)$ are the membership functions of the fuzzy sets associated to the terms $l_i$ and $c_k$, respectively.

Therefore, the result of $\tau_{AS_T}$ for any linguistic value of $A$ is a fuzzy set defined in the BLTS, $S_T$.

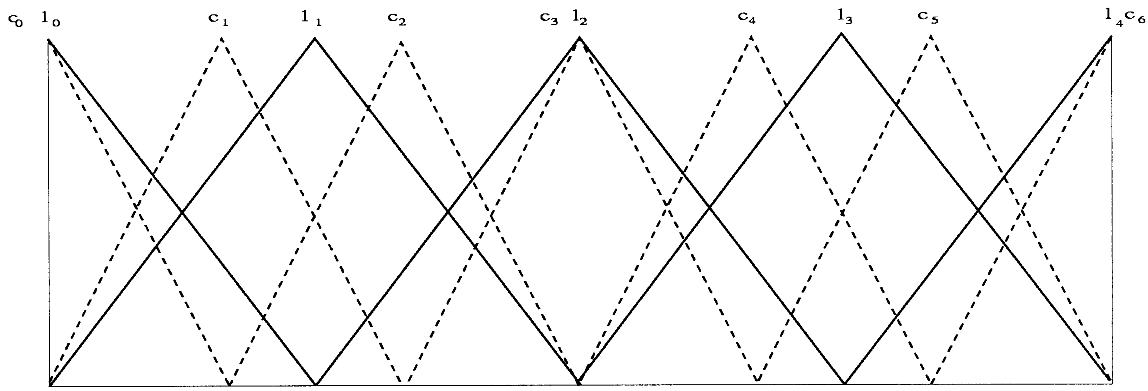


Fig. 4. Term sets $A$ and $S_T$.

**Remark 2.** As was mentioned earlier, we consider that all the information sources use the same scale ([0,1]). Then, we have chosen the max–min operation in this definition because it is a classical tool to set the matching degree between fuzzy sets [13, 43]. However, many other specifications could be chosen, e.g., to use a t-conorm and a t-norm [13] instead of max and min operations, respectively.

**Example.** Let $A=\{l_0,l_1,\ldots,l_4\}$ and $S_T=\{c_0,c_1,\ldots,c_6\}$ be two term sets, with 5 and 7 labels, respectively, and with the following semantics associated (see Fig. 4):

| | | | |
|---|---|---|---|
| $l_0$ | $(0,0,0.25)$ | $c_0$ | $(0,0,0.16)$ |
| $l_1$ | $(0,0.25,0.5)$ | $c_1$ | $(0,0.16,0.34)$ |
| $l_2$ | $(0.25,0.5,0.75)$ | $c_2$ | $(0.16,0.34,0.5)$ |
| $l_3$ | $(0.5,0.75,0.1)$ | $c_3$ | $(0.34,0.5,0.66)$ |
| $l_4$ | $(0.75,1,1)$ | $c_4$ | $(0.5,0.66,0.84)$ |
| | | $c_5$ | $(0.66,0.84,1)$ |
| | | $c_6$ | $(0.84,1,1)$. |

The fuzzy sets obtained after applying $\tau_{AS_T}$ for $l_0$ and $l_1$ are:

$$\tau_{AS_T}(l_0)=\{(c_0,1),(c_1,0.58),(c_2,0.18),(c_3,0),(c_4,0),(c_5,0),(c_6,0)\},$$

$$\tau_{AS_T}(l_1)=\{(c_0,0.39),(c_1,0.85),(c_2,0.85),(c_3,0.39),(c_4,0),(c_5,0),(c_6,0)\}.$$

In Fig. 5, we can see the conversion process for both labels graphically.

Using the *multi-granularity transformation functions* $\{\tau_{S_jS_T},\ \forall j\}$ a conversion process consists of the transformation of all the linguistic performance profiles provided by the multiple information sources

$$\{p^{1j},\ldots,p^{nj}\},\quad \forall p_j\ (p^{ij}\in S_j)$$

into $S_T$. Then we represent each linguistic performance value $p^{ij}$ as a fuzzy set defined on $S_T=\{c_0,\ldots,c_g\}$ characterized by the following expression:

$$\tau_{S_jS_T}(p^{ij})=\{(c_0,\alpha_0^{ij}),\ldots,(c_g,\alpha_g^{ij})\}.$$

Thus, the performance profile of a source $p_j$ is represented as a set of fuzzy sets on $S_T$,

$$\{\tau_{S_jS_T}(p^{1j}),\ldots,\tau_{S_jS_T}(p^{nj})\}.$$

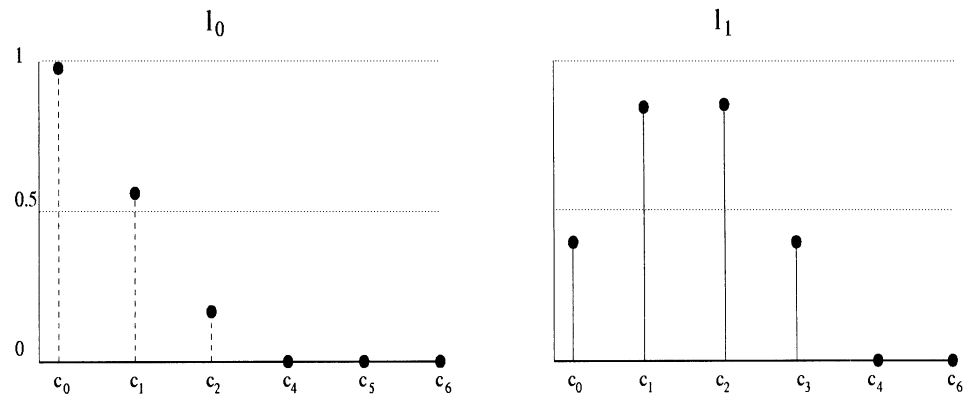


Fig. 5. $l_0$ and $l_1$ as fuzzy sets on $S_T$.

We denote $\tau_{S_j S_T}(p^{ij})$ as $r^{ij}$, and represent each fuzzy set of performance, $r^{ij}$, by means of its respective membership degrees, i.e.,

$$r^{ij} = (\alpha_0^{ij}, \ldots, \alpha_g^{ij}).$$

In the following subsection, we present how to obtain the collective performance values of each alternative assuming the given representation of the individual performance values.

### 3.1.2. Computing collective performance values

As we have just shown, the individual performance value over an alternative $x_i$ provided by a information source $p_j$ is defined as the performance fuzzy set $r^{ij}$ on the BLTS, $S_T$. Then, the collective performance value of an alternative $x_i$ according to all the source evaluations $\{r^{ij}, \forall j\}$ is obtained by means of the aggregation of these fuzzy sets. This collective performance value, denoted $r^i$, is a new fuzzy set defined on $S_T$, i.e.,

$$r^i = (\alpha_0^i, \ldots, \alpha_g^i),$$

characterized by the following membership function:

$$\alpha_k^i = f(\alpha_k^{i1}, \ldots, \alpha_k^{im}),$$

where $f$ is an "aggregation operator".

Therefore, the result of this step in our decision process is a set of collective evaluations, that provides the collective performance value of each alternative according to all the source evaluations supplied for those alternatives, i.e.,

$$\{r^1, \ldots, r^n\}.$$

In the following subsection, we show how to achieve a solution set of alternatives from the collective evaluation of the alternatives.

## 3.2. Choosing the best alternatives

The goal of the decision process is to reach a set with the best alternative(s) according to the performance values of all the sources. In our case, the performance values of the alternatives are fuzzy sets on the BLTS, $r^i$, and in this framework, it is not an easy task to define a choice method. We solve this difficulty by changing the representation of the collective evaluations based on fuzzy sets in $S_T$, for a representation based on a fuzzy preference relation. We have decided to use a fuzzy preference relation because it contains a large quantity of information for the choice of alternatives. It is a well studied and applied preference structure in the literature [7, 13, 21, 23, 28, 29, 31]. Then, we use a ranking method of pairs of fuzzy sets in the setting of Possibility Theory. We apply the degree of possibility of dominance on fuzzy numbers [12] acting on fuzzy sets ($r^i$) in a discrete universe (the BLTS). Therefore, we present a method to obtain the solution set of alternatives composed by two steps:

1. Computing a fuzzy preference relation.
2. Applying a choice degree to this relation in order to rank the alternatives and to choose the best one(s).

*3.2.1. Computing a fuzzy preference relation*

We present the definition of the degree of possibility of dominance and its particularization to act on fuzzy sets defined in the BLTS.

**Definition 3** (Dubois and Prade [12]). Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be two fuzzy numbers, the degree of possibility of dominance of $u$ over $v$ is:

$$P(u \geqslant v) = \max_{z} \min_{y \leqslant z} \{\mu_u(z), \mu_v(y)\}.$$

**Definition 4.** Let $x_i, x_j \in X$ $(i \neq j)$ be two alternatives with their respective collective performance fuzzy sets $r^i, r^j \in F(S_T)$; then the degree of possibility of dominance of $x_i$ over $x_j$, $b_{ij}$, is obtained according to the following expression:

$$b_{ij} = \max_{c_l} \min_{c_h \leqslant c_l} \{\mu_{r^i}(c_l), \mu_{r^j}(c_h)\}, \quad c_l, c_h \in S_T,$$

where $\mu_{r_i}(c_l) = \alpha_l^i$ and $\mu_{r_j}(c_h) = \alpha_h^j$.

Applying this definition over all the possible pairs of the alternatives $(i \neq j)$, we obtain a fuzzy preference relation $B = [b_{ij}]$.

**Remark 3.** The collective performance values on the alternatives are fuzzy sets, and then, for obtaining a ranking of the alternatives, a fuzzy set comparison procedure is needed. When we have to rank various fuzzy sets there are at least two ways of achieving this purpose:

1. to extend the pairwise possibilistic indices to $n$-ary versions, and
2. to build fuzzy relations through pairwise comparison of the fuzzy sets, and then to process these relations so as to obtain some final rankings.

We have decided to use the second approach because the fuzzy preference relations contain the maximum information for the choice of alternatives. For computing the fuzzy preference relation we have proposed the method based on Possibility Theory because this has demonstrated to be a natural framework for the derivation of comparison indices aiming at ranking fuzzy numbers. It is applied in decision-analysis problems where the attractiveness of alternatives must be evaluated and compared [12]. Furthermore, we have simplified it using only the fuzzy relation obtained from a particular index of possibility of dominance ($\max_z \min_{y \geqslant z}$). We find that it gives the maximum information for computing a fuzzy preference relation [12].

*3.2.2. Applying a choice degree*

Finally, the decision process finds the solution set of alternatives applying a choice degree or function to the fuzzy preference relation, $B$. Different choice functions can be found in [13, 31]. Using one of them we can rank the alternatives and choose those ones with the maximum value of choice degree.

The following section presents a particular example of this described decision process.

## 4. Example of a decision process under multiple sources of multi-granularity linguistic information

### 4.1. Example of application

Suppose an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives of where to invest the money:

- $x_1$ is a car industry,
- $x_2$ is a food company,
- $x_3$ is a computer company,
- $x_4$ is an arms industry.

The investment company has a group of four consultancy departments.

- $p_1$ is the risk analysis department,
- $p_2$ is the growth analysis department,
- $p_3$ is the social-political analysis department,
- $p_4$ is the environmental impact analysis department.

Each department is directed by an expert, and thus, each expert is a information source. These experts use, to provide their preferences over the alternative set, different linguistic term sets. Specifically:

- $p_1$ provides his preferences in the set of 9 labels, *A*.
- $p_2$ provides his preferences in the set of 7 labels, *B*.
- $p_3$ provides his preferences in the set of 5 labels, *C*.
- $p_4$ provides his preferences in the set of 9 labels, *D*.

| Label set A | | Label set B | |
|---|---|---|---|
| $a_0$ | (0, 0, 0.12) | $b_0$ | (0, 0, 0.16) |
| $a_1$ | (0, 0.12, 0.25) | $b_1$ | (0, 0.16, 0.33) |
| $a_2$ | (0.12, 0.25, 0.37) | $b_2$ | (0.16, 0.33, 0.5) |
| $a_3$ | (0.25, 0.37, 0.5) | $b_3$ | (0.33, 0.5, 0.66) |
| $a_4$ | (0.37, 0.5, 0.62) | $b_4$ | (0.5, 0.66, 0.83) |
| $a_5$ | (0.5, 0.62, 0.75) | $b_5$ | (0.66, 0.83, 0.1) |
| $a_6$ | (0.62, 0.75, 0.87) | $b_6$ | (0.83, 1, 1) |
| $a_7$ | (0.75, 0.87, 0.1) | | |
| $a_8$ | (0.87, 1, 1) | | |

| Label set C | | Label set D | |
|---|---|---|---|
| $c_0$ | (0,0,0.25) | $d_0$ | (0,0,0,0) |
| $c_1$ | (0,0.25,0.5) | $d_1$ | (0,0.01,0.02,0.07) |
| $c_2$ | (0.25,0.5,0.75) | $d_2$ | (0.04,0.1,0.18,0.23) |
| $c_3$ | (0.5,0.75,1) | $d_3$ | (0.17,0.22,0.36,0.42) |
| $c_4$ | (0.75,1,1) | $d_4$ | (0.32,0.41,0.58,0.65) |
| | | $d_5$ | (0.58,0.63,0.80,0.86) |
| | | $d_6$ | (0.72,0.78,0.92,0.97) |
| | | $d_7$ | (0.93,0.98,0.99,1) |
| | | $d_8$ | (1,1,1,1) |

After an in depth study each expert provides the following performance values:

| | | Alternatives | | | |
|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
| Experts | $p_1$ | $a_4$ | $a_6$ | $a_3$ | $a_5$ |
| | $p_2$ | $b_3$ | $b_4$ | $b_3$ | $b_5$ |
| | $p_3$ | $c_2$ | $c_3$ | $c_2$ | $c_1$ |
| | $p_4$ | $d_4$ | $d_5$ | $d_3$ | $d_5$ |

In the following subsection, we present a particular decision process which allows us to solve this example.

### 4.2. *A decision process under multiple sources of multi-granularity linguistic information based on the OWA operator and the non-dominance choice degree*

This particular decision process follows the same scheme as the proposed general decision process, but it presents the following two peculiarities:

1. The collective performance values $r^i$ are obtained using as the aggregation operator $f$ the OWA operator guided by a fuzzy linguistic quantifier [36, 39], representing the concept of "fuzzy majority".
2. The choice scheme is guided by the "non-dominance choice degree" defined by Orlovski [28].

Both elements are presented in Appendix A.

Then, the decision process with multiple sources of multi-granularity linguistic information may be summarized by the following steps:

1. *Fusion of multi-granularity linguistic information*

It is performed in the following two phases:

- *Making the information uniform.* In this phase, we have to choose the appropriate BLTS, $S_T = \{c_0, \ldots, c_g\}$. In this case, there are two term sets with the maximum granularity and different semantics, then, we choose as $S_T$ the special term set of 15 labels given in Fig. 3. All the assessments must be converted to $S_T$ by means of the set of muti-granularity tranformation functions $\{\tau_{AS_T}, \tau_{BS_T} \tau_{CS_T}, \tau_{DS_T}\}$. We obtain the following results:

$r^{11}$ $(0, 0, 0, 0, 0.05, 0.45, 0.8, 0.82, 0.48, 0.23, 0, 0, 0, 0, 0)$

$r^{12}$ $(0, 0, 0, 0, 0.11, 0.45, 0.65, 0.95, 0.68, 0.39, 0.1, 0, 0, 0, 0)$

$r^{13}$ $(0, 0, 0, 0.22, 0.35, 0.59, 0.8, 0.98, 0.75, 0.52, 0.32, 0.1, 0, 0, 0)$

$r^{14}$ $(0, 0, 0, 0, 0.3, 0.77, 1, 1, 1, 0.51, 0, 0, 0, 0, 0)$

$r^{21}$ $(0, 0, 0, 0, 0, 0, 0, 0, 0.25, 0.99, 0.7, 0.31, 0.01, 0, 0)$

$r^{22}$ $(0, 0, 0, 0, 0, 0, 0, 0.35, 0.63, 0.94, 0.76, 0.46, 0.2, 0, 0)$

$r^{23}$ $(0, 0, 0, 0, 0, 0, 0.01, 0.25, 0.5, 0.7, 0.9, 0.9, 0.65, 0.45, 0.2)$

$r^{24}$ $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0.55, 0, 0)$

$r^{31}$ $(0, 0, 0, 0.18, 0.55, 0.95, 0.7, 0.35, 0, 0, 0, 0, 0, 0, 0)$

$r^{32}$ $(0, 0, 0, 0, 0.1, 0.45, 0.65, 0.95, 0.68, 0.39, 0.1, 0, 0, 0, 0)$

$r^{33}$ $(0, 0, 0, 0.22, 0.35, 0.59, 0.8, 0.98, 0.75, 0.52, 0.32, 0.1, 0, 0, 0)$

$r^{34}$ $(0, 0, 0.41, 1, 1, 0.99, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

$r^{41}$ $(0, 0, 0, 0, 0, 0, 0, 0.36, 0.71, 0.91, 0.56, 0.22, 0, 0, 0)$

$r^{42}$ $(0,0,0,0,0,0,0,0,0,0.23,0.54,0.84,0.86,0.58,0.3)$

$r^{43}$ $(0.25,0.4,0.7,0.9,0.87,0.65,0.4,0.2,0,0,0,0,0,0,0)$

$r^{44}$ $(0,0,0,0,0,0,0,0,0,1,1,1.55,0,0)$.

- *Computing the collective performance values.* In this phase, for each alternative $x_i$, we compute its collective performance value using the OWA operator, $F_Q$, guided by a fuzzy linguistic quantifier $Q$ (see Appendix A) as the aggregation operator $f$. Particularly, we use the quantifier "As many as possible" with its parameters (0.5,1), i.e., with weighting vector $W=\{0,0,0.5,0.5\}$. Then, the collective performance values obtained are:

  $r^1$ $(0,0,0,0,0.08,0.45,0.72,0.88,0.58,0.31,0,0,0,0,0)$

  $r^2$ $(0,0,0,0,0,0,0,0,0.12,0.82,0.73,0.38,0.1,0,0)$

  $r^3$ $(0,0,0,0.05,0.23,0.52,0.32,0.17,0,0,0,0,0,0,0)$

  $r^4$ $(0,0,0,0,0,0,0,0,0,0.12,0.27,0.11,0,0,0)$,

  where, for example, the value $\alpha_6^1$ is obtained according to this expression:

  $\alpha_6^1 = F_Q(0.8,0.65,0.8,1) = 0.72.$

2. *Choosing the best alternatives*

It is also performed in the following two phases:

- *Computing a fuzzy preference relation.* In this phase, we obtain the collective preference relation $B=[b_{ij}]$ according to Definition 4. Therefore, from the above collective evaluations we find out the following fuzzy preference relation $B$:

$$B = \begin{pmatrix} - & 0.31 & 0.52 & 0.12 \\ 0.82 & - & 0.52 & 0.27 \\ 0.45 & 0 & - & 0 \\ 0.27 & 0.27 & 0.27 & - \end{pmatrix}.$$

For example, we show how the preference degrees $b_{12}$ and $b_{13}$ are computed:

$$b_{12} = \max_{c_i} \min_{c_j \leqslant c_i} \{\mu_{r^1}(c_i), \mu_{r^2}(c_j)\} = (0.58 \wedge 0.12) \vee (0.31 \wedge 0.12) \vee (0.31 \wedge 0.82) = 0.31$$

$$\begin{aligned} b_{13} = {} & (0.08 \wedge 0.05) \vee (0.08 \wedge 0.23) \vee (0.45 \wedge 0.05) \vee (0.45 \wedge 0.23) \\ & \vee (0.45 \wedge 0.52) \vee (0.72 \wedge 0.05) \\ & \vee (0.72 \wedge 0.23) \vee (0.72, 0.52) \vee (0.72 \wedge 0.32) \vee (0.88 \wedge 0.05) \\ & \vee (0.88 \wedge 0.23) \vee (0.88 \wedge 0.52) \vee (0.88 \wedge 0.32) \\ & \vee (0.88 \wedge 0.17) \vee (0.58 \wedge 0.05) \vee (0.58 \wedge 0.23) \vee (0.58 \wedge 0.52) \\ & \vee (0.58 \wedge 0.32) \vee (0.58 \wedge 0.17) \vee (0.31 \wedge 0.05) \vee (0.31 \wedge 0.23) \\ & \vee (0.31 \wedge 0.52) \vee (0.31 \wedge 0.32) \vee (0.31 \wedge 0.17) = 0.52, \end{aligned}$$

where $\vee$ stands for "max" and $\wedge$ stands for "min".

- *Applying the non-dominance choice degree.* For each alternative $x_i$, we calculate its non-dominance choice degree $NDD_i$ as is shown in the Appendix A. First, the strict preference relation $B^s$ is computed:

$$B^s = \begin{pmatrix} - & 0 & 0.07 & 0 \\ 0.51 & - & 0.52 & 0 \\ 0.0 & 0 & - & 0 \\ 0.15 & 0 & 0.27 & - \end{pmatrix}.$$

Then, we compute the non-dominance choice degree of each alternative:

$$\{NDD_1 = 0.49,\ NDD_2 = 1,\ NDD_3 = 0.48,\ NDD_4 = 1\},$$

where, for example $NDD_1$ is computed as:

$$NDD_1 = \min\{(1-0.51),(1-0),(1-0.15)\} = 0.49.$$

And finally, we obtain the maximal solution set of alternatives, which is the set of maximal non-dominated alternatives, according to the following expression:

$$X^{ND} = \left\{ x_i / x_i \in X,\ NDD_i = \sup_{x_j \in X} \{NDD_j\} \right\}.$$

Therefore, our decision process ends obtaining the following solution set of alternatives:

$$X^{ND} = \{x_2, x_4\}.$$

**Remark 4.** The alternatives $x_2$ and $x_4$ are non-dominated with degree 1 and are the chosen alternatives. If we want to achieve a more specific solution set of the alternatives, we could also apply other different choice functions over the set $X$, as for example, it occurs in [7] where the total solution is obtained as the intersection of the partial solutions. Of course, the last consideration does not imply to do an iterative sequence of application of a same choice function over the maximal solution sets, because we may find non-monotone choice functions which, in such a case, obtain contradictory solutions (for example, it may occur with the non-dominance choice degree, see [30]).

## 5. Concluding remarks

In this paper, we have presented a fusion tool of multi-granularity linguistic information applied in a decision making problem with multiple information sources (purposes or experts or criteria) that provide the linguistic performance values on the alternatives using linguistic fuzzy choice subsets assessed on linguistic term sets with different multi-granularity and/or semantic.

We are setting frameworks in which the sources that participate in the decision processes may express their judgments by means of information of a different nature according to their preferences. In [11], we studied group decision making problems in which the experts used numerical ($[0,1]$) and linguistic (a term set $S_i$) expression domains to give their preferences. We proposed techniques and operators for combining the numerical and linguistic information to solve the group decision making processes. Therefore, by joining both proposals, we shall be able to manage multi-source decision making problems with numerical and multi-granularity linguistic information.

Finally, an aspect of this paper that is worth to be pointed out, is the introduction of a tool for managing multi-granularity linguistic information, which represents a first approach to model multi-granularity linguistic frameworks. We have shown its application in decision theory, but, of course, it may be used in other fields, e.g., information retrieval, diagnosis, etc.

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## Appendix A. The OWA operator and the non-dominance degree

### *A.1. The OWA operator as quantifier guided aggregation operator*

In any decision process with multiple information sources the final decisions must be made according to the majority of performance profiles given by the different sources. Traditionally, the majority is defined as a threshold number of individuals. However the majority is itself a fuzzy nature concept. Fuzzy majority is a soft majority concept, which is manipulated via a fuzzy logic based calculus of linguistically quantified propositions [42]. In [21] Kacprzyk specified fuzzy majority rule by means of a fuzzy linguistic quantifier [42] to derive various solution concepts for group decision making problems. Here, we work in a similar way, but in the field of quantifier guided aggregations as in [39].

**Definition 5** (Yager [36]). Let $A=\{a_1,\dots,a_n\}$ be a set of values to be aggregated; the ordered weighted averaging (OWA) operator $F$ is defined as

$$F(a_1,\dots,a_n)=WB^{\mathrm{T}}=\sum_{i=1}^{n} w_i b_i,$$

where $W=\{w_1,\dots,w_n\}$ is a weighting vector, such that, $w_i\in[0,1]$ and $\sum_i w_i=1$ and $B$ is the associated ordered value vector, where $b_i\in B$ is the $i$th largest value in $A$.

Given that we are interested in the area of quantifier guided aggregations, following Yager's method [36], we may calculate weights of the OWA operator using fuzzy linguistic quantifiers, which, for a non-decreasing relative quantifier, $Q$, is given by

$$w_i=Q(i/m)-Q((i-1)/m),\quad i=1,\dots,m.$$

The non-decreasing relative quantifier, $Q$, is defined as [42]

$$Q(y)=\begin{cases}0 & \text{if } y<a,\\ \dfrac{y-a}{b-a} & \text{if } a\leqslant y\leqslant b,\\ 1 & \text{if } y>b,\end{cases}$$

with $a,b,y\in[0,1]$, and $Q(y)$ indicating the degree to which the proportion $y$ is compatible with the meaning of the quantifier it represents. Some examples of non-decreasing relative quantifiers are shown in Fig. 6, where the parameters $(a,b)$ are $(0.3,0.8)$, $(0,0.5)$ and $(0.5,1)$, respectively.

In the following, $F_Q$ denotes the OWA operator whose weights are computed using the linguistic quantifier, $Q$.

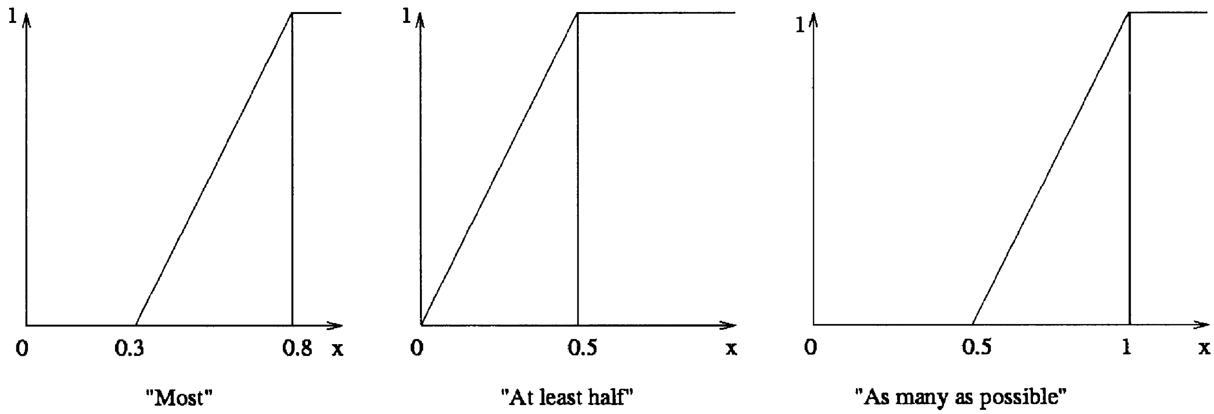


Fig. 6. Non-decreasing relative quantifiers.

### A.2. The non-dominance degree acting over fuzzy preference relations

This degree is obtained for each alternative $x_i$ from a fuzzy preference relation and it indicates the degree in which the alternative $x_i$ is not dominated by the remaining alternatives. Its definition is given as:

**Definition 6** (Orlovski [28]). Let $B=[b_{ij}]$ be a fuzzy preference relation defined over a set of alternatives $X$. For the alternative $x_i$, its non-dominance degree, $NDD_i$, is obtained as

$$NDD_i = \min_{x_j}\{1 - b^s_{ji},\ j \neq i\},$$

where $b^s_{ji} = \max\{b_{ji} - b_{ij}, 0\}$ represents the degree to which $x_i$ is strictly dominated by $x_j$.

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