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Fuzzy sets and operations research: Perspectives

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Abstract

Operations Research present state and future trends are analyzed from the Fuzzy-Sets-based methodologies point of view. Then, in a more particular perspective, some specific topics of Fuzzy-Sets-based models in Operations Research, such as fuzzy optimization, preference modelling, linguistic modelling and decision models, as well as some other well known Operations Research specific topics, are also reviewed and prospected. © 1997 Elsevier Science B.V.

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1. Introduction

Most concisely, but with a more than general meaning, it may be said that Operations Research (OR) studies the art of modelling and solving management and decision making problems in real contexts. Therefore the art covered by OR is so large that, without a shadow of doubt, it may be stated that in one way or another it is to be found in every field of Science and Engineering and that applications may be found for its techniques and models in every branch of human knowledge.

This enormous variety justifies the practical impossibility of being able to successfully sum up in just one article a real, objective and true vision of the achievements made, the interests involved and the future outlook for OR and Fuzzy Sets-based models in OR. However, that impossibility is also manifest when we think about the outlook form which we could approach this work, since the result would be very different if the approach were made from a methodological or practical perspective, by presenting general trends or specific topics, etc. Bearing in mind this huge context, here we have decided to consider a dual framework. On the one hand, the one referring to the practice of OR as a whole, at the present time, and on the other hand, that concerning some specific topics, undoubtedly biased by the training and the research interests of the authors of this article, which we believe will play an important role in the very near future for Fuzzy Sets-based OR developments. Anyway, we should point out that the role played by Fuzzy Sets in the field of OR is absolutely essential, since, as is well known, they allow us to model more than adequately those situations in which certain ambiguity arises of a non-probabilistic type, as well as in a large part of human beings' reasoning mechanisms.

So, now that the context in which we shall move has been set, the incidence and repercussion of all kinds which Fuzzy-Sets-based models have had in a field like OR become clear. Undoubtedly, this field

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has been the one in which developments and models based on Fuzzy Sets have received the most attention and efforts from researchers all over the world. But, although since the early seventies a great deal of work was dedicated to this type of models, progressively, and especially since the so-called Fuzzy Boom, it seems that interest in them has been decreasing, with the result that other kinds of applications and results have come to the forefront.

With this framework and this background, what we intend in this article is to make a prospective approach to the field of OR relating to Fuzzy Sets, by analyzing their possible interrelationship with other related fields in such a way that, finally, we may give an answer, though only a partial one, to some questions that are considered interesting such as: (1) Where are we going? (2) Is this research area open? (3) Is it promising?

Therefore below we concentrate on the current situation of OR in practice in order to, henceforth, try to make a forecast about its future, which will clearly be linked to that of other scientific technical areas, with Fuzzy Sets appearing as a basic tool for future developments. Then we shall comment upon the much more specific fields of fuzzy optimization, preference modelling, linguistic modelling and decision models, as well as some other well known OR specific topics.

2. OR in the knowledge society

Although the theoretical developments, practical applications and specific results based on OR techniques have been plentiful and spectacular since the early 1940s, one gets the impression that at the beginning of the 1980s, this methodology (meaning the broad approach adopted for addressing these issues) was affected by a crisis, mainly caused by the profound and diverse social and technological changes which have occurred since then. Amongst other effects, this crisis might have encouraged the opinion that the OR's future might not be as rosy as its past. Aware of this fact, the Council of the Operational Research Society decided, in 1983, to establish a Commission on the Future Practice of OR, with the following terms of reference:

> To investigate for Council the changing state of OR in practice; to advise Council on how the OR Society might help in equipping the practitioners of OR and supporting them in meeting the challenges of the last 15 years of the 20th century [51].

As for the future of OR in practice is concerned the Commission concluded that:

> OR in the future will be like OR in the immediate past, responding to issues that arise and developing and introducing new methods as and where appropriate. Some, perhaps many, issues may be related to Information Technologies (IT), but it is quite possible to foresee other issues which could attain importance.

In particular, major preoccupations for the next 10–15 yr (the present time) like social issues and the physical rebuilding of some economic infrastructure were pointed out.

Doubtless this perspective was considered during an economic and work situation which, though not as buoyant as in the period between the 1940s and the beginning of the 1970s, we could call reasonably stable. However, at the present time our economies are growing at a slower rate, if indeed at all, they have tremendous trade deficits and, worst of all, millions of people cannot find work. This may force a profound social change which we sense to be fairly imminent, and will lead to a society with different habits from the present ones: The Knowledge Society in the era of IT.

The IT have burst into our daily lives in such a big way that, without danger of exaggerating at all, nowadays we depend on the developments that have been achieved and commercialized, and we await the things that may be guessed in the near future. In particular, Artificial Intelligence (AI) has achieved goals that only a few years ago seemed unthinkable and conventional computers have now become so everyday amongst us that they may almost be considered as household electrical appliances.

In this respect, it seems that the trend towards further development in the field of IT is impossible to stop. It appears that all the efforts spent in R&D in this field are few, and it is undeniable that there has been and still is a migratory process from OR to IT. What is happening to OR then? Will its models and techniques be needed in the future? Has the limit been reached? and if so, will new experts be needed in this subject? Certainly it would be most outlandish to

Table 1
List of the occupations expected to grow

USA	Thousands employed		Projected increase (%)
	1990	2005	
System analysts, computer scientists	463	829	78.9
Physical therapists	88	155	76.0
Operations research analysts	57	100	73.2
Psychologists	125	204	63.6
Computer programmers	565	882	56.0
Occupational therapists	36	56	55.2
Management analysts	151	230	52.3
Marketing, advertising,...	427	630	47.4

offer a general response which might be valid in any context, i.e., for every country or group, but perhaps what appears is going to happen in the US might serve as guide for us.

The American Bureau of Labor Statistics has published a list of the occupations expected to grow and contract most rapidly in the years ahead (Newsweek 14 June 1993). In the coming decade the biggest gainers with a college Degree or more required are shown in Table 1.

Although these figures only refer to forecasts in the US, they are more than explicit enough and usable to reach conclusions about many of the specialities mentioned in this list. As far as OR is concerned, the aforesaid data show unreservedly the present and future needs for highly qualified professionals in this field. In view of these figures, as is obvious, it appears that the future of OR is assured and that it may not just cohabit with future developments in the field of IT, but it may also be stated, without risking too much, that its methodology may also be very important for the progress thereof.

As was pointed out in [20], "this fact derives, perhaps surprisingly from the remarriage of two disciplines that were once united, having arisen from a common origin, but which became separated and maintained only loose ties for several decades: OR and AI. This renewed union is highlighting limitations in the framework of each (as commonly applied), and promises fertile elaborations to the strategies each has believed fruitful".

But this serves to clear the future for OR, both from the point of view of isolation and in collaboration with other disciplines, as is the specific case that we have just mentioned, that of AI, what role shall be played in that context by Fuzzy-Sets-based models and technologies?.

3. The role of Fuzzy-Sets-based models

We believe that one outstanding aspect of the new economic and management models, and therefore of OR in general on the one hand and of new technologies for processing information, one of whose facets is AI, on the other hand, is their quest to achieve a level of approach and working as near as possible to human beings normal way of acting, because the basis of present computers depends on a theoretical conception based on "yes and no" logic, i.e., on Boolean logic, which is far removed, in almost every case, from human beings' normal way of processing information and reasoning: people's way of reasoning is based on common sense and on the issuing of judgments on flexible evaluations, which, for the time being, cannot be carried out by computers, and this is a crucial problem.

However proof that present and future efforts are headed in that direction may be found both in the European Union's Fourth Work Programme, which includes the use of Fuzzy-Sets-based models in a wide number of fields and in the Real World Computing Programme, which is being developed in Japan [40]. To be more explicit, this program aims "at establishing the basis for flexible and advanced information IT which are closely allied to humans and are capable of processing a variety of diversified information in the real world. They would be essential for the advanced information society of the 21st century for creating a cooperative relationship between human beings and computers, and for producing innovative and generic technologies".

Bearing in mind that Fuzzy Sets have proven to be an extraordinary tool in this field and to have sufficient theoretical resources for developing magnificent practical applications, the future augured for that field of interface between OR and IT, seen from the Fuzzy Sets point of view, in our opinion is, at least, as good as its splendid past. In fact, as Prof. L.A. Zadeh pointed out [69]: "A fundamental contribution of fuzzy logic is a methodology for computing with words (CW) which mimics human reasoning. It is this methodology that in one form or another is already used in most of the applications of fuzzy logic. In coming years, however, computing with words, based on fuzzy logic, is likely to emerge as a field of key importance in its own right".

From this point of view, with which we all agree, and provided that OR, and the interface OR-IT, is a rather wide field of work, it is very difficult to forecast to any extent what will be the future for each of the OR key topics. Therefore below we shall focus on different fuzzy and/or CW based OR topics that, although well studied in the recent past, may need some redefinition-reformulation or extensions: fuzzy optimization, preference modelling, linguistic modelling and decision models, and other OR specific topics.

3.1. Fuzzy optimization

There is no doubt that the very origin of fuzzy optimization is the seminal paper by R.E. Bellman and L.A. Zadeh: Decision Making in a Fuzzy Environment [2]. Simply, but masterfully explained, in that article the concepts of fuzzy constraint, fuzzy objective and fuzzy decision which were later so profusely used and applied, are introduced. It is clear that since the publication of that article, fuzzy optimization as a whole has been one of the topics that has received the greatest attention within the general field of Fuzzy Sets. Similarly to what happens in conventional optimization, in fuzzy optimization different kinds of models have been developed, distinct theoretical extensions have been drawn up and practical applications have been carried out. But as new topics have appeared (combinatorial models and meta-heuristic techniques may be good examples) there still remain work to do. In the following we will refer to four specific topics, the well known fuzzy mathematical programming, fuzzy-sets-based models of combinatorial optimization, the use of meta-heuristic techniques, and a particular problem that we have highlighted for its outstanding interest and applicability, the fuzzy scheduling problem.

3.1.1. Fuzzy mathematical programming

In the concrete and general case of Mathematical Programming, and not wishing to cover every possible angle but rather just to review the past and thereby be able to orientate the future somewhat, the contributions in the field have almost all touched on the same subjects, models of Fuzzy Linear Programming. But the fuzzy models and techniques relating to multiobjective, dynamic, fractional or possibilistic, have also been the motive for numerous and very good contributions. Among the applications we may quote water resources, economic planning, air pollution regulation, development software, forestry, etc. Using this broad catalogue of theoretical developments and practical applications, we may easily check the enormous amount of work that has been carried on Fuzzy Mathematical Programming since the early 1970s [12].

The topic of Fuzzy Mathematical Programming may be understood to have been well, broadly and seriously researched into, but there still remain many questions to be resolved ranging from what we understand by fuzzy solution and how it is implemented in practice, to the most sophisticated multiobjective models, but additionally it makes complete sense [65] to consider the application of Fuzzy Mathematical Programming techniques and models to the case of fuzzy-knowledge-based systems [7].

Specially relevant in this field, because of their basic character, may be those models and techniques relating to Integer Programming [30], and especially the Boolean Programming problems [32, 31] as foundation basis for solving combinatorial optimization problems and fuzzy-knowledge-based systems [7].

One field yet to be explored is the use of fuzzy control rules for terminating Mathematical Programming algorithms, since in cases of highly complex problems for which an optimal solution is quite difficult to obtain, as is the case of many fuzzy optimization problems, the use of this type of rules might help solutions to be found which, although they may not be fully optimal, may well be good enough for the decision-maker [14].

3.1.2. Combinatorial optimization

The combination of constraint satisfaction techniques and concepts handling vagueness and uncertainty offers solutions to combinatorial optimization problems, where the concepts taken from Fuzzy Sets allow a more accurate characterization of the applicability of combinatorial optimization.

The application of Fuzzy Sets allows us to relax conditions and constraints in order to find easier solutions in a realistic sense by fuzzyfication. When coefficients cannot be determined due to the vagueness of the judgment or the imprecise knowledge of a specialist then it is more realistic to express the vagueness of coefficients with fuzzy numbers. Also, typical constraints from the application domain of combinatorial optimization problems are usually vaguely defined, and thus lend themselves perfectly for being reformulated as fuzzy constraints. From this point of view, the study of combinatorial optimization problems with special structures, where Fuzzy Sets are useful for handling the inherent uncertainty and imprecision, deserves special attention.

Some Fuzzy-Sets-based combinatorial optimization problems have already been studied, this is the case for instance of the well known knapsack problem [46, 47], discrete location model with fuzzy accessibility measures [11], set covering problem [71, 64, 72], etc.

3.1.3. Meta-heuristic techniques

Optimization problems almost always involves huge computational efforts, therefore it would appear appropriate to develop approximate optimization techniques for them which could be used efficiently. On the other hand, most of combinatorial optimization problems are NP complete problems which do not have efficient methods. It would be particularly interesting to study the combination of techniques imported from the field of the Fuzzy Sets used for modelling the uncertainty and imprecision with advanced optimization techniques such as the metaheuristics algorithms (Genetic Algorithms, Simulated Annealing, Tabu Search, etc.) used for obtaining good approximate solutions.

Metaheuristics are a class of approximate methods that are developed in the early 1980s. They are designed to attack complex optimization problems where classical heuristics and optimization methods have failed to be effective and efficient. A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions [48, 49]. These families of approaches include, but are not limited to: ejection chains, genetic algorithms, greedy random adaptive search procedures, neural networks, problem and heuristic space-search, simulated annealing, tabu search, threshold algorithms and their hybrids. They incorporate concepts based on biological evolution, intelligent problem solving, mathematical and physical sciences, nervous systems and statistical mechanics.

Over the last few years, different metaheuristic techniques have been combined with Fuzzy Sets for solving fuzzy optimization problems, this is the case, for instance, of neural networks for solving the fuzzy multiobjective 0–1 programming [54] and the maximum cut in a non-directed graph with fuzzy weights [3], simulated annealing for solving fuzzy flowshop scheduling [34], and genetic algorithms for solving maximum flow in a network with fuzzy capacities [5, 27], fuzzy flowshop scheduling problems [36], vehicle routing problem with fuzzy due-time [9], etc. Let us consider, for instance, the interface between Genetic Algorithms (GAs) and Fuzzy Sets Theory.

GAs are search algorithms that use operations found in natural genetics to guide the trek through a search space. GAs use a direct analogy of natural behavior. They work with a population of individuals (chromosomes), each one representing a possible solution to a given problem. Each individual is assigned a fitness score according to how good a solution to the problem it is. GAs are theoretically and empirically proven to provide robust search in complex spaces, offering a valid approach to problems requiring efficient and effective search. Much of the interest in GAs is due to the fact that they provide a set of efficient domain-independent search heuristics which are a significant improvement over traditional methods without the need for incorporating highly domain-specific knowledge. The basic principles of GAs were first laid down rigorously by Holland [33], and are well described in many books such as [23, 42].

It is known that recently numerous papers and applications combining fuzzy concepts and GAs have appeared, and there is an increasing interest in the integration of these two topics. From this combination, among others, the so-called Fuzzy Genetic Algorithms (FGA) arise. Two main approaches dealing with FGA have been presented in the literature [28]:

(a) *The use of Fuzzy-Sets-based techniques to model different GA components.* The use of Fuzzy-Sets-based techniques permits GA behavior to be improved in different ways such as: representation models for dealing with more complex genotype-phenotype relationships, fuzzy logic control systems for introducing a self-control parameter process according to some performance measures, fuzzy operators and fuzzy connectives for designing genetic operators that introduce different population diversity levels and mechanisms, that allow GA performance to be analyzed from a human point of view.

(b) *To manage problems in an imprecise environment*, where the inherent imprecision is modelled by means of Fuzzy Sets, solving fuzzy optimization problems. Two ways to manage fuzzy information with GAs have been presented, [5, 27], both with a similar root, but considering a different way to obtain a real value for defining the selection process. The differences lie in the conception of the variables, the first proposal considers variables with fuzzy values representing these in a chromosome [5] and the second one considers non fuzzy variables with a fuzzy evaluation [27], having a fuzzy fitness. Different fuzzy optimization problems can be considered for applying them as is the case of the problem of maximum flow in a network with fuzzy capacities [5, 27].

It should be remarked that the use of GAs may offer great potential for the Fuzzy-Sets-based optimization approaches for representing uncertainty and approximation in relationships between system variables, given the potential of GAs in a fuzzy environment as a flexible tool for optimization and search.

In fact, GAs have been used for solving different fuzzy optimization problems, this is the case for instance of fuzzy flowshop scheduling problems [36], vehicle routing problems with fuzzy due-time [9], fuzzy mixed integer programming applied to resource distribution [43], interactive fuzzy satisfycing method for multiobjective 0–1 [53], fuzzy optimal reliability design problems [55], job-shop scheduling problem with fuzzy processing time [61], fuzzy optimization of distribution networks [6], etc.

3.1.4. Fuzzy scheduling problems

Roughly speaking the process of determining a schedule is a decision-making process in which the scheduling is the allocation of resources over time, to carry out a set of tasks attending to a variety of constraints and objectives. Among the constraints, two main types are to be remarked [39]: On the one hand there are constraints defining the space of admissible solutions, and on the other hand there are constraints characterizing the quality of scheduling decisions. Typically the first constraints are referred to release dates, operation durations and precedences, transfer and setup times, resource availability and resource sharing, and the second are relaxable preference constraints related to due dates, productivity, frequency of tool changes, inventory levels and shop stability.

Usually some of the constraints must be satisfied for a schedule to be valid, but other constraints may not always be satisfied and might need to be relaxed. Thus a good schedule is one that satisfies all hard constraints while selectively relaxing soft constraints to optimize performance [16]. This implies that rigid scheduling procedures, designed to provide optimal or near-optimal schedules in particular circumstances, are in general not satisfactorily applicable in other circumstances.

A way to waive this malfunction is to consider AI techniques in the scheduling process, as AI techniques, as a whole, permit the management of interactions between the former theoretical system and the empirical knowledge coming from the system experts.

Although scheduling problems have been studied in the OR literature since the early fifties, it is surprising that the AI community did not recognize the relevance of the topic until 1982, when scheduling was introduced to the AI community [19]. Basically, in the AI field scheduling is meant as the process of selecting between alternative plans and assigning resources and times to the set of activities in the plan. More specifically, [18], in AI terms, the job-shop scheduling may mean a planning problem with the following features:

(a) It is a time-based planning problem in which activities must be selected, sequenced and assigned resources and an execution time.

(b) It is a multi-agent planning problem, in which, for each different agent, one must create a plan (schedule). Additionally, the agents are uncooperative, and hence each one will attempt to optimize its own goals.

(c) Resource competition is high, and therefore closely coupling decisions.

But in any case, the main problem associated with scheduling is that of having an excessive combinatorial complexity, as the number of feasible schedules can grow exponentially along each variable (jobs, machines, etc.). In fact scheduling is NP-complete and has proven to be a difficult task for human planners and schedulers. Even under ideal conditions such as simplified goals and deterministic hypotheses, it is quite difficult to look for optimal solutions. Hence algorithms looking for exact solutions are generally useless since they will not serve to solve real practical problems. Instead, practical approaches to production scheduling are heuristic in nature. From this point of view, different solution methods for scheduling problems providing approximate solutions have been proposed: Simulated Annealing [1,34,50,44,45,56], Tabu Search [21,22,59,63,62], Genetic Algorithms [58,36,35,56,61], etc. It is just here where Fuzzy-Sets-based techniques can play an outstanding role, as Approximate Reasoning techniques, as we pointed out above, may be a rather satisfactory way to improve all of these approaches [8,38,41,15,24,34,36,35,16,61,57]. But furthermore, concepts arising from the Approximate Reasoning field may be used to open up new problems more human-consistent than the former classical ones.

For instance, and as far as the well known m-machine and n-job sequencing problem is concerned, linguistic due dates could be assumed, as we are convinced that in practical problems the experts are more comfortable in expressing their knowledge in a linguistic manner than in a numerical one. In this way, and only as an illustration, if $t(j)$ denotes the completion time for the job j, and $\lambda(j)$ is the linguistic label associated with the due date of the job j, $\mu(t(j))$ would represent the degree of accomplishment of the linguistic property expressed by $\lambda(j)$ when the completion time is $t(j)$. It is clear that, in this case, to find the best sequencing, solution methods using linguistic aggregation operators ought to be considered.

3.2. Preference modelling

Preference modelling is an implicit problem in a lot of OR problems where the preferences of the decision-maker are often represented by a preference structure to be managed for obtaining solutions in decision contexts.

From a general point of view, the so-called "preference structure" can be represented according to different guidelines such as: graph representation, numerical or functional representation, geometrical representation of preferences, expected utility theory, etc. [52].

Since human judgments including preferences are often vague, the Fuzzy Sets Theory plays an important role in preference modelling and their associated decision processes. Fuzzy Sets Theory applications to preference modelling and decision problems allows a more flexible framework where it is possible to deal with the fuzziness of human judgments and therefore incorporate more human consistency in the preference models [70, 17]. Therefore, the analysis of "preference structures" incorporating fuzzy information is of great interest.

Together with the preference structure analysis, there are two basic problems, the aggregation of preferences and the exploitation of preference models. The first ones are necessary for aggregating the information obtained from different sources, experts, objectives, etc. The second ones are necessary for obtaining the solution sets from the preference structures in the decision problem.

Another relevant problem in this field may be the use of different preference structures in a decision problem, since in cases of multicriteria or multiperson decision problems, every expert or criterion may use a different preference structure, then the use of either transformation methods between preference structures or of combination methods of different preference structures may be necessary [10].

The qualitative linguistic opinions are very typical of situations in human sciences, and the information can be supplied in that form, that is, with linguistic assessments. From this point of view, it deserves attention to study the extension of preference modelling structures to linguistic preferences structures, that is, to introduce a methodology for CW in decision models.

3.3. Linguistic modelling and decision models

The linguistic approach considers the variables which participate in the problem assessed by means of linguistic terms instead of numerical values [68]. This approach is appropriate for a lot of problems, since it allows a representation of the individuals' information in a more direct and adequate form, whether they are unable of expressing that with precision.

A linguistic variable differs from a numerical one in that its values are not numbers, but words or sentences in a natural or artificial language. Since words, in general, are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a means of approximated characterization of phenomena, which are too complex or ill-defined for its description to be amenable in conventional quantitative terms.

A linguistic variable is associated with two rules:
(1) *a syntactic rule*, which may take the form of a grammar for generating the names of the values of the variable, and
(2) *a semantic rule*, which defines an algorithmic procedure for computing the meaning of each value.

Usually, the semantic of the elements of the term set is given by fuzzy numbers defined on the [0, 1] interval, which are described by membership functions. Because the linguistic assessments are just approximate ones given by the individuals, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible or unnecessary to obtain more accurate values. This representation is achieved by the 4-tuple $(a_i, b_i, \alpha_i, \beta_i)$, the first two parameters indicate the interval in which the membership value is 1; the third and fourth parameters indicate the left and right width.

We need a term set defining the uncertainty granularity, that is the level of distinction between different countings of uncertainty. The elements in the term set will determine the granularity of the uncertainty. For instance, in [4] the use of term sets with odd cardinals was studied, representing the middle term an assessment of "approximately 0.5", with the rest being terms placed symmetrically around it and the limit of granularity being 11 or no more than 13.

In this way the following topics may be pointed out.

(a) *Linguistic preference structures.* The use of linguistic preference structures in decision making situations to voice experts' opinions about an alternative set, with respect to certain criteria, appears to be useful in modelling the decision process.

In the following we show one of their possibilities, the *linguistic preference relations* [29]:

Let $X = \{x_1, \ldots, x_n\}$ be a set of alternatives over which the fuzzy preference attitude of a decision-maker is defined. A *fuzzy preference relation* is described by a fuzzy binary relation R on X, i.e., a fuzzy set on the product set $X \times X$, characterized by a membership function $\mu_R : X \times X \to [0, 1]$, where $\mu_R(x_i, x_j)$ denotes the preference degree of the alternative x_i over x_j.

As was introduced in [29], assuming a linguistic framework, i.e. a label set $S = \{s_i\}$, $i \in H = \{0, \ldots, T\}$, and a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, the experts' preference attitude about X can be defined as an $n \times n$ linguistic preference relation R, such that, $R = (r_{ij})$, $i, j = 1, \ldots, n$, where $r_{ij} \in S$ denotes the preference degree of alternative x_i over x_j, linguistically assessed, with

$$s_0 \leqslant r_{ij} \leqslant s_T \quad (i, j = 1, \ldots, n),$$

and where
- $r_{ij} = s_T$ indicates the maximum degree of preference of x_i over x_j.
- $s_{T/2} < r_{ij} < s_T$ indicates a definite preference of x_i over x_j.
- $r_{ij} = s_{T/2}$ indicates indifference between x_i and x_j.

(b) *Linguistic decision process.* It is necessary to manage the decision processes associated with different linguistic preference structures. As an example, a group decision making situation may be considered as follows.

In a group decision making situation there are basically two problems to solve:
(i) *alternatives selection problem*, i.e., how to obtain solution alternative(s) set, and
(ii) *consensus reaching problem*, i.e., how to achieve the maximum consensus degree from a group of experts for a solution alternative(s) set.

The first way consists of establishing *a group choice process* which obtains a decision scheme as a solution to group decision-making problems. The second one consists of establishing *a group consensus process* for different decision schemata until achieving the possible maximum consensus degree about solution alternative(s) set. These processes may be combined in the following resolution scheme. Firstly, the consensus process is applied. In each step, the degree of existing consensus among experts' opinions is measured. If the consensus degree is satisfactory, then a choice process is applied in order to obtain a solution. Otherwise, the experts are persuaded to update their opinions. In this way, a group decision-making process may be defined as a dynamic and iterative process where the experts, via the exchange of information and rational arguments, update their opinions until they become sufficiently similar. The consensus degrees and choice processes under linguistic assessments are the basis of this resolution scheme [25, 26].

(c) *Linguistic aggregation operators.* It is clear that the aggregation of information plays a central role in the choice processes of the multicriteria or group decision-making problems, and more specifically in our linguistic context, aggregation operators of linguistic labels are needed.

Clearly, there are two types of linguistic information:
1. *Non-weighted linguistic information.* This is the situation in which we have only one set of linguistic values to aggregate.
2. *Weighted linguistic information.* This is the situation in which we have a set of linguistic values to aggregate, and each value is characterized by an importance degree, indicating its weight in the overall set of values.

In both cases, linguistic aggregation operators are needed that appropriately combine the information, in such a way, that the final aggregation is the "best" representation of the overall opinions. In the literature various aggregation operators of linguistic information have been proposed [60, 13, 29, 67, 66]. Hence, going deeper into this area and developing new linguistic operators deserve attention.

(d) *Some extensions.* In the following we briefly describe some extensions of the work published in this topic. These can be summarized in the following points:

- To go deeper into the development of rational linguistic operators for non-weighted and weighted linguistic information and to apply them to linguistic decision-making models.
- To extend the linguistic approach to different decision-making problems and to different preference structure models.
- To develop *decision support systems* according to the linguistic decision processes, being a flexible tool for managing linguistic information in decision making.
- The application of the linguistic decision processes to human related sciences, economics, law, science, medicine, sociology, etc.

3.4. Other specific topics

Other specific topics that deserves to be pointed out in a brief description are:

(a) *Solving games and conflicts.* This is a field which has not received much attention and that would have to be developed more deeply. Games, understood as conflicts, are very important in a lot of practical applications. Usually, in fuzzy sets setting, they have been solved assuming the fuzziness is, in some way, on the players (number, coalitions, etc.), but it is also realistic to assume the fuzziness is on the rewards. In this way, again, it could be considered using linguistic labels to model the pay-offs, and then obtain theoretical results, similar to those in the classical case, permitting us to solve and implement these models in a very human consistent way. Then to link up these models with those from the broader area of the conflict analysis could serve, for example, as help to engineers because of the increasing importance of social and political influences in engineering decision making.

(b) *Problems relating to graphs.* A key aspect to be taken into account, from a very general point of view, may be that of graphs as problems relating to graphs for modelling a variety of systems, where the relations between the objects in the system play a dominant role, are present in a lot of cases. Reasoning and management in linguistic terms is the usual daily way for human beings. As OR is mainly a discipline oriented to solving practical problems for managers, the necessity of solving these OR models from the CW point of view appears clear. Therefore, they should be considered and/or re-considered from the CW point of view in the near future.

(c) *Fuzzy-Sets-based AI techniques in OR.* The important results achieved from integrating problem solving techniques, typically associated with OR, with those typically associated with AI to represent and solve complex decision problems are of great interest. The use of Fuzzy-Sets-based AI techniques in OR problems permits the use of AI techniques for managing uncertainty and imprecision, this is the case, for instance, of the well known fuzzy rule knowledge based systems.

(d) *Intelligent decision support systems.* In this same way, another tendency which could become normal in OR is that of Intelligent Decision Support Systems, combining and extending techniques associated with AI and Decision Support Systems. Fuzzy Sets Theory, and hence CW-based methodology, which seems to be an excellent tool for creating a friendship interface between OR and AI models for the development of Intelligent Decision Support Systems.

4. Conclusions

Firstly, we realize that it is not easy to draw a synthetic picture of both the present and the future of OR and Fuzzy-Sets-based models in OR in so few pages. We have presented some specific topics, but obviously other tendencies which become noticeable in OR are omitted from this contribution, and of course, in all these topics, Fuzzy Sets are useful for handling inherent uncertainty and imprecision.

Secondly and finally, we want to remark that in our opinion we have shown in this paper that, from a scientific research viewpoint and from a practical viewpoint, as far as applications are concerned, and from the job market for future practitioners, the subject situated in the interface among OR, AI and Fuzzy-Sets-based methods has a promising future in the very short term, because Fuzzy Sets are useful tools for handling inherent uncertainty and imprecision in OR problems, and they seem to be an excellent tool for creating an interface between OR and AI models.

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