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A Sequential Selection Process in Group Decision Making with a Linguistic Assessment Approach

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ABSTRACT

In this paper, a sequential selection process in group decision making under linguistic assessments is presented, where a set of linguistic preference relations represents individuals preferences. A collective linguistic preference is obtained by means of a defined linguistic ordered weighted averaging operator whose weights are chosen according to the concept of fuzzy majority, specified by a fuzzy linguistic quantifier. Then we define the concepts of linguistic nondominance, linguistic dominance, and strict dominance degrees as parts of the sequential selection process. The solution alternative(s) is obtained by applying these concepts.

1. INTRODUCTION

A group decision making process can be defined as a decision situation in which (i) there are two or more individuals who differ in their preferences (value systems), but have the same access to information, each of them characterized by his or her own perceptions, attitudes, motivations, and personalities; (ii) who recognize the existence of a common problem; and (iii) who attempt to reach a collective decision [2].

The use of preference relations is normal in group decision making. Moreover, since human judgments including preferences are often vague, fuzzy sets play an important role in decision making. Several authors have provided interesting results on group decision making or social choice

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theory with the help of fuzzy sets. They have proven that fuzzy sets provided a more flexible framework for the discussion of group decision making [8, 10–14, 16, 18].

In a fuzzy environment, it is supposed that there exists a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ as well as a finite set of individuals $N = \{1, \ldots, m\}$, and each individual $k \in N$ provides his fuzzy preference relation on X, i.e., $p_k \subset XxX$, and $\mu_{P_k}(x_i, x_j)$ denotes the degree of preference of alternative x_i over x_j, $\mu_{P_k}(x_i, x_j) \in [0,1]$. In this framework, to make decisions consists of choosing one or more alternatives of the mentioned alternatives set according to the individuals' fuzzy preference relations.

Sometimes, however, an individual could have vague information about the preference degree of alternative x_i over x_j, and cannot estimate his preference with an exact numerical value. Then a more realistic approach may be to use linguistic assessments instead of numerical values, i.e., by supposing that the variables (preference relations) which participate in the problem are assessed by means of linguistic terms [4, 6, 7, 9, 15, 20]. A scale of certainty expressions (linguistically assessed) would be presented to the individual, who could then use it to describe his degree of certainty in a preference.

Assuming a set of alternatives or decisions, the basic question is how to relate to different decision schemata. According to [3], there are (at least) two possibilities, a group selection process and a consensus process. The first, a calculation of some mean value decision schema of a set of decisions, would imply the choice of an algebraic consensus as a mapping $l : DxD \to D$, whereas the second, the measure of distance between schemata, could be called topological consensus involving a mapping $k : DxD \to L$, where L is a complete lattice.

Here, we shall focus on the first possibility, for developing a group choice process under linguistic preferences.

Assuming a set of linguistic preferences representing individuals preferences, the group choice process develops according to the following scheme: a linguistic ordered weighted averaging (LOWA) operator is defined for linguistic labels, based on the ordered weighted averaging operator [19], and the convex combination of linguistic labels [5]. The concepts of fuzzy majority, represented by means of linguistic quantifiers and the LOWA operator, are used in order to obtain a collective linguistic preference. Finally, a sequential selection process acting on the collective preference relation is defined according to the following two steps:

1. Using the concept of nondominated alternatives [17] for defining a nondominance linguistic degree, and obtaining the set of maximal nondominated alternatives from the collective linguistic preference.

2. Defining the concepts of linguistic dominance degree and strict dominance degree with linguistic labels, and applying them to the set of maximal nondominated alternatives for obtaining the best alternatives.

Graphically, see Figure 1.

The aim of this paper is to present the group choice process using a sequential selection process. To do so, Section 2 shows the linguistic approach in group decision making, Section 3 presents the linguistic ordered weighted averaging operator, Section 4 shows how to obtain the collective linguistic preference relation under a fuzzy majority, Section 5 is devoted to developing the sequential selection process, Section 6 presents examples, and at the end, some conclusions are pointed out.

2. THE LINGUISTIC APPROACH IN GROUP DECISION MAKING

The linguistic approach considers the variables which participate in the problem assessed by means of linguistic terms instead of numerical values [21]. This approach is appropriate for many problems since it allows a representation of the experts' information in a more direct and adequate form, whether they are unable to express that with precision.

A linguistic variable differs from a numerical variable in that its values are not numbers; they are words or sentences in a natural or artificial language. Since words, in general, are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a means of approximated characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms.

A linguistic variable is associated with two rules:

1. *a syntactic rule*, which may take the form of a grammar for generating the names of the values of the variable, and
2. *a semantic rule*, which defines an algorithmic procedure for computing the meaning of each value.

DEFINITION [21]. A linguistic variable is characterized by a quintuple $(H, T(H), U, G, M)$ in which H is the name of the variable; $T(H)$ (or simply T) denotes the term set of H, i.e., the set of names of linguistic values of H, with each value being a fuzzy variable denoted generically by X and ranging across a universe of discourse U which is associated with the base variable u; G is a syntactic rule (*which usually takes the form of a grammar*) for generating the names of values of H; and M is a semantic rule for associating its meaning with each $H, M(X)$, which is a fuzzy subset

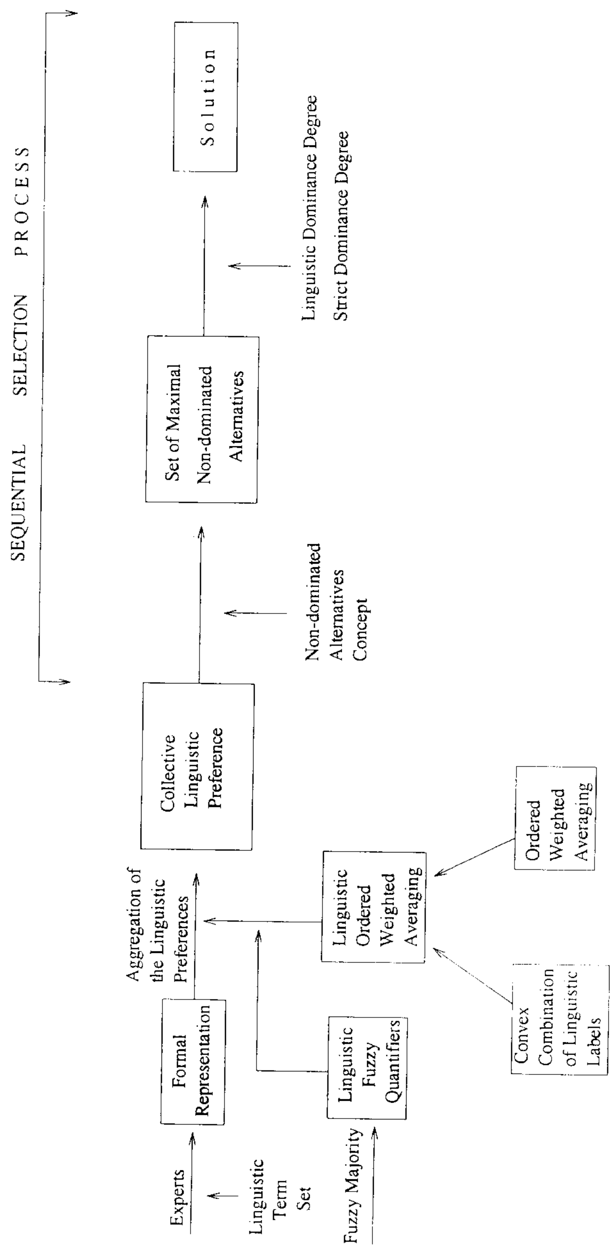

Fig. 1. Group choice process.

of U. Usually, the semantic of the elements of the term set is given by fuzzy numbers defined on the $[0, 1]$ interval, which are described by membership functions.

Provided that the linguistic assessments are just approximate ones given by the experts or decision-makers, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments since obtaining more accurate values may be impossible or unnecessary.

Therefore, we need a term set defining the uncertainty granularity, i.e., the finest level of distinction among different quantifications of uncertainty. The elements of the term set determine the granularity of the uncertainty. In [1], the use of term sets was studied with odd cardinals, representing the middle term a probability of "approximately 0.5," the remaining being terms placed symmetrically around it and the limit of granularity 11 or with no more than 13.

Let $S = \{s_i\}$, $i \in H = \{0, \ldots, T\}$ be a finite and totally ordered term set on $[0, 1]$ in the usual sense [1, 4, 21]. Any label s_i represents a possible value for a linguistic real variable, i.e., a vague property or constraint on [0, 1]. We consider the term set with an odd cardinal, where the middle label represents an uncertainty of "approximately 0.5" and the remaining terms are placed symmetrically around it. Moreover, we require the following properties for the term set:

1) The set is ordered: $s_i \geq s_j$ if $i \geq j$.
2) The negation operator is defined: $\mathrm{Neg}(s_i) = s_j$ such that $j = T - i$.
3) Maximization operator: $\mathrm{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$.
4) Minimization operator: $\mathrm{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.

Since aggregation of uncertainty information is a recurrent need in the decision process, combinations of linguistic values are needed. Two main different approaches can be found in order to aggregate and compare linguistic values: the first acts by direct computation on labels [5], and the second uses the associated membership functions [1, 21].

Most available techniques belong to the latter kind; however, the final results of those methods are fuzzy sets which do not correspond to any label in the original term set. If one finally wants to have a label, then a "linguistic approximation" is needed [1, 7, 21, 22]. The process of linguistic approximation consists of finding a label whose meaning is the same or the closest (according to some metric) to the meaning of an unlabeled membership function generated by some computational model. A simplified solution is the following. For each element of the term set and for the unlabeled membership function representing the result of some arithmetic

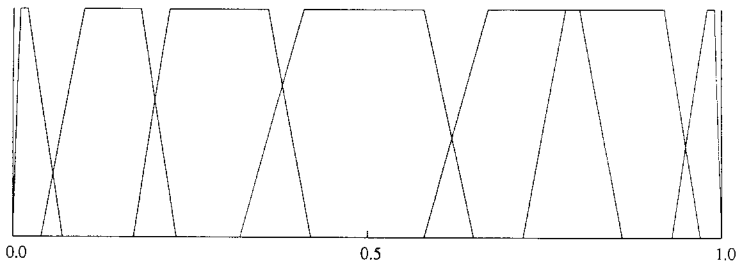

Fig. 2. Distribution of the nine linguistic term set.

operation, two features are extracted: the first moment of the distribution, and the area beneath the curve. A weighted Euclidean distance, where the weights reflect the relevance of the two parameters in determining semantic similarity, provides the metric required to select the element of the term set that most closely represents the result. There is neither a general criterion to evaluate the goodness of an approximation nor a general method for associating a label to a fuzzy set, so that specific problems may require to develop tailored methods.

Consider the following nine linguistic term set with the associated semantic [1] (the first two parameters indicate the interval in which the membership value is 1.0; the third and fourth parameters indicate the left and right width of the distribution):

C	$Certain$	$(1, 1, 0, 0)$
EL	$Extremely\_Likely$	$(0.98, 0.99, 0.05, 0.01)$
ML	$Most\_Likely$	$(0.78, 0.92, 0.06, 0.05)$
MC	$Meaningful\_Chance$	$(0.63, 0.80, 0.05, 0.06)$
IM	It_May	$(0.41, 0.58, 0.09, 0.07)$
SC	$Small\_Chance$	$(0.22, 0.36, 0.05, 0.06)$
VLC	$Very\_Low\_Chance$	$(0.1, 0.18, 0.06, 0.05)$
EU	$Extremely\_Unlikely$	$(0.01, 0.02, 0.01, 0.05)$
I	$Impossible$	$(0, 0, 0, 0)$

and shown graphically in Figure 2.

Formally speaking, it seems difficult to accept that all individuals should agree on the same membership function associated to linguistic terms, and therefore there are not any universality distribution concepts. For example, the two close perceptions shown in Figure 3 for the evaluation could be considered.

It is well known and accepted that the tuning of membership functions is a crucial issue in process control. In our context, we consider an environment where experts can discriminate perfectly the same term set under

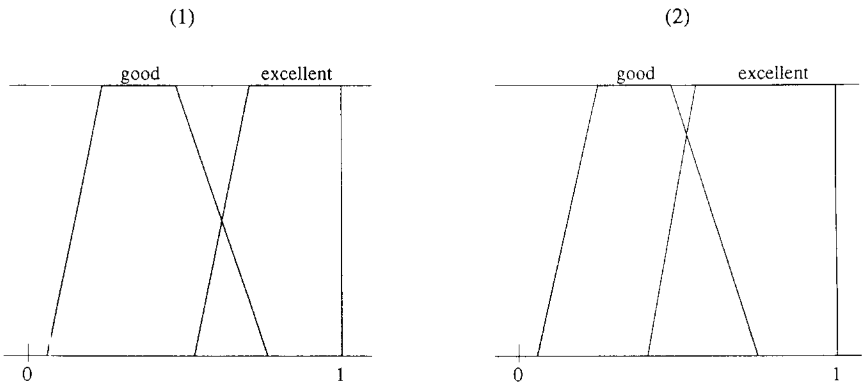

Fig. 3. Different distribution concepts.

a similar conception, taking into account that the concept of a linguistic variable serves the purpose of providing a means of approximated characterization of imprecise preference information.

On the other hand, we shall focus on the second approach, which is independent of the semantic of the term set, considering a similar discrimination of the experts. We present an aggregation operator of linguistic labels by direct computation on labels, which is based on the ordered weighted averaging (OWA) operator [19] and the convex combination of linguistic labels defined by Delgado *et al.* [5]. We call it a linguistic ordered weighted averaging (LOWA) operator.

3. THE LINGUISTIC ORDERED WEIGHTED AVERAGING OPERATOR

A mapping F,

$$F : [0,1]^n \to [0,1]$$

is called an OWA operator of dimension n if associated with F is a weighting vector W, $W = [w_1, \ldots, w_n]$, such that, i) $w_i \in [0,1]$, ii) $\sum_i w_i = 1$, and $F(a_1, \ldots, a_n) = w_1 \cdot b_1 + w_2 \cdot b_2 + \cdots + w_n \cdot b_n$, where b_i is the ith largest element in the collection $a_1, \ldots, a_n$. Denoting B as the vector consisting of the arguments of F put in descending order,

$$F(a_1, \ldots, a_n) = W \cdot B^T$$

provides an aggregation type operator that always lies between the "and" and the "or" aggregation. Its properties are presented in [19] and its first use in group decision making in [8].

This operator can be extended to linguistic arguments using the convex combination of linguistic labels defined in [5]. In fact, let M be a collection of linguistic labels, $p_k \in M$, $k = 1, \ldots, m$, and assume $p_m \leq p_{m-1} \leq \cdots \leq p_1$ without loss of generality. For any set of coefficients $\{\lambda_k \in [0,1], k = 1, 2, \ldots, m, \sum \lambda_k = 1\}$, the convex combination of these m generalized labels is the one given by

$$\mathbf{C}\{\lambda_k, p_k, \ k = 1, \ldots, m\} = \lambda_1 \odot p_1 \oplus (1 - \lambda_1) \odot \mathbf{C}\{\beta_h, p_h, \ h = 2, \ldots, m\}$$

$$\beta_h = \lambda_h \Big/ \sum_2^n \lambda_k; \ h = 2, \ldots, m.$$

In [5], the aggregation of labels was defined by addition, the difference of generalized labels, and the product by a positive real number over a *generalized label space* $\mathcal{S}$, based on S, i.e., the Cartesian product $\mathcal{S} = Sx\mathbf{Z}^+$, with the basic label set $S = \{(s_i, 1), \ i \in H\}$. In our context, all the operations are carried out over the basic set S. Briefly, the result of the expression $\lambda \odot s_j \oplus (1 - \lambda) \odot s_i$, $j \geq i$, is the s_k such that $k = \min\{T, i + \text{round}(\lambda \cdot (j - i))\}$.

An example to clarify of this operation is the following. Suppose the term set

$$\begin{aligned} S = \{&s_8 = C, s_7 = EL, s_6 = ML, s_5 = MC, \\ &s_4 = IM, s_3 = SC, s_2 = VLC, s_1 = EU, s_0 = I\} \end{aligned}$$

and $\lambda = 0.4$,

		$1-\lambda = 0.6$			
		SC	**VLC**	**C**	**EL**
	ML	IM	IM	EL	EL
$\lambda = 0.4$	**I**	VLC	EU	MC	IM
	IM	SC	SC	ML	ML
	VLC	SC	VLC	ML	MC

where, for example,

$$\begin{aligned} k_{11} &= min\{8, 3 + round(0.4 * (6 - 3))\} = 4 \text{ (IM)} \\ k_{21} &= min\{8, 0 + round(0.6 * (3 - 0))\} = 2 \text{ (VLC)}. \end{aligned}$$

Therefore, the LOWA operator can be defined as

$$\begin{aligned} F(a_1, \ldots, a_m) &= W \cdot B^T = \mathbf{C}\{w_k, b_k, \ k = 1, \ldots, m\} \\ &= w_1 \odot b_1 \oplus (1 - w_1) \odot \mathbf{C}\{\beta_h, b_h, \ h = 2, \ldots, m\} \end{aligned}$$

where $\beta_h = w_h/\Sigma_2^m w_k$, $h = 2, \ldots, m$, and B is the associated ordered label vector. Each element $b_i \in B$ is the ith largest label in the collection $a_1, \ldots, a_m$.

4. THE COLLECTIVE LINGUISTIC PREFERENCE RELATION UNDER A FUZZY MAJORITY

Suppose we have a set of n alternatives $X = \{x_1, \ldots, x_n\}$ and a set of individuals $N = \{1, \ldots, m\}$. Each individual $k \in N$ provides a preference relation linguistically assessed in the term set S,

$$\phi_{P^k} : XxX \to S,$$

where $\phi_{P^k}(x_i, x_j) = p_{ij}^k \in S$ represents the linguistically assessed preference degree of the alternative x_i over x_j. We assume that p_k is reciprocal in the sense, $p_{ij}^k = \text{Neg}(p_{ji}^k)$, and by the definition $p_{ii}^k = None$ (the minimum label in S).

As is now known, basically two approaches may be considered. A direct approach

$$\{P^1, \ldots, P^m\} \to \text{ solution}$$

according to which, on the basis of the individual preference relations, a solution is derived, and an indirect approach

$$\{P^1, \ldots, P^m\} \to P \to \text{ solution}$$

providing the solution on the basis of a collective preference relation, P, which is a preference relation of the group of individuals as a whole.

Here, we consider the indirect derivation, and hence we must derive a collective linguistic preference relation.

First, we introduce the concept of a fuzzy quantifier [23], used in order to specify the fuzzy majority concept as proposed Kacprzyk [10]. The fuzzy linguistic quantifiers were introduced by Zadeh [23]. Linguistic quantifiers are typified by terms such as *most*, *at least half*, *all*, *as many as possible*, and assuming a quantifier Q is a fuzzy set in $[0, 1]$. Zadeh distinguished between two types of quantifiers, absolute and proportional or relative. Absolute quantifiers are used to represent amounts that are absolute in nature. These quantifiers are closely related to the concepts of the counting of the number of elements. Zadeh suggested that these absolute quantifiers values can be represented as fuzzy subsets of the nonnegative real numbers, R^+. In particular, he suggested that an absolute quantifier can be represented by a fuzzy subset Q, where for any $r \in R^+$, $Q(r)$ indicates the degree to which the value r satisfies the concept represented by Q. And, relative quantifiers represent proportion type statements. Thus, if Q is a relative quantifier, then Q can be represented as a fuzzy subset of $[0,1]$ such that for each $r \in [0,1]$, $Q(r)$ indicates the degree to which r portion of objects satisfies the concept denoted by Q.

An absolute quantifier satisfies

$$Q(0) = 0,$$
$$\exists k \text{ such that } Q(k) = 1.$$

A relative quantifier

$$Q : [0,1] \to [0,1]$$

satisfies

$$Q(0) = 0,$$
$$\exists r \in [0,1] \text{ such that } Q(r) = 1.$$

A nondecreasing quantifier satisfies

$$\forall a, b \text{ if } a > b \text{ then } Q(a) \geq Q(b).$$

The membership function of a relative quantifier can be represented as

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \dfrac{r-a}{b-a} & \text{if } a \leq r \leq b \\ 1 & \text{if } r > b \end{cases}$$

with $a, b, rx \in [0,1]$.

Some examples of relative quantifiers are shown in Figure 4, where the parameters (a,b) are $(0.3, 0.8), (0, 0.5)$, and $(0.5, 1)$, respectively.

By means of the concept of *fuzzy majority* specified by a fuzzy linguistic quantifier and the use the LOWA operator, the collective preference relation, P, is obtained as

$$P = F(P^1, \ldots, P^m)$$

with $p_{ij} = F(p_{ij}^1, \ldots, p_{ij}^m)$ and the weight vector, W, obtained from the nondecreasing fuzzy linguistic quantifier representing the fuzzy majority over the individuals [19].

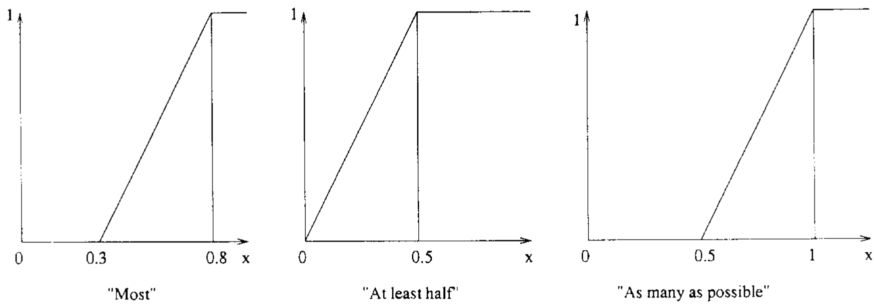

Fig. 4. Linguistic quantifiers.

Yager computes the weights w_i of the aggregation from the function Q describing the quantifier [19]. In the case of an absolute proportional quantifier

$$w_i = Q(i) - Q(i-1), \qquad i = 1, \ldots, m,$$

and in the case of a relative proportional one,

$$w_i = Q(i/m) - Q((i-1)/m), \qquad i = 1, \ldots, m.$$

5. THE SEQUENTIAL SELECTION PROCESS

As we said earlier, the sequential selection process acts on the collective preference relation in two steps:

1. Using the concept of nondominated alternatives [17] for defining the concept of linguistic nondominance degree and obtaining the set of maximal nondominated alternatives from the collective linguistic preference.
2. Defining the concepts of dominance and strict dominance with linguistic labels, and applying them to the set of maximal nondominated alternatives for obtaining the best alternative.

5.1. *LINGUISTIC NONDOMINANCE DEGREE*

Suppose the linguistic collective preference $P = (p_{ij}), i, j = 1, \ldots, n$; then let P^s be a *linguistic strict preference relation* $\mu_{P^s}(x_i, x_j) = p_{ij}^s$ such that

$$\begin{aligned} p_{ij}^s &= None && \text{if } p_{ij} < p_{ji}, \\ \text{or } p_{ij}^s &= s_k \in S && \text{if } p_{ij} \geq p_{ji} \text{ with } p_{ij} = s_l, p_{ji} = s_t \text{ and } l = t + k. \end{aligned}$$

The *linguistic nondominance degree* of x_i is defined as

$$\mu_{ND}(x_i) = \operatorname*{Min}_{x_j \in X}[\text{Neg}(\mu_{P^s}(x_j, x_i))]$$

where the value $\mu_{ND}(x_i)$ is meant as a linguistic degree to which the alternative x_i is not dominated by any of the elements in X.

Finally, a set of *maximal nondominated alternatives*, $X^{ND} \subset X$, is obtained as

$$X^{ND} = \left\{x \in X \;/\; \mu_{ND}(x) = \operatorname*{Max}_{y \in X}[\mu_{ND}(y)]\right\}.$$

Therefore, aggregating the knowledge of the experts, X^{ND} is selected as the set of preferred alternatives in our choice process.

5.2. LINGUISTIC DOMINANCE DEGREE

We define a linguistic degree which acts on the alternatives of X^{ND},

$$LDD(x_i) = F_{Q_{i \neq j}}(p_{ij}),$$

where F_Q is a LOWA operator whose weights are defined the using relative quantifier Q, and whose components are the elements of the corresponding row of P, i.e., for x_i, the set of $n-1$ labels $\{p_{ij} \;/\; j = 1, \ldots, n \text{ and } i \neq j\}$.

This measure allows us to define the set of nondominated alternatives with maximum linguistic dominance degree:

$$X^{LDD} = \left\{x \in X^{ND} \;/\; LDD(x) = \operatorname*{Max}_{y \in X^{ND}}[LDD(y)]\right\}.$$

5.3. STRICT DOMINANCE DEGREE

We define this degree as a real degree which acts on the alternatives of X^{LDD},

$$SDD(x_i) = Q\left(\frac{r_i}{n-1}\right),$$

where

$$r_i = card\{p_{iq} \in S \;/\; p_{iq} > p_{qi}\}.$$

Thus, we obtain the final solution to the selection process as the following set of alternatives,

$$X^{SDD} = \left\{x \in X^{LDD} \;/\; SDD(x) = \operatorname*{Max}_{y \in X^{LLD}}[SDD(y)]\right\}.$$

REMARK. The last two dominance degrees use the concept of fuzzy majority in their definition represented by a fuzzy linguistic quantifier.

6. EXAMPLES

EXAMPLE 1. Consider the above nine linguistic term set (Figure 2) and an associated semantic, and four individuals whose linguistic preferences using the above nine element term set are

$$P^1 = \begin{bmatrix} - & SC & MC & VLC \\ MC & - & IM & IM \\ SC & IM & - & VLC \\ ML & IM & ML & - \end{bmatrix} P^2 = \begin{bmatrix} - & IM & IM & VLC \\ IM & - & MC & IM \\ IM & SC & - & VLC \\ ML & IM & ML & - \end{bmatrix}$$

$$P^3 = \begin{bmatrix} - & IM & MC & I \\ IM & - & ML & IM \\ SC & VLC & - & VLC \\ C & IM & ML & - \end{bmatrix} P^4 = \begin{bmatrix} - & IM & MC & SC \\ IM & - & IM & SC \\ SC & IM & - & VLC \\ MC & MC & ML & - \end{bmatrix}.$$

Using the linguistic quantifier "At least half" with the pair $(0.0, 0.5)$, and the corresponding LOWA operator with $W = (0.5, 0.5, 0, 0)$, the collective linguistic preference is

$$P = \begin{bmatrix} - & IM & MC & VLC \\ IM & - & MC & IM \\ SC & IM & - & VLC \\ EL & IM & ML & - \end{bmatrix}.$$

Next, we apply the sequential selection process: the linguistic strict preference relation is

$$P^s = \begin{bmatrix} - & I & VLC & I \\ I & - & EU & I \\ I & I & - & I \\ MC & I & IM & - \end{bmatrix}$$

and the linguistic nondominance degree

$$\mu_{ND}(x_1, x_2, x_3, x_4) = [SC, C, IM, C].$$

The set of maximal nondominated alternatives, X^{ND}, is

$$X^{ND} = \{x_2, x_4\}.$$

As we have obtained a nondominated alternative set with more than one element, then we apply the linguistic dominance degree on the alternatives of X^{ND}. To do so, we use the same linguistic quantifier, obtaining

$$(LDD(x_2), LDD(x_4)) = [ML, EL].$$

Therefore,

$$X^{LDD} = \{x_4\}.$$

Since X^{LDD} has only one alternative, this is the best alternative for the group selection process.

EXAMPLE 2. Consider the following 13 linguistic term set with an associated semantic, [1]:

C	*Certain*	$(1, 1, 0, 0)$
EL	*Extremely_Likely*	$(0.98, 0.99, 0.05, 0.01)$
VHC	*Very_High_Chance*	$(0.87, 0.96, 0.04, 0.03)$
ML	*Most_Likely*	$(0.78, 0.92, 0.06, 0.05)$
HC	*High_Chance*	$(0.75, 0.87, 0.04, 0.04)$
MC	*Meaningful_Chance*	$(0.63, 0.80, 0.05, 0.06)$
L	*Likely*	$(0.53, 0.69, 0.09, 0.12)$
IM	*It_May*	$(0.41, 0.58, 0.09, 0.07)$
SC	*Small_Chance*	$(0.22, 0.36, 0.05, 0.06)$
VLC	*Very_Low_Chance*	$(0.1, 0.18, 0.06, 0.05)$
NL	*Not_Likely*	$(0.05, 0.15, 0.03, 0.03)$
EU	*Extremely_Unlikely*	$(0.01, 0.02, 0.01, 0.05)$
I	*Impossible*	$(0, 0, 0, 0)$

and shown graphically in Figure 5.

Consider the same four individuals whose linguistic preferences, using the above thirteen element term, are

$$P^1 = \begin{bmatrix} - & SC & HC & NL \\ HC & - & MC & MC \\ SC & IM & - & VLC \\ VHC & IM & ML & - \end{bmatrix} \quad P^2 = \begin{bmatrix} - & IM & MC & VLC \\ MC & - & HC & IM \\ IM & SC & - & NL \\ ML & MC & VHC & - \end{bmatrix}$$

$$P^3 = \begin{bmatrix} - & IM & HC & I \\ MC & - & ML & IM \\ SC & VLC & - & VLC \\ C & MC & ML & - \end{bmatrix} \quad P^4 = \begin{bmatrix} - & IM & HC & SC \\ MC & - & IM & SC \\ SC & MC & - & NL \\ HC & HC & VHC & - \end{bmatrix}.$$

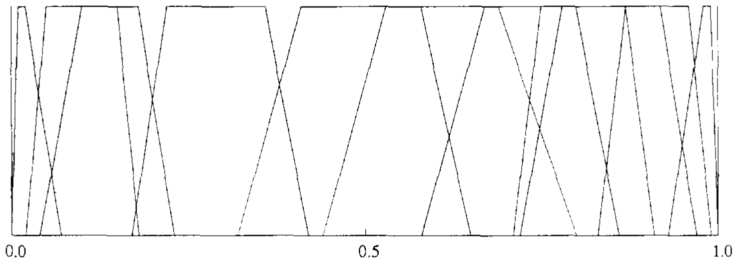

Fig. 5. Distribution of linguistic term set.

Note that these preferences represent the same problem as before (Example 1), but with a finest granularity in the linguistic term set.

As in the earlier example, the linguistic quantifier "At least half" and the corresponding LOWA operator are used in order to obtain the collective linguistic preference:

$$P = \begin{bmatrix} - & IM & HC & VLC \\ MC & - & HC & L \\ SC & L & - & VLC \\ EL & MC & VHC & - \end{bmatrix}.$$

The linguistic strict preference relation is

$$P^s = \begin{bmatrix} - & I & SC & I \\ NL & - & NL & I \\ I & I & - & I \\ HC & EU & MC & - \end{bmatrix}$$

and the linguistic nondominance degree is

$$\mu_{ND}(x_1, x_2, x_3, x_4) = [SC, EL, IM, C].$$

The set of maximal nondominated alternatives is

$$X^{ND} = \{x_4\}.$$

In the first step, the sequential selection process has found the best alternative, i.e., x_4. This can be explained by the finest granularity of the second term set.

7. CONCLUSIONS

In this paper, we have presented a sequential decision process in group decision making where the preferences of the individuals are represented by linguistic preference relations. The sequential decision process has been based on the concepts of *fuzzy majority, fuzzy linguistic quantifiers, linguistic ordered weighted averaging operators, nondominance, and dominance degree.*

We have developed one of the two possibilities in a decision schema, the group selection process. The second possibility, a consensus process under a linguistic assessment approach, ought to be developed in order to obtain a linguistic consensus degree and to cooperate in the solution of the group decision problem. This is a problem to be discussed in future work.

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