Direct approach processes in group decision making using linguistic OWA operators

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Abstract

In a linguistic framework, several group decision making processes by direct approach are presented. These processes are designed using the linguistic ordered weighted averaging (LOWA) operator. To do so, first a study is made of the properties and the axiomatic of LOWA operator, showing the rationality of its aggregation way. And secondly, we present the use of LOWA operator to solve group decision making problems from individual linguistic preference relations.

Keywords: Group decision making; Linguistic preference relation; Fuzzy linguistic quantifier; OWA operators

1. Introduction

Decision making is a usual task in human activities. It consists of finding the best option from a feasible set. Many decision making processes, in the real world, take place in an environment in which the goals, constraints and consequences of possible actions are not precisely known. In these cases, probability theory has always allowed one to deal quantitatively with that lack of precision. However, when the lack of precision is of a qualitative nature too, the use of other techniques is necessary.

Fuzzy set theory applied to decision making allows a more flexible framework, where by it is possible to simulate humans' ability to deal with

the fuzziness of human judgments quantitatively, and therefore to incorporate more human consistency or "human intelligence" in decision making models. Different fuzzy decision making models have been proposed. A classification for all of them, depending on the number of stages before the decision is reached, is shown in [20]. We are interested in one fuzzy model in one-stage decision making, i.e., a fuzzy multi-person decision making model applied in group decision theory.

A group decision making process may be defined as a decision situation in which (i) there are two or more individuals, each of them characterized by his or her own perceptions, attitudes, motivations, and personalities, (ii) who recognize the existence of a common problem, and (iii) attempt to reach a collective decision.

In a fuzzy environment, a group decision problem is taken out as follows. It is assumed that there

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exists a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ as well as a finite set of experts $E = \{e_1, \ldots, e_m\}$, and each expert $e_k \in E$ provides his preference relation on X, i.e., $p_k \subset X \times X$, and $\mu_{P_k}(x_i, x_j)$ denotes the degree of preference of alternative x_i over x_j, $\mu_{P_k}(x_i, x_j) \in [0, 1]$.

Sometimes, an individual may have vague knowledge about the preference degree of the alternative x_i over x_j and cannot estimate his preference with an exact numerical value. Then, a more realistic approach may be to use linguistic assessments instead of numerical values, i.e., supposing that the variables which participate in the problem are assessed by means of linguistic terms [6, 11, 21, 29, 31]. A scale of certainty expressions (linguistically assessed) is presented to the individuals, who could then use it to describe their degrees of certainty in a preference. In this environment we have linguistic preference relations for providing individuals' opinions.

On the other hand, assuming a set of alternatives or decisions, two ways to relate to different decision schemata are known. The first way, called *the algebraic way*, consists in establishing a group selection process which obtains a decision scheme as solution to group decision making problem. The second one, called *the topological way*, consists in establishing a measure of distance between different decision schemata. Several authors have dealt with both proposals in fuzzy environments [3, 16–19, 23]. We have proposed various models for both possibilities under linguistic assessments in [11, 14, 15], respectively. Here, we will focus on the first possibility, and we develop various group decision making processes under linguistic preferences, based on the linguistic information aggregation operation carried out by the Linguistic Ordered Weighted Averaging (LOWA) operator [10]. To do this, we study the properties and the preference aggregations axiomatic of the LOWA operator, showing its rational aggregation way. Then we present how to use the LOWA operator for solving a group decision making problem from individual linguistic preference relations.

The paper is structured as follows: Section 2 shows the use of linguistic preference relation and some properties; Section 3 analyzes the properties and axiomatic of LOWA operator; Section 4 pre-

sents the proposed group decision making processes; Section 5 illustrates its application with some examples; and finally, Section 6 presents our conclusions.

2. Linguistic preference relations in group decision making

Let X be a set of alternatives over which the fuzzy preference attitude of a decision-maker is defined. Then, according to Tanino [27, 28], the fuzzy preference may be represented as:

1. *A fuzzy choice set to represent his total preference attitude*. It is described by a fuzzy subset of X, i.e., by a membership function μ on X, whose value $\mu(x)$ denotes the preference degree of x, or degree to which x is chosen as a desirable alternative.

2. *A fuzzy utility function*. It is described as fuzzy mapping v, which associates a fuzzy subset of the utility values space (usually the space of real numbers R) with each alternative x, $v: X x R \rightarrow [0, 1]$, where $v(x, t)$ denotes the degree to which the utility value of the alternative x is equal to t.

3. *A fuzzy preference relation*. It is described by a fuzzy binary relation R on X, i.e., a fuzzy set on the product set $X x X$, characterized by a membership function $\mu_R: X x X \rightarrow [0, 1]$, where $\mu_R(x_i, x_j)$ denotes the preference degree of the alternative x_i over x_j.

The use of fuzzy preference relations in decision making situations to voice experts' opinions about an alternative set, with respect to certain criteria, appears to be a useful tool in modelling decision processes. Among others, they appear in a very natural way when we want to aggregate experts' preferences into group ones, that is, in the processes of group decision making.

As we have mentioned above, in many cases an expert is not able to estimate his preference degrees with exact numerical values. Then, another possibility is to use linguistic labels, that is, to voice his opinions about alternatives by means of a *linguistic preference relation*. Therefore, to fix previously a label set it is absolutely essential to voice the experts' preferences.

In [2] the use of label sets with odd cardinals was studied, the middle label representing a probability

of "approximately 0.5", the remaining labels being placed symmetrically around it and the limit of granularity is 11 or no more than 13. The semantic of the labels is given by fuzzy numbers defined on the [0,1] interval, which are described by membership functions. As the linguistic assessments are merely approximate ones given by the individuals, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since obtaining more accurate values may be impossible or unnecessary. This representation is achieved by the 4-tuple $(a_i, b_i, \alpha_i, \beta_i)$ (the first two parameters indicate the interval in which the membership value is 1.0; the third and fourth parameters indicate the left and right widths of the distribution).

We shall consider a finite and totally ordered label set $S = \{s_i\}$, $i \in H = \{0, \ldots, T\}$, in the usual sense and with odd cardinality as in [2], where each label s_i represents a possible value for a linguistic real variable, i.e., a vague property or constraint on [0,1]. We shall require the following properties:

(1) The set is ordered: $s_i \geqslant s_j$ if $i \geqslant j$.
(2) There is the negation operator: $\text{Neg}(s_i) = s_j$ such that $j = T - i$.
(3) Maximization operator: $\max(s_i, s_j) = s_i$ if $s_i \geqslant s_j$.
(4) Minimization operator: $\min(s_i, s_j) = s_i$ if $s_i \leqslant s_j$.

Assuming a linguistic framework and a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, the experts' preference attitude over X can be defined as a $n \times n$ linguistic preference relation R, such that $R = (r_{ij})$, $i, j = 1, \ldots, n$, where $r_{ij} \in S$ denotes the preference degree of alternative x_i over x_j, linguistically assessed, with

$$s_0 \leqslant r_{ij} \leqslant s_T \quad (i, j = 1, \ldots, n),$$

and where:

1. $r_{ij} = s_T$ indicates the maximum degree of preference of x_i over x_j.
2. $s_{T/2} < r_{ij} < s_T$ indicates a definite preference of x_i over x_j.
3. $r_{ij} = s_{T/2}$ indicates the indifference between x_i and x_j.

So that the linguistic relation reflects a preference, it would be desirable to satisfy some of the following properties, proposed by Tanino in fuzzy

environment [27, 28], and interpreted here in a linguistic environment:

1. Reciprocity: $r_{ij} = \text{Neg}(r_{ji})$, and $r_{ii} = s_0 \ \forall i, j$.
2. max–min   Transitivity: $r_{ik} \geqslant \text{Min}(r_{ij}, r_{jk})$, $\forall i, j, k$.
3. max–max   Transitivity: $r_{ik} \geqslant \max(r_{ij}, r_{jk})$, $\forall i, j, k$.
4. Restricted max-min transitivity: $r_{ij} \geqslant s_{T/2}, r_{jk} \geqslant s_{T/2}, \Rightarrow r_{ik} \geqslant \min(r_{ij}, r_{jk})$, $\forall i, j, k$.
5. Restricted max–max transitivity: $r_{ij} \geqslant s_{T/2}, r_{jk} \geqslant s_{T/2}, \Rightarrow r_{ik} \geqslant \max(r_{ij}, r_{jk})$, $\forall i, j, k$.

In order to make good use of the *linguistic preference relations* for aggregating experts' preferences, an aggregation operator of linguistic information is needed. Various operators have been proposed [2, 10, 29, 32]. It is important that these operators satisfy a well defined axiomatic. In the next section, we study some postulated axiomatics of an intuitively acceptable fuzzy aggregation operator, and more concretely, we analyze the properties and axioms that the linguistic aggregation operator, as defined by Herrera and Verdegay in [10], verifies.

3. Preferences aggregation axiomatic

The main problem in fuzzy logic-based group decision making is how to aggregate the experts' opinions to obtain a group decision in such a way that some rational criteria are satisfied. The problem may be discussed as a special case of information aggregation in multi-person and multi-criteria decision making or social choice theories (see, for instance, the surveys in books [8, 19, 25] and papers [7, 26, 30]).

Many papers on "classical" theory of group decision make use of Arrow's work [1] as a starting point and a basic guide. Arrow proposed a qualitative setting composed by a set of axioms, which any acceptable aggregation tool of group decision making should satisfy. *Arrow's impossibility theorem* was an important result thereof. According to this theorem, it is impossible to aggregate individual preferences into group preference in a completely rational way. This is a problem that dissappears in cardinal setting in a fuzzy context, introducing preference intensities, which provides additional degrees of freedom to any aggregation model [7, 5].

3.1. Preferences aggregation axiomatic in fuzzy environment

Therefore an axiomatic to model the aggregation processes in fuzzy set area is needed. Some axiomatic approaches have been partially taken by Fung and Fu [9], Montero [22] and Cholewa [4]. Fung and Fu presented a set of axioms to rational fuzzy group decision making in order to justify the minimum and maximum operations. Montero introduces fuzzy counterparts of veto and dictatorship, and Cholewa offers a collection of axioms for the aggregation of fuzzy weighted opinions and indicates that the weighted mean satisfies these axioms.

In [7], a very detailed analysis about proposed axiomatic approaches to rational group fuzzy decision making is presented. A complete set of axioms in the fuzzy set setting for homogeneous groups is reviewed. These axioms are natural properties of a voting procedure that include the ones proposed by Arrow. Some of these are: *unrestricted domain, unanimity, neutrality, ...* A three-group classification has been established:

- *Imperative axioms,* whose violation leads to counterintuitive aggregation modes, e.g.: neutrality.
- *Technical axioms,* that facilitate the representation and the calculation of the aggregation operator, e.g.: unrestricted domain.
- *Facultative axioms,* that are applied in special circumstances but are not universally accepted, e.g.: unanimity.

Obviously a particular aggregation operator ϕ does not have to satisfy all axioms together, it must satisfy those that its special application circumstances require. For more information about the axiomatic of fuzzy aggregation, see [7].

In the next sections, we study some properties and axioms that the aggregation operator of linguistic opinions, LOWA, verifies.

3.2. The linguistic ordered weighted averaging operator

The LOWA operator is based on the ordered weighted averaging (OWA) operator defined by Yager [30], and on the convex combination of linguistic labels defined by Delgado et al. [6].

Let $\{a_1, \ldots, a_m\}$ be a set of labels to aggregate, then the LOWA operator ϕ is defined as

$$\phi(a_1, \ldots, a_m) = W \cdot B^{\mathrm{T}} = C^m\{w_k, b_k, k = 1, \ldots, m\}$$
$$= w_1 \odot b_1 \oplus (1 - w_1)$$
$$\odot C^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\},$$

where $W = [w_1, \ldots, w_m]$, is a weighting vector, such that, $w_i \in [0, 1]$ and $\sum_i w_i = 1$; $\beta_h = w_h/\sum_2^m w_k$, $h = 2, \ldots, m$, and B is the associated ordered label vector. Each element $b_i \in B$ is the ith largest label in the collection $a_1, \ldots, a_m$. C^m is the convex combination operator of m labels and if $m = 2$ then it is defined as

$$C^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k,$$
$$s_j, s_i \in S \quad (j \geqslant i)$$

such that

$$k = \min\{T, i + \mathrm{round}(w_1 \cdot (j - i))\},$$

where *round* is the usual round operation, and $b_1 = s_j, b_2 = s_i$.

If $w_j = 1$ and $w_i = 0$ with $i \neq j$ $\forall i$, then the convex combination is defined as:

$$C^m\{w_i, b_i, i = 1, \ldots, m\} = b_j.$$

3.2.1. How to calculate the weights of LOWA operator

Quantifiers can be used to represent the amount of items satisfying a given predicate. Classic logic is restricted to the use of the two quantifiers, *there exists* and *for all,* that are closely related respectively to the *or* and *and* connectives. Human discourse is much richer and more diverse in its quantifiers, e.g. *about 5, almost all, a few, many, most, as many as possible, nearly half, at least half.* In an attempt to bridge the gap between formal systems and natural discourse and, in turn, to provide a more flexible knowledge representation tool, Zadeh introduced the concept of linguistic quantifiers [34].

Zadeh suggested that the semantic of a linguistic quantifier can be captured by using fuzzy subsets for its representation. He distinguished between two types of linguistic quantifiers, *absolute* and *proportional.* Absolute quantifiers are used to represent amounts that are absolute in nature such as

about 2 or *more than* 5. These absolute linguistic quantifiers are closely related to the concept of the count or number of elements. He defined these quantifiers as fuzzy subsets of the non-negative real numbers, $\mathbb{R}^+$. In this approach, an absolute quantifier can be represented by a fuzzy subset Q, such that for any $r \in \mathbb{R}^+$ the membership degree of r in Q, $Q(r)$, indicates the degree to which the amount r is compatible with the quantifier represented by Q. Proportional quantifiers, such as *most, at least half*, can be represented by fuzzy subsets of the unit interval, $[0, 1]$. For any $r \in [0, 1]$, $Q(r)$ indicates the degree to which the proportion r is compatible with the meaning of the quantifier it represents. Any quantifier of natural language can be represented as a proportional quantifier or given the cardinality of the elements considered, as an absolute quantifier. Functionally, linguistic quantifiers are usually of one of three types, *increasing, decreasing,* and *unimodal.* An increasing type quantifier is characterized by the relationship

$$Q(r_1) \geqslant Q(r_2) \quad \text{if } r_1 > r_2.$$

These quantifiers are characterized by values such as *most, at least half.* A decreasing type quantifier is characterized by the relationship

$$Q(r_1) \leqslant Q(r_2) \quad \text{if } r_1 < r_2.$$

The quantifiers characterize terms such as *a few, at most* α. Unimodal type quantifiers have the property that

$$Q(a) \leqslant Q(b) \leqslant Q(c) = 1 \geqslant Q(d)$$

for some $a \leqslant b \leqslant c \leqslant d$. These are useful for representing terms like *about q.*

A natural question in the definition of the LOWA operator is how to obtain the associated weighting vector. In [30, 33], Yager proposed two ways to obtain it. The first approach is to use some kind of learning mechanism using some sample data; and the second approach is to try to give some semantics or meaning to the weights. The final possibility has allowed multiple applications on areas of fuzzy and multi-valued logics, evidence theory, designing of fuzzy controllers, and the quantifier guided aggregations. We are interested in the area of quantifier guided aggregations, because our idea is to calculate weights using linguistic

quantifiers for representing the concept of *fuzzy majority* in the aggregations that are made in our group decision making processes. Therefore, in the aggregations of LOWA operator the concept of fuzzy majority is underlying by means of the weights.

In [30, 33], Yager suggested an interesting way to compute the weights of the OWA aggregation operator using linguistic quantifiers, which, in the case of a non-decreasing proportional quantifier Q, is given by this expression:

$$w_i = Q(i/n) - Q((i - 1)/n), \quad i = 1, \dots, n$$

being the membership function of a non-decreasing proportional quantifier Q, as follows:

$$Q(r) = \begin{cases} 0 & \text{if } r < a, \\ \dfrac{r - a}{b - a} & \text{if } a \leqslant r \leqslant b, \\ 1 & \text{if } r > b. \end{cases}$$

with $a, b, r \in [0, 1]$. When it is used a fuzzy linguistic quantifier Q to compute the weights of LOWA operator ϕ, it is symbolized by ϕ_Q.

In Section 4, we explain how to use the LOWA operator for solving a group decision making problem from individual linguistic preference relations by the direct approach, according to two types of fuzzy majority:

- *Fuzzy majority of dominance,* used to quantify the dominance that one alternative has over all the others, according to one expert's opinions.
- *Fuzzy majority of experts,* used to quantify the dominance that one alternative has over all the others, according to the experts' opinions, considered as a whole.

3.3. Properties of the LOWA operator

The LOWA operator has some properties of the OWA operators investigated by Yager in [30], i.e.: *monotonicity property, the commutativity property, and the property to be an "orand" operator.* Before demonstrating the properties, we are going to demonstrate the following theorem.

Theorem. *Let* $A = [a_1, a_2, \ldots, a_n]$ *be an ordered labels vector* $(a_1 \geqslant a_2 \geqslant \cdots \geqslant a_n)$ *and* $a_i \in S$, *then*

$$a_n \leqslant \phi(a_1, a_2, \ldots, a_n) \leqslant a_1.$$

Proof. In [6] the following property was demonstrated: *if* k *is the resulting label from the convex combination of two labels,* i *and* j $(i \leqslant j)$, *then* $i \leqslant k \leqslant j$.

When observing that property, obviously the theorem is a consequence thereof.

Property 1. *The LOWA operator is* increasing monotonous *with respect to the argument values, in the following sense:*

Let $A = [a_1, a_2, \ldots, a_n]$ *be an ordered argument vector, let* $B = [b_1, b_2, \ldots, b_n]$ *be a second ordered argument vector, such that* $\forall j, a_j \geqslant bj$ *then*

$$\phi(A) \geqslant \phi(B).$$

Proof. By induction over the number of arguments to aggregate.

For $n = 2$. Let s_j, s_i, s_p, s_q, be the ordered labels from S, corresponding to a_1, a_2, b_1, b_2, respectively. Clearly $j \geqslant p$ and $i \geqslant q$, and therefore given any $w_1 \in [0, 1]$ then

$$j \cdot w_1 \geqslant p \cdot w_1,$$

and

$$i \cdot (1 - w_1) \geqslant q \cdot (1 - w_1),$$

and thus

$$j \cdot w_1 + i \cdot (1 - w_1) \geqslant p \cdot w_1 + q \cdot (1 - w_1).$$

As *round* is an increasing monotonous function, then

$$\text{round}(j \cdot w_1 + i \cdot (1 - w_1))$$
$$\geqslant \text{round}(p \cdot w_1 + q \cdot (1 - w_1))$$
$$\Rightarrow \text{round}((j - i) \cdot w_1) + i))$$
$$\geqslant \text{round}((p - q) \cdot w_1) + q));$$

and as $i \in \mathbb{Z}^+$ and $((j - i) \cdot w_1) > 0$ then

$$i + \text{round}((j - i)w_1) \geqslant q + \text{round}((p - q)w_1)$$
$$\Rightarrow \phi(a_1, a_2) \geqslant \phi(b_1, b_2).$$

Suppose that it is true for $n - 1$, i.e.,

$$\phi(a_1, a_2, \ldots, a_{n-1}) \geqslant \phi(b_1, b_2, \ldots, b_{n-1}).$$

For n,

$$\phi(a_1, a_2, \ldots, a_n) = w_1 \odot a_1 \oplus (1 - w_1)$$
$$\odot C^{n-1}\{\beta_h, a_h, h = 2, \ldots, n\},$$

and

$$\phi(b_1, b_2, \ldots, b_n) = w_1 \odot b_1 \oplus (1 - w_1)$$
$$\odot C^{n-1}\{\beta_h, b_h, h = 2, \ldots, n\}.$$

As

$$C^{n-1}\{\beta_h, a_h, h = 2, \ldots, n\} = \phi(a_2, a_3, \ldots, a_n),$$

and

$$C^{n-1}\{\beta_h, b_h, h = 2, \ldots, n\} = \phi(b_2, b_3, \ldots, b_n),$$

then by induction hypothesis

$$\phi(a_2, a_3, \ldots, a_n) \geqslant \phi(b_2, b_3, \ldots, b_n).$$

Let s_j, s_i, be the labels corresponding to $\phi(a_2, a_3, \ldots, a_n)$, and $\phi(b_2, b_3, \ldots, b_n)$, respectively, then using induction hypothesis $s_j \geqslant s_i$ and as $a_1 \geqslant b_1$, and as the theorem $a_1 \geqslant s_j$ and $b_1 \geqslant s_i$, then

$$\phi(a_1, a_2, \ldots, a_n) = \phi(a_1, s_j),$$

and

$$\phi(b_1, b_2, \ldots, b_n) = \phi(b_1, s_i).$$

We know that

$$\phi(a_1, s_j) \geqslant \phi(b_1, s_i),$$

as it is proven for $n = 2$, and therefore

$$\phi(a_1, a_2, \ldots, a_n) \geqslant \phi(b_1, b_2, \ldots, b_n).$$

Property 2. *The LOWA operator is* commutative, *i.e.,*

$$\phi(a_1, a_2, \ldots, a_n) = \phi(\pi(a_1), \pi(a_2), \ldots, \pi(a_n)),$$

where π *is a permutation over the set of arguments.*

Proof. Clearly the commutativity property is verified, because we use an "ordered" weighted average of the arguments.

Property 3. *The LOWA operator is an "orand" operator. That is, for any weighting vector W and ordered labels vector* $A = [a_1, a_2, \ldots, a_n]$, *then*

$$\min(a_1, a_2, \ldots, a_n) \leqslant \phi(a_1, a_2, \ldots, a_n)$$

$$\leqslant \max(a_1, a_2, \ldots, a_n)$$

Proof. Clearly it is a consequence of the aforementioned theorem.

3.4. Axiomatic of the LOWA operator

In what follows we are going to study some of the proposed axioms in fuzzy setting considering the LOWA operator which works with linguistically valued preferences. Before this, we include the following linguistic notation that we shall use.

Let $A = \{x_1, \ldots, x_n\}$ be a finite non-empty set of alternatives.

Let $E = \{e_1, \ldots, e_m\}$ be a panel of experts.

Let $S = \{s_i: i = 0, \ldots, T\}$ be a label set to voice experts' opinions.

Let $x_{ij} \in S$ be the linguistic rating of alternative x_i by expert e_j.

Let F_j be the linguistic rating set over alternatives by expert e_j.

Let μ_{F_j} be the linguistic membership function of F_j such that $x_{ij} = \mu_{F_j}(x_i)$.

Let F be the linguistic rating set such that $F = \phi(F_1, \ldots, F_n)$.

Axiom I. *Unrestricted domain.* For any set of individual preference patterns $\{F_j, j = 1, \ldots, m\}$ there is a social preference pattern F, which may be constructed,

$$\forall F_1, \ldots, F_m \in S^n, \exists F \in S^n \text{ such that}$$

$$F = \phi(F_1, \ldots, F_m).$$

It is basically technical, and clearly it is satisfied in accordance with the LOWA operator definition.

Axiom II. *Unanimity or idempotence.* If everyone agrees on a preference pattern, it must be chosen as the social choice pattern,

$$F_j = F, \forall j \Rightarrow F = \phi(F, F, \ldots, F).$$

Following this definition, the LOWA operator can immediately be verified.

Axiom III. *Positive association of social and individual values.* If an individual increases his linguistic preference intensity for x_i then the sociallinguistic preference for x_i cannot decrease. This means that if F'_j and F_j are such that $\mu_{F_j} \leqslant \mu_{F'_j}$, then if $\phi(F_1, \ldots, F_j, \ldots, F_m) = F$ and $\phi(F_1, \ldots, F'_j, \ldots, F_m) = F'$, then

$$\mu_F \leqslant \mu_{F'}.$$

Clearly it is satisfied, because it is a consequence of increasing monotonicity property of the LOWA operator.

Axiom IV. *Independence of irrelevant alternatives.* The social preference intensity for x_i only depends on the individual preference intensity for x_i, and not for x_k, $k \neq i$,

$$\mu_{\phi(F_1, \ldots, F_m)}(x_i) = \varphi(x_{i1}, \ldots, x_{im}).$$

It is basically technical, and is satisfied by the definition of the LOWA operator. Clearly this axiom does not extend strictly speaking, since for preference relations the independence of irrelevant alternatives deals with pairs of alternatives.

Axiom V. *Citizen sovereignty.* It means that any social preference pattern can be expressed by the society of individuals; in other words

$$\forall F, \exists F_1, \ldots, F_m \text{ such that } F = \phi(F_1, \ldots, F_m).$$

A weaker form of citizen sovereignity called Non-Dictatorship [22] is as follows: there is no individual e_j such that

$$\phi(F_1, \ldots, F_j, \ldots, F_m) = F_j.$$

This requirement prohibits any individual from acting as a veto or dictator under any circumstances.

Obviously, this axiom is satisfied in its general form, because it is a consequence of axiom II (unanimity). Clearly, as the LOWA operator is commutative, then it also satisfies the weaker form of the axiom.

Axiom VI. *Decomposability of the voting procedure.* This means that it is possible to split the set of individuals into disjoint subgroups, build the social preference pattern for each subgroup and then combine the local social preference patterns to obtain the result. Some forms of this axiom are:

1. *Strongest form, associativity:*

$$\phi(\phi(F_1, F_2), F_3) = \phi(F_1, \phi(F_2, F_3)).$$

2. *Weaker form, auto-distributivity:*

$$\phi(F_1, \phi(F_2, F_3)) = \phi(\phi(F_1, F_2), \phi(F_1, F_3)).$$

This axiom is not verified for any of its forms. Thus, the LOWA operator is not associative, example:

Suppose, there is a set of nine labels, and we want to aggregate the labels s_7, s_6, s_5. If we fix two weights $w_1 = 0.3$, $w_2 = 0.7$ then

$$\phi(\phi(s_7, s_6), s_5) = s_5 \neq s_6 = \phi(s_7, \phi(s_6, s_5)).$$

And the LOWA operator is not auto-distributive, example:

Let s_1, s_2, s_4 be the opinions to be aggregated. Using the same weights as before,

$$\phi(s_1, \phi(s_2, s_4)) = s_2 \neq s_1 = \phi(\phi(s_1, s_2), \phi(s_1, s_4)).$$

Axiom VII. *Neutrality.* The neutrality axiom refers to the invariance properties of the voting procedure. There are three types:

1. *Neutrality with respect to alternatives.* If x_i and x_k are such that $x_{ij} = x_{kj}$, $\forall j$, then $\mu_{\phi(F_1, \ldots, F_m)}(x_i) = \mu_{\phi(F_1, \ldots, F_m)}(x_k)$.

2. *Neutrality with respect to voters.* In a homogeneous group, this is the anonymity property, i.e., the commutativity of ϕ.

3. *Neutrality with respect to the intensity scale or Neutrality of Complement.* If F_j^c is the complement to F_j, such that $F_j^c = \text{Neg}(F_j)$, the social pattern $\phi(F_1^c, \ldots, F_m^c)$ should be the complement of the social preference pattern,

$$\phi(F_1, \ldots, F_m)^c = \phi(F_1^c, \ldots, F_m^c).$$

Clearly it is verified for the form of neutrality respect to alternatives. As the LOWA operator is commutative, then it also verifies the neutrality with respect to voters. However it does not verify the neutrality with respect to intensity scale. Example:

Consider a label set with eight elements S. Let $s_3, s_2 \in S$ be the labels to be aggregated and its

complement labels s_4, s_5, and $w_1 = 0.1$, then

$$\text{Neg}(\phi(s_3, s_2)) = s_5 \neq s_4 = \phi(s_4, s_5).$$

In conclusion, the LOWA operator verifies the following axioms: *Unrestricted domain, Unanimity or Idempotence, Positive association of social and individual values, Independence of irrelevant alternatives, Citizen sovereignty, Neutrality.* The fulfilment of those axioms provides evidence of rational aggregation using the LOWA operator in particular frameworks.

4. Direct approach to group decision making under linguistic assessments

Suppose we have a set of n alternatives $X = \{x_1, \ldots, x_n\}$ and a set of experts $E = \{e_1, \ldots, e_m\}$. Each expert $e_k \in E$ provides a preference relation linguistically assessed into the label set S,

$$\omega_{P^k} : X \times X \to S,$$

where $\omega_{P^k}(x_i, x_j) = p_{ij}^k \in S$ represents the linguistically assessed preference degree of the alternative x_i over x_j. We assume that P^k is reciprocal without loss of generality.

As is known, basically two approaches may be considered. A direct approach

$$\{P^1, \ldots, P^m\} \to \text{solution}$$

according to which, on the basis of the individual preference relations, a solution is derived, and an indirect approach

$$\{P^1, \ldots, P^m\} \to P \to \text{solution}$$

providing the solution on the basis of a collective preference relation, P, which is a preference relation of the group of individuals as a whole.

In [14], we considered a group decision making process by indirect approach. Here, we consider the direct approach. We present three direct ways to solve a group decision making process in a linguistic framework: (i) *Dominance Process (DP)*, (ii) *Strict-Dominance Process (SDP)*, (iii) *Non-Dominance Process (NDP)*.

These processes are based on *linguistic dominance, linguistic strict-dominance,* and *linguistic*

non-dominancenance degrees respectively, as defined in [13]. The direct approach processes are calculated on two levels of action:

- *Level of Preference or Level of Individual.* The different linguistic degrees are calculated for each alternative according to the opinions of each expert, considered individually, i.e., *individual degrees* are calculated on this level.

- *Level of Degrees or Level of Group.* Here, the different linguistic degrees are calculated for each alternative according to the opinions of the expert group, considered as a whole, i.e., *social degrees* are calculated on this level.

Note that between both levels the concept of fuzzy majority is different, dominance majority and experts' majority respectively, and therefore we can use different fuzzy linguistic quantifiers on each level.

The following subsections are devoted to developing each one of the aforementioned models.

4.1. Dominance process

This is a resolution model that uses the linguistic dominance degree of each alternative to decide which of them to choose as a solution to the group decision making problem. The linguistic dominance degree of one alternative gives a measure of the averaging preference degree that the overall ones in function of the experts' opinions.

After fixing a label set S and the concepts of fuzzy majority of dominance and fuzzy majority of experts by means of two fuzzy quantifiers, Q_1 and Q_2, respectively, the process is described in the following steps:

1. For each linguistic preference relation of each expert, P^k, using the LOWA operator ϕ_{Q_1}, find out the *individual linguistic dominance degree* of each alternative x_i, called ID_i^k, according to this expression:

$$\text{ID}_i^k = \phi_{Q_1}(p_{ij}^k, j = 1, \ldots, n, j \neq i)$$

with $k = 1, \ldots, m; i = 1, \ldots, n.$

2. For each alternative x_i, calculate the *social linguistic dominance degree*, called SD_i, as follows

$$\text{SD}_i = \phi_{Q_2}(\text{ID}_i^k, k = 1, \ldots, m)$$

with $i = 1, \ldots, n.$

3. Obtain the set of alternatives with maximum linguistic dominance degree $X_{\max}^d$, as follows

$$X_{\max}^d = \{x_i \in X/\text{SD}_i = \max_j(\text{SD}_j)\}$$

and the solution set, i.e., those alternatives with maximum degree.

This process is shown in Fig. 1.

4.2. Strict-dominance process

This proposal of direct approach selects the alternative(s) solution according to its respective linguistic strict-dominance degree. The linguistic strict-dominance degree gives a linguistic measure of the number of times that one alternative is preferred overall ones according to all experts' opinions.

In this process, as we shall see, fuzzy majority of dominance is not underlying in the weights of an LOWA operator. Here, it is used to quantify linguistically the number of times that one alternative is preferred to overall ones. Therefore, we need to define a fuzzy quantifier of the $Q': [0, 1] \rightarrow L$ type, being L a label set. This can be done by means of this expresion:

$$Q'(r) = \begin{cases} l_0 & \text{if } r < a, \\ l_i & \text{if } a \leqslant r \leqslant b, \\ l_U & \text{if } r > b, \end{cases}$$
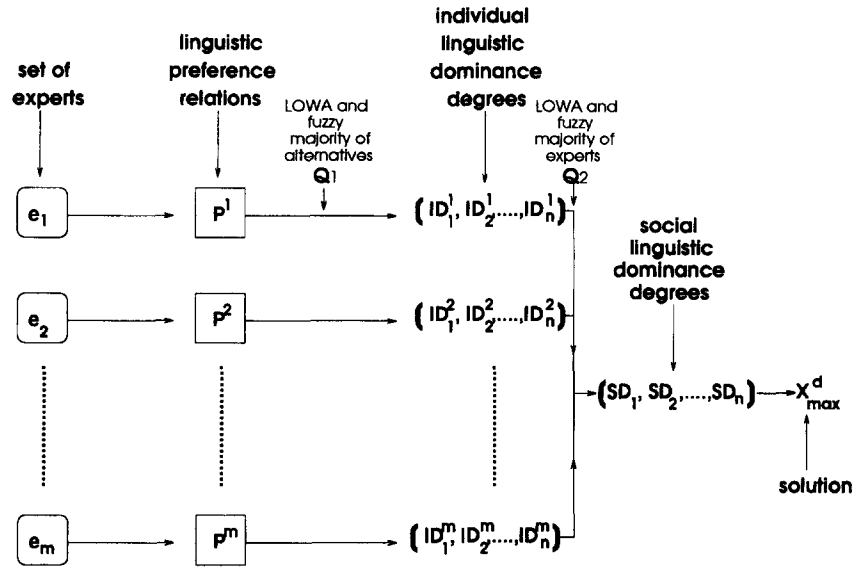
l_0 and l_U are the minimum and maximum labels in L, respectively, and

$$l_i = \sup_{l_q \in M} \{l_q\},$$

with

$$M = \left\{ l_q \in L: \mu_{l_q}(r) = \sup_{t \in J} \left\{ \mu_{l_t}\left(\frac{r-a}{b-a}\right) \right\} \right\},$$
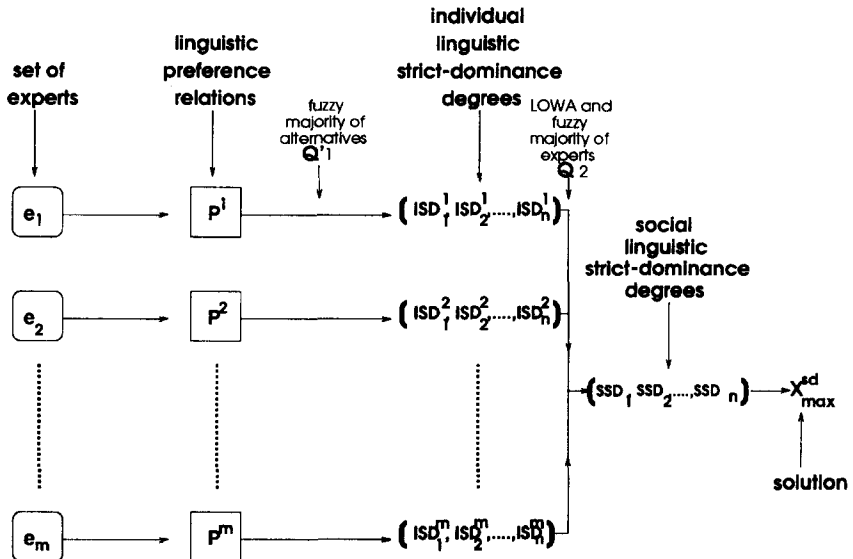
with $a, b, r \in [0, 1]$. Another definition of Q' can be found in [32].

Fixed two label sets, S and L, and the concepts of fuzzy majority of alternatives and experts by means of two fuzzy quantifiers, Q'_1 and Q_2, respectively, the process is shown in Fig. 2, and described in the

DIRECT APPROACH BASED ON LINGUISTIC DOMINANCE DEGREE

Fig. 1. Dominance process.

DIRECT APPROACH BASED ON LINGUISTIC STRICT-DOMINANCE DEGREE

Fig. 2. Strict-dominance process.

following steps:

1. For each linguistic preference relation of each expert P^k, find out the *individual linguistic strict-dominance degree* of each alternative x_i, called ISD_i^k, according to this expresion:

$$\mathrm{ISD}_i^k = Q_1'\left(\frac{r_i^k}{n-1}\right),$$

with $k = 1, \ldots, m; i = 1, \ldots, n$, and where,

$$r_i^k = \#\{x_j \in X \text{ such that} \\ p_{ij} > p_{ji}, j = 1, \ldots, n, j \neq i\},$$

and $\#$ stands for the cardinal of the term set.

2. For each alternative x_i, calculate the *social linguistic strict-dominance degree*, called SSD_i, as follows

$$\mathrm{SSD}_i = \phi_{Q_2}(\mathrm{ISD}_i^k, \quad k = 1, \ldots, m)$$

with $i = 1, \ldots, n$.

3. Obtain the set of alternative with maximum linguistic strict-dominance degree X_{max}^{sd}, as follows

$$X_{max}^{sd} = \{x_i \in X / \mathrm{SSD}_i = \max_j(\mathrm{SSD}_j)\}$$

the solution set.

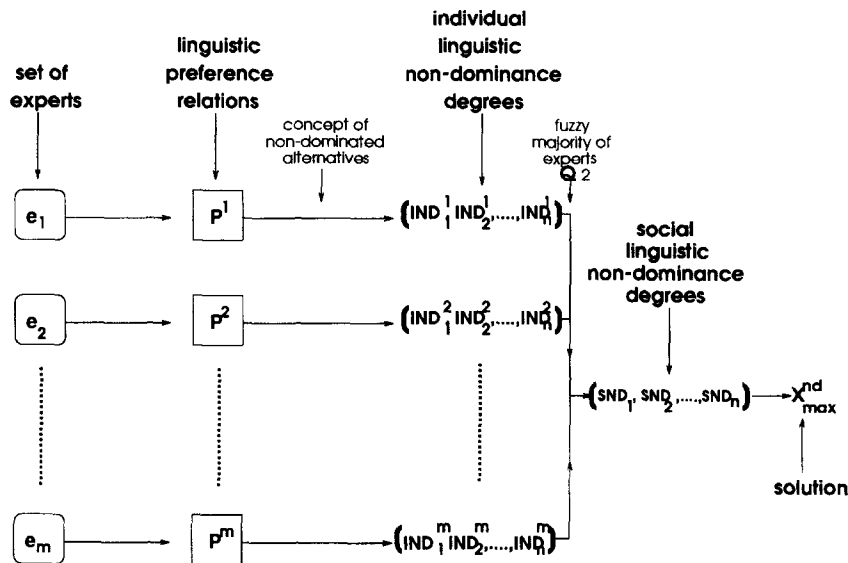### 4.3. Non-dominance process

Here, we present a third model using the direct approach, which selects the alternative(s) solution according to its respective linguistic non-dominance degree. The linguistic non-dominance degree of one alternative expresses a linguistic measure in which that alternative is not dominated by any, according to the experts' opinions. It is calculated on the basis of the concept of non-dominated alternatives described by Orlovsky [24], extended here to linguistic environment.

After fixing a label set, S, and the concept of fuzzy majority of experts by means of the fuzzy quantifiers, Q_2, the group decision making process is shown in Fig. 3, and described in the following steps:

1. For each linguistic preference relation of each expert, P^k, find its respective linguistic strict preference relation, P_s^k, with $\omega_{p_s^k} = p_{s\,ij}^k$, such that,

$$p_{s\,ij}^k = s_0 \quad \text{if } p_{ij}^k < p_{ji}^k,$$

or $p_{s\,ij}^k = s_h \in S$ if $p_{ij}^k \geqslant p_{ji}^k$ with $p_{ij}^k = s_l$, $p_{ji}^k = s_t$ and $l = t + h$.

DIRECT APPROACH BASED ON LINGUISTIC NON-DOMINANCE DEGREE

Fig. 3. Non-dominance process.

2. For each linguistic strict preference relation of each expert P_s^k, find the *individual linguistic non-dominance degree* of each alternative x_i, called IND_i^k, according to this expression:

$$\mathrm{IND}_i^k = \min_{x_j \in X} [\mathrm{Neg}(\omega_{P_s^k}(x_j, x_i))]$$

with $k = 1, \ldots, m; i = 1, \ldots, n$.

3. For each alternative x_i, calculate the *social linguistic non-dominance degree*, called SND_i, as follows

$$\mathrm{SND}_i = \phi_{Q_2}(\mathrm{IND}_i^k, k = 1, \ldots, m)$$

with $i = 1, \ldots, n$.

4. Obtain the set of alternatives with maximum linguistic non-dominance degree $X_{\max}^{\mathrm{nd}}$, as follows

$$X_{\max}^{\mathrm{nd}} = \{x_i \in X / \mathrm{SND}_i = \max_j(\mathrm{SND}_j)\}$$

and the solution set.

4.4. Sequential process

Sometimes, any of above solution set of alternatives can be formed by various alternatives. This may happen because of the existence of balance between all the alternatives, or the existence of an inconsistency situation between all the experts' opinions. In these cases, the combined application of the three models could be very interesting, because it can help to avoid the inconsistency situation and to identify the best solution set of alternatives.

In line with the aforementioned, we propose a group decision making process which combines the ones described earlier, called *The Sequential Process*. This model consists in applying each one of them in sequence, according to a previously established order. There is no criterion to establish an order, e.g., we can establish an order based on the order of the presentation of our work. From this supposition, the sequential process is shown in Fig. 4 and developed in three steps:

1. Apply the first model, DP over X, and obtain $X_{\max}^d$. If $\#(X_{\max}^d) = 1$ then End, and this is the solution. Otherwise continue, using the following step.

2. Apply the second model, SDP over X, and obtain $X_{\max}^{\mathrm{sd\_}d} \subseteq X_{\max}^d$. If $\#(X_{\max}^{\mathrm{sd\_}d}) = 1$ then End, and this is the solution. Otherwise continue, using the following step.
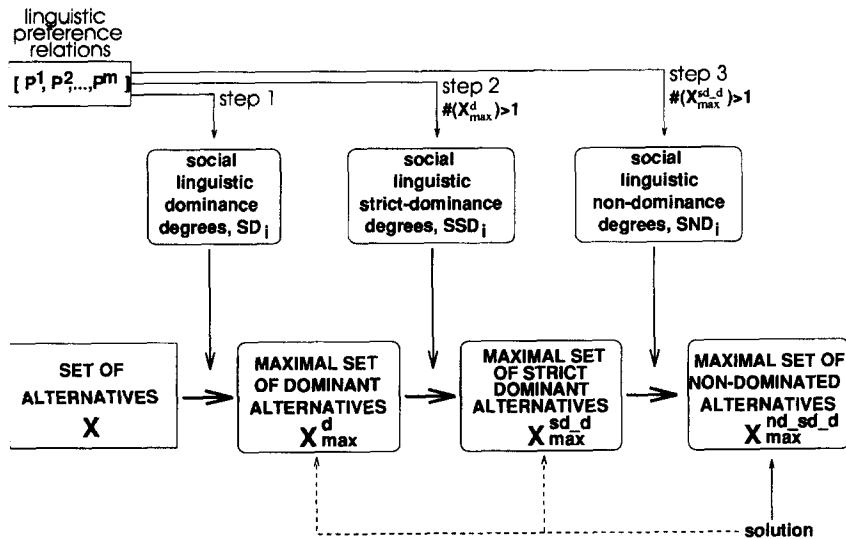
Fig. 4. Sequential process.

3. Apply the third model, NDP over X, and obtain $X_{\max}^{\text{nd\_sd}\_d} \subseteq X_{\max}^{\text{sd}\_d}$, and this is the best solution.

5. Examples

In this section the application of aforementioned models is shown in the following conditions:
Let the nine linguistic label set be S:

C	Certain	(1, 1, 0, 0)
EL	Extremely_likely	(0.98, 0.99, 0.05, 0.01)
ML	Most_likely	(0.78, 0.92, 0.06, 0.05)
MC	Meaningful_chance	(0.63, 0.80, 0.05, 0.06)
IM	It_may	(0.41, 0.58, 0.09, 0.07)
SC	Small_chance	(0.22, 0.36, 0.05, 0.06)
VLC	Very_low_chance	(0.1, 0.18, 0.06, 0.05)
EU	Extremely_unlikely	(0.01, 0.02, 0.01, 0.05)
I	Impossible	(0, 0, 0, 0)

Suppose a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$, as well as a set of four experts $E = \{e_1, e_2, e_3, e_4\}$, whose respective opinions are the following linguistic preference relations, expressed over X, using the labels of S:

$$P^1 = \begin{bmatrix} - & SC & C & I \\ MC & - & EU & EL \\ I & EL & - & VLC \\ C & EU & ML & - \end{bmatrix},$$

$$P^2 = \begin{bmatrix} - & IM & C & EU \\ IM & - & EU & C \\ I & EL & - & VLC \\ EL & I & ML & - \end{bmatrix},$$

$$P^3 = \begin{bmatrix} - & IM & EL & I \\ IM & - & I & EL \\ EU & C & - & VLC \\ C & EU & ML & - \end{bmatrix},$$

$$P^4 = \begin{bmatrix} - & IM & C & EU \\ IM & - & EU & C \\ I & EL & - & VLC \\ EL & I & ML & - \end{bmatrix}.$$

Using the linguistic quantifier "As many as possible" with the pair (0.5, 1) for all the operations, and assuming that $Q_1 = Q_2$ and $S = L$, then the direct models are applied as follows:

5.1. Dominance process

Step 1. The individual linguistic dominance degrees of each alternative using the LOWA operator with $W = (0, 0.334, 0.666)$ are:
Expert one,

$(\text{ID}_1^1, \text{ID}_2^1, \text{ID}_3^1, \text{ID}_4^1) = [\text{EU}, \text{VLC}, \text{EU}, \text{SC}]$.

Expert two,

$(\text{ID}_1^2, \text{ID}_2^2, \text{ID}_3^2, \text{ID}_4^2) = [\text{VLC}, \text{VLC}, \text{EU}, \text{VLC}]$.

Expert three,

$(\text{ID}_1^3, \text{ID}_2^3, \text{ID}_3^3, \text{ID}_4^3) = [\text{EU}, \text{EU}, \text{EU}, \text{SC}]$.

Expert four,

$(\text{ID}_1^4, \text{ID}_2^4, \text{ID}_3^4, \text{ID}_4^4) = [\text{VLC}, \text{VLC}, \text{EU}, \text{VLC}]$.

Step 2. The social linguistic dominance degree of each alternative using the LOWA operator with $W = (0, 0, 0.5, 0.5)$ is:

$(\text{SD}_1, \text{SD}_2, \text{SD}_3, \text{SD}_4 = [\text{EU}, \text{VLC}, \text{EU}, \text{VLC}])$.

Step 3. The set of alternatives with maximum linguistic dominance degree is:

$X_{\max}^d = \{x_2, x_4\}$.

5.2. Strict-dominance process

Step 1. The individual linguistic strict-dominance degrees of each alternative are:
Expert one,

$(\text{ISD}_1^1, \text{ISD}_2^1, \text{ISD}_3^1, \text{ISD}_4^1)$

$= [Q_1'(1/3), Q_1'(2/3), Q_1'(1/3), Q_1'(2/3)]$

$= [I, \text{SC}, I, \text{SC}]$.

Expert two,

$(\text{ISD}_1^2, \text{ISD}_2^2, \text{ISD}_3^2, \text{ISD}_4^2)$

$= [Q_1'(1/3), Q_1'(1/3), Q_1'(1/3), Q_1'(2/3)]$

$= [I, I, I, \text{SC}]$.

Expert three,

$(ISD_1^3, ISD_2^3, ISD_3^3, ISD_4^3)$

$= [Q_1'(1/3), Q_1'(1/3), Q_1'(1/3), Q_1'(2/3)]$

$= [I, I, I, SC].$

Expert four,

$(ISD_1^4, ISD_2^4, ISD_3^4, ISD_4^4)$

$= [Q_1'(1/3), Q_1'(1/3), Q_1'(1/3), Q_1'(2/3)]$

$= [I, I, I, SC].$

Step 2. The social linguistic strict-dominance degree of each alternative using the LOWA operator with $W = (0, 0, 0.5, 0.5)$ is:

$(SSD_1, SSD_2, SSD_3, SSD_4) = [I, I, I, SC].$

Step 3. The set of alternatives with maximum linguistic dominance degree is:

$X_{max}^{sd} = \{x_4\}.$

5.3. Non-dominance process

Step 1. Find the respective strict linguistic preference relations of each expert.

$$P_s^1 = \begin{bmatrix} - & I & C & I \\ VLC & - & I & ML \\ I & ML & - & I \\ C & I & IM & - \end{bmatrix},$$

$$P_s^2 = \begin{bmatrix} - & I & C & I \\ I & - & I & C \\ I & ML & - & I \\ ML & I & IM & - \end{bmatrix},$$

$$P_s^3 = \begin{bmatrix} - & I & ML & I \\ I & - & I & ML \\ I & C & - & I \\ C & ML & IM & - \end{bmatrix},$$

$$P_s^4 = \begin{bmatrix} - & I & C & I \\ I & - & I & C \\ I & ML & - & I \\ ML & I & IM & - \end{bmatrix}.$$

Step 2. The individual linguistic non-dominance degrees of each alternative are:
Expert one,

$(IND_1^1, IND_2^1, IND_3^1, IND_4^1) = [I, VLC, I, VLC].$

Expert two,

$(IND_1^2, IND_2^2, IND_3^2, IND_4^2) = [VLC, VLC, I, I].$

Expert three,

$(IND_1^3, IND_2^3, IND_3^3, IND_4^3) = [I, I, VLC, VLC].$

Expert four,

$(IND_1^4, IND_2^4, IND_3^4, IND_4^4) = [VLC, VLC, I, I].$

Step 3. The social linguistic non-dominance degree of each alternative using the LOWA operator with $W = (0, 0, 0.5, 0.5)$ is:

$(SND_1, SND_2, SND_3, SND_4 = [I, EU, I, I]).$

Step 4. The set of alternatives with maximum linguistic non-dominance degree is:

$X_{max}^d = \{x_2\}.$

5.4. Sequential process

Step 1. After applying the DP model we obtain the following set of alternatives with maximum linguistic dominance degree:

$X_{max}^d = \{x_2, x_4\}.$

As $\#(X_{max}^d) > 1$, then continue.

Step 2. Apply the SSP model over X. As $X_{max}^d = \{x_2, x_4\}$ then only its respective strict-dominance degrees are considered,

$(SSD_2, SSD_4) = [I, SC],$

and therefore the $X_{max}^{sd\_d}$ is

$X_{max}^{sd\_d} = \{x_4\},$

and as $\#(X_{max}^{sd\_d}) = 1$ then End.

In consequence, by applying the proposed models together, it is possible to distinguish better amongst alternatives.

Remark. In the sequential process, the order of application of the different processes is chosen by the user.

6. Conclusions

The paper has developed basically two ideas:
- finding out evidence of the rationality of the LOWA operator,
- showing its usefulness in processes of group decision making in a linguistic environment.

We have checked the first goal, expressing the properties of the LOWA operator and examining some of the axioms of an acceptable aggregation operator that it satisfies. Then we combined the LOWA operator with other fuzzy tools, such as fuzzy linguistic quantifier, linguistic preference relations, concept of dominance and non-dominance, to show its use in the field of group decision. We presented three models of group decision making based on rational properties of the LOWA operator.

This paper shows how to use fuzzy techniques to incorporate more human consistency in decision models. In the future, we are interested in developing decision models that allow a closer link between the computer world and human beings world in fields, such as multi-criteria decision making and multi-stage decision making, using linguistic elements.

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