Representation Models for Aggregating Linguistic Information: Issues and Analysis

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Abstract. The linguistic information has been used successfully in many areas. The aggregation of linguistic information is a crucial aspect. In the literature we can find different linguistic computational models that present linguistic aggregation operators as: (i) The computational model based on the Extension Principle, which operates over the fuzzy numbers that supports the semantics of the linguistic labels. (ii) The symbolic one makes the computations directly over the order index of the linguistic labels. And, (iii) the model based on the linguistic 2-tuple representation, which uses the symbolic translation to make the linguistic computations.

Depending upon the linguistic aggregation model, different results can be obtained. In this chapter we shall make a comparative analysis of the aggregation approaches according to the results obtained in a decision-making application.

Keywords: linguistic variables, linguistic aggregation.

1 Introduction

Usually, the most of the problems we solve deal with quantitative aspects. These are easily assessed by means of precise numerical values that specify in a simply manner the importance or preference of these aspects in the problem. However, in practice we find a lot of these models are difficult to apply because they fail to assess the aspects which describe the problem under consideration. This is due to the fact that these problems deal with qualitative aspects that are complex to assess by means of precise values. In order to overcome this drawback the use of the fuzzy linguistic approach [26] has provided very good results in different areas, as information retrieval [3,25], economics [13,23], planning [1], consensus reaching processes [4,12], decision-making [11,22], etc.

We focus our study in those linguistic problems that deal with multiple criteria or experts manage linguistic preferences and are solved following a common resolution scheme with the two following processes [21]:

- *Aggregation process.* The individual linguistic preference values are combined to obtain collective preference values.

- *Exploitation process.* The collective preference values are analysed to obtain the best alternative/s for the problem.

This resolution scheme implies processes of computing with words, that is, processes of aggregation of linguistic information. In the literature we can find different linguistic computational models to accomplish these processes:

- *The approximative computational model based on the Extension Principle* [2,6]. This model uses fuzzy arithmetic based on the Extension Principle to make computations over the linguistic variables. This model can present the results in two ways:
  1. By means of the fuzzy numbers obtained from the fuzzy arithmetic computations based on the Extension Principle.
  2. Or by means of linguistic labels computed from the fuzzy numbers obtained using a linguistic approximation process.
- *The ordinal linguistic computational model [7].* This symbolic model makes direct computations on labels, using the ordinal structure of the linguistic term sets. Its results are inherently linguistic labels due to either the operators used, basicaly max and min operators [24] or because in the computations on the order index there exist an approximation by means of the *round operator* [10].
- *The 2-tuple linguistic computational model.* It uses the 2-tuple linguistic representation model and its characteristics to make linguistic computations [14,17], obtaining as results linguistic 2-tuples. A linguistic 2-tuple is a pair of values, where the first one is a linguistic label and the second one is a real number that represents the value of the symbolic translation, that it is the basic concept of the linguistic 2-tuple representation model that will be introduced in the following section.

The aim of this contribution is to make a comparative analysis among the linguistic computational models for the aggregation of linguistic information from different points of view. To do so, we shall solve a simple decision-making problem and compare the results obtained by each model. This study will help us to decide what linguistic computational model is most adequated to make computations over linguistic information.

This contribution is structured as follows: in Section 2 we shall make a brief review of the fuzzy linguistic approach and introduce a simple decision-making problem. In Section 3 we shall review the different linguistic computational models and solve the above decision-making problem with each one. In Section 4 a comparative study of the different linguistic computational models is presented. Finally some concluding remarks are pointed out.

2 Preliminaries

In this section we review the fuzzy linguistic approach and present a decision-making problem that will be solved along this contribution using different linguistic methods.

2.1 Fuzzy Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [26]. This approach is adequated in some situations, for example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language. This may arise for different reasons. There are some situations where the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car, terms like "bad", "poor", "tolerable", "average", "good" can be used [19]). In other cases, precise quantitative information may not be stated because either it is not available or the cost of its computation is too high, then an "approximate value" may be tolerated (e.g., when evaluating the speed of a car, linguistic terms like "fast", "very fast", "slow" are used instead of numerical values).

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In order to accomplish this objective, an important aspect to analyze is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9 [20], where the mid term represents an assessment of "approximately 0.5", and with the rest of the terms being placed symmetrically around it [2].

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [24]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: Neg(s_i) $= s_j$ such that $j = g$-i (g+1 is the cardinality of S).
2. $s_i \leq s_j \iff i \leq j$. Therefore, there exists a minimization and a maximization operator.

The semantics of the linguistic terms is given by fuzzy numbers defined in the [0,1] interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [2]. The linguistic assessments given by the users are just approximate ones, some authors [2,6,8,11] consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). An example may be the following (Figure 1) :

$$N = (0, 0, .25) \;\; VL = (0, .2, .4) \;\; L = (.25, .4, .50)$$
$$M = (.4, .5, .6) \;\; H = (.5, .6, .75) \;\; VH = (.6, .8, 1)$$
$$P = (.75, 1, 1).$$

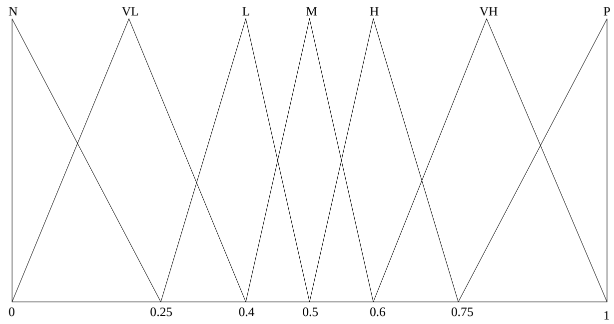

Fig. 1. A set of seven linguistic terms with its semantics

Other authors use a non-trapezoidal representation, e.g., Gaussian functions [3].

2.2 A Decision-Making problem

Here, we propose a simple decision-making problem to be solved with different linguistic computational models following the resolution scheme presented in the introduction.

- **Description**

  A distribution company needs to renew its computing system, so it contracts a consulting company to carry out a survey of the different possibilities existing on the market, to decide which is the best option for its needs. The alternatives are the following:

x_1	x_2	x_3	x_4
LINUX	WINDOWS-2000	VMS	SOLARIS

The consulting company has a group of four consultancy departments

p_1	p_2	p_3	p_4
Cost analysis	System analysis	Risk analysis	Techonology analysis

- **Initialization**

  Each department provides a performance vector expressing its preferences for each alternative. The preferences are assessed in the term set $S = \{N, VL, L, M, H, VH, P\}$ (see Figure 1).

		alternatives			
	L_{ij}	x_1	x_2	x_3	x_4
experts	p_1	L	M	M	M
	p_2	L	L	VL	M
	p_3	VL	VL	M	M
	p_4	H	VL	L	M

  whose membership functions, L_{ij}, are assumed to be of the triangular type $L_{ij} = (a_{ij}, b_{ij}, c_{ij})$.

3 Linguistic Modelling in Decision-Making Problems

In this section we apply the different linguistic computational models to the linguistic decision problem presented just above, following the resolution scheme presented in the introduction.

3.1 Solution based on the Extension Principle

The Extension Principle is a basic concept in the fuzzy sets theory [8] which is used to generalize crisp mathematical concepts to fuzzy sets, i.e., it allows non-fuzzy function to be fuzzified in the sense that if the function arguments are made into fuzzy sets, then the function value is also a fuzzy set. One of the several formulations of this concept is the following [26]:

Definition 1. Let X be the cartesian product of the universes $X_1, ..., X_r$ and $\tilde{A}_1, ..., \tilde{A}_r$ be fuzzy sets in $X_1, ..., X_r$, respectively. Let f be a function defined from X, $(X = X_1 \times ... \times X_r)$ to Y, $y = f(x_1, ..., x_r)$. The Extension Principle allows us to define a fuzzy set $\tilde{B}$ in Y, from the fuzzy sets $\tilde{A}_1, ..., \tilde{A}_r$ representing its value from f, according to the following expression:

$$\tilde{B} = \{(y, \mu_{\tilde{B}}(y))/y = f(x_1, ..., x_r), (x_1, ..., x_r) \in X\}$$

where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup_{(x_1,...,x_r) \in f^{-1}(y)} \min\{\mu_{\tilde{A}_1}(x_1), ..., \mu_{\tilde{A}_r}(x_r)\}, & if \;\; f^{-1}(y) \neq \emptyset \\ 0, & otherwise. \end{cases}$$

For $r=1$, the extesion principle is:

$$\tilde{B} = f(A) = \{(y, \mu_{\tilde{B}}(y))/y = f(x), x \in X)$$

where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_{\tilde{A}}(x), & if \;\; f^{-1}(y) \neq \emptyset \\ 0, & otherwise. \end{cases}$$

The resolution scheme for linguistic problems, presented in the introduction, is divided in two processes: *aggregation* and *exploitation*. Depending on the process, this computational model can deal with information from different nature.

- *Aggregation process.* The approximative model based on the Extension Principle uses the membership functions of the linguistic terms, μ_{s_i} where $s_i \in S$, for computing collective (aggregated) values that are expressed by means of fuzzy numbers. A formal scheme can be:

$$S^n \xrightarrow{\tilde{F}} F(\mathcal{R})$$

  where S^n symbolizes the n cartesian product of S, $\tilde{F}$ is an aggregation operator based on the Extension Principle, and $F(\mathcal{R})$ the set of fuzzy sets over the Real line $\mathcal{R}$.
  Now, we apply this model to the decision-making problem. There exist a lot of aggregation operators, an usual one is the mean operator, that will be used in this paper.

$$L_j = \tilde{x}(L_{ij}) = (\frac{1}{m}\sum_{i=1}^{m} a_{ij}, \frac{1}{m}\sum_{i=1}^{m} b_{ij}, \frac{1}{m}\sum_{i=1}^{m} c_{ij})$$

  being m the number of experts, $\tilde{x}$ the fuzzy mean operator based on the extension principle and L_j the collective value for x_j. We obtain the following collective preference vector:

L_1	L_2	L_3	L_4
(.25,.4,.53)	(.16,.32,.47)	(.26,.4,.52)	(.4,.5,.6)

- *Exploitation process.* The objective of this process is to order the collective values obtained in the aggregation process. To do so, this computational model can act on two different ways:
  1. *Ranking fuzzy numbers obtained on the aggregation process.* Therefore a fuzzy ranking function must be applied to them, obtaining an alternatives order according to the real numbers computed by the fuzzy ranking:

$$F(\mathcal{R}) \xrightarrow{\tilde{O}} \mathcal{R}$$

In the specialized literature we can find different fuzzy ranking functions [27], in this paper we shall use the following two different ones.

(a) The González fuzzy ranking [9]. Let F_j (j=1,...,m) be m fuzzy numbers whose membership functions are $F_j = (a_j, b_j, c_j)$. Define the following fuzzy ranking function:

$$V^{\alpha}(F_j) = \alpha(c + \frac{b}{2}) + (1 - \alpha)(c - \frac{a}{2})$$

The value α is an index of rating attitude.

Applying this fuzzy ranking to the collective fuzzy numbers with $\alpha = 0.1$ we obtain the following ranking values:

$V^{0.1}(L_1)$	$V^{0.1}(L_2)$	$V^{0.1}(L_3)$	$V^{0.1}(L_4)$
.442	.414	.423	.445

Therefore, the best alternative for the problem according to this fuzzy ranking is $\{x_4\}$, i.e., the *Solaris based System*.

(b) Another fuzzy ranking was presented by Chang in [5], where:

$$Ch(\tilde{F}_j) = \frac{1}{6}(c_i - a_i)(a_i + b_i + c_i)$$

This fuzzy ranking obtains the following ranking for the collective values:

$Ch(L_1)$	$Ch(L_2)$	$Ch(L_3)$	$Ch(L_4)$
.055	.049	.051	.05

The solution set of alternatives in this case is $\{x_1\}$, i.e., *the Linux based system*

2. *Applying a linguistic approximation process to the fuzzy numbers obtained in the aggregation process.* This approximation process obtains the linguistic term closest to the unlabelled fuzzy number in the initial term set. Once the aggregated values are expressed by means of linguistic terms these have an inner order defined by the structure of the linguistic term set:

$$F(\mathcal{R}) \xrightarrow{app_1(\cdot)} S$$

Different linguistic approximation functions, $app_1(\cdot)$, can be used. In this paper we shall use the following one. Let $F_j = (a_j, b_j, c_j)$, $(j = 1, ..., m)$ be the collective fuzzy numbers obtained in the aggregation process the linguistic approximation process, $app_1(\cdot)$, comes back the closest linguistic term, $s_l \in S$, to the unlabelled fuzzy number, F_j.

$$d(s_l, F_j) = \sqrt{P_1(a_l - a_j)^2 + P_2(b_l - b_j)^2 + P_3(c_l - c_j)^2}$$

being (a_l, b_l, c_l) and (a_j, b_j, c_j) the membership functions of "s_l" and "F_j" respectively, with P_1, P_2, P_3 being the weights that represent the importance of a, b *and* c. Therefore, $app_1(\cdot)$ chooses s_l^* ($app_1(F_j) = s_l^*$), such that, $d(s_l^*, F_j) \leq d(s_l, F_j) \;\; \forall s_l \in S$, where S is the linguistic term set used as expression domain.
Applying this linguistic approximation process to the collective vector, we shall obtain a collective linguistic vector that is easy to order and thus to choose the best alternative/s.
This linguistic approximation process is applied to the collective values, with $P_1 = 0.2$, $P_2 = 0.6$, $P_3 = 0.2$. We select these values because of the parameter "b_i" is the most representative of the membership function and "a_i", "c_i" are equally representative.
The collective preference vector obtained after the linguistic approximation is:

$app_1(L_1)$	$app_1(L_2)$	$app_1(L_3)$	$app_1(L_4)$
M	L	L	M

The exploitation process applies a choice degree to the collective performance vector for obtaining the solution set of alternatives. In this case we obtain as solution set:

$$\{x_1, x_4\}$$

that is a solution set with multiple alternatives.

3.2 Solution based on the Linguistic Computational Symbolic Model

Here, we shall apply the same decision process for solving the problem, but in this case we shall deal with the symbolic approach presented in [7].

- *Aggregation process.* The symbolic operator we shall use to aggregate linguistic variables is the Convex Combination [7]:

  Definition 2.[7]*Let $A = \{a_1, ..., a_m\}$ be a set of linguistic terms to be aggregated, the Convex Combination is defined in a recursive way as:*

  For m=2:

  $$C^2\{\{w_1, 1-w_1\}, \{b_1, b_2\}\} = w_1 \odot s_j \oplus (1-w_1) \odot s_i = s_k, \;\; s_j, \; s_i \in \; S, \; (j \geq i)$$

  such that,

  $$k \;=\; \min\{g, i \;+\; round(w_1 \cdot (j-i))\},$$

where $g+1$ *is the cardinality of* S*,* $round(\cdot)$ *is the usual round operation, and* $b_1 = s_j,\ b_2 = s_i$.

For m > 2:

$C^m\{w_k, b_k, k = 1, ..., m\} = w_1 \odot b_1 \oplus (1 - w_1) \odot C^{m-1}\{\eta_h, b_h, h = 2, \ldots, m\} = C^2\{\{w_1, 1 - w_1\}, \{b_1, C^{m-1}\{\eta_h, b_h, h = 2, \ldots, m\}\}\}$

where $W = [w_1, \ldots, w_m]$ *is a weighting vector associated to* A*, such that, (i)* $w_i \in [0,1]$*, and (ii)* $\sum_i w_i = 1$*; and* $B = \{b_1, \ldots, b_m\}$ *is a vector, such that,* $B = \{a_{\sigma(1)}, \ldots, a_{\sigma(m)}\}$*, where,* $a_{\sigma(j)} \leq a_{\sigma(i)}\ \forall\ i \leq j$*, with* σ *being a permutation over the values* a_i*.* $\eta_h = w_h / \Sigma_2^m w_k, h = 2, \ldots, m$*. With* $\odot$ *and* $\oplus$ *being the product of a number by a label and the addition of two labels respectively.*

In our example the weighting vector is $\{.25, .25, .25, .25\}$, then the collective performance values obtained are:

L_1	L_2	L_3	L_4
M	M	L	M

- *Exploitation process.*The alternatives with the highest collective performance are:

$$\{x_1, x_2, x_4\}$$

Here again we find a multiple alternative solution set.

3.3 Solution based on linguistic 2-tuples

This model is based on the representation model presented in [14,17] and it is based on the concept of symbolic translation and uses it for representing the linguistic information by means of 2-tuples, (s_i, α), where s is a linguistic term and α is a numerical value representing the symbolic translation.

Let $S = \{s_0, ..., s_g\}$ be a linguistic term set, and $\beta \in [0, g]$ a numerical value in its interval of granularity (e.g.: let β be a value obtained from a symbolic aggregation operation [2,7]).

Definition 3. *Let* β *be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set* S*, i.e., the result of a symbolic aggregation operation.* $\beta \in [0, g]$*, being* $g+1$ *the cardinality of* S*. Let* $i = round(\beta)$ *and* $\alpha = \beta - i$ *be two values, such that,* $i \in [0, g]$ *and* $\alpha \in [-.5, .5)$ *then* α *is called a Symbolic Translation.*

Roughly speaking, the symbolic translation of a linguistic term, s_i*, is a numerical value assessed in* $[-.5, .5)$ *that supports the "difference of information" between a counting of information* $\beta \in [0, g]$ *obtained after a symbolic aggregation operation and the closest value in* $\{0, ..., g\}$ *that indicates the index of the closest linguistic term in* S $(i = round(\beta))$*.*

From this concept we shall develop a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-.5, .5)$:

- s_i represents the linguistic label of the information, and
- α_i is a numerical value expressing the value of the translation from the original result β to the closest index label, i, in the linguistic term set (s_i), i.e., the Symbolic Translation.

This model defines a set of transformation functions between linguistic terms and 2-tuples, and between numeric values and 2-tuples.

Definition 4. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \ with \ \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where $round(\cdot)$ is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof.
It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark: From definitions 2 and 3 and from proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \Longrightarrow (s_i, 0)$$

This representation model has a computational technique based on 2-tuples presented in [14,17], that it will be used in the aggregation and exploitation processes:

1. Aggregation of 2-tuples

The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. In [14] we can find some 2-tuple aggregation operators, that are based on classical aggregation operators.

2. Comparison of 2-tuples

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:

- if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2)
- if $k = l$ then
  1. if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information
  2. if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2)
  3. if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2)

Here, we shall use the linguistic representation model with 2-tuples to solve the decision making problem presented in Subsection 2.4. To do so, we follow the same selection model.

The performance vectors of the experts are transformed into 2-tuples:

		alternatives			
		x_1	x_2	x_3	x_4
experts	p_1	$(P, 0)$	$(M, 0)$	$(M, 0)$	$(M, 0)$
	p_2	$(L, 0)$	$(L, 0)$	$(VL, 0)$	$(M, 0)$
	p_3	$(VL, 0)$	$(VL, 0)$	$(M, 0)$	$(M, 0)$
	p_4	$(H, 0)$	$(H, 0)$	$(L, 0)$	$(M, 0)$

Now we apply the resolution scheme using linguistic 2-tuples.

1. *Aggregation process.* We aggregate these 2-tuples, using the 2-tuple arithmetic mean [14].

   Definition 6. *Let $x = \{(r_1, \alpha_1), \ldots, (r_m, \alpha_m)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean $\overline{x}^e$ is computed as,*

$$\overline{x}^e = \Delta(\sum_{i=1}^{m} \frac{1}{m} \Delta^{-1}(r_i, \alpha_i)) = \Delta(\frac{1}{m} \sum_{i=1}^{m} \beta_i)$$

   Obtaining the following collective values:

$\overline{x}_1^e(L_1)$	$\overline{x}_2^e(L_2)$	$\overline{x}_3^e(L_3)$	$\overline{x}_4^e(L_4)$
(M,.25)	(M,-.5)	(L,.25)	(M,0)

2. *Exploitation process.* We obtain as solution set of alternatives:

$$\{x_1\}.$$

4 Analysis of the linguistic computational models

Here, we shall analyse the linguistic computational models from different points of view studying the results obtained in the above decision-making problem and which can be observed in the following table:

	Ranking Fuzzy Numbers		Ling. Approx.	Symbolic Model	2-tuples
	González	Chang	S	S	S
x_1	.442	**.055**	**M**	**M**	**(M,.25)**
x_2	.414	.049	*L*	**M**	(M,-.5)
x_3	.423	.051	*L*	L	(L,.25)
x_4	**.445**	.050	**M**	**M**	(M,.0)

Table I. Results

The bold typed values in each column represent the solution set of alternatives correspondent to each model.

The different points of view we shall use in this comparative analysis are:

1. *Linguistic description.* This point of view analyses the expression domain used to express the results. This criterion focus in the closeness of the results to the initial linguistic expression domain.
2. *Consistency.* This criterion measures if from the same inputs the linguistic computational models obtaining the same final results.
3. *Accuracy.* This point of view studies the precision of the linguistic operations and the granularity of uncertainty of the results obtained by the different linguistic computational models.

We have chosen these points of view for analysing the different linguistic computational models becuase, they are critical criteria to decide if the reached solution is good and easy to understand by the users that want to solve a problem by means of linguistic information.

To carry out this analysis we shall make a table in which we describe the score of each linguistic computational model according to the different points of view. This score is expressed in the following linguistic term set $S = \{LOW, MEDIUM, HIGH\}$.

	Approx. model (Fuzzy Numbers)	*Approx. model (Labels)*	*Ordinal model*	*2-tuple model*
Ling. Descrip.	LOW	HIGH	HIGH	HIGH
Consistency	MEDIUM	HIGH	HIGH	HIGH
Accuracy	HIGH	LOW	LOW	HIGH

Table II. Analysis of the linguistic computational models

Now we explain the results of the Table II by models (columns):

- *Linguistic approximative model with fuzzy numbers.* This model has a LOW score in the linguistic description because its results are expressed by means of numerical values, a value MEDIUM for consistency due to although it is not usual sometimes it is possible that with the same inputs obtain different results due to the fact that fuzzy numbers have not an inner order, and must be ordered by fuzzy ranking functions. Finally, it has a HIGH value for accuracy because it does not lose information in its computation processes.
- *Linguistic approximative model with linguistic labels.* In this case, the linguistic description is HIGH because the results are expressed in the initial linguistic domain, the consistency has an score of HIGH because from the same input always obtains the same results, while the accuracy is LOW due to this model loses information in the linguistic approximation processes.
- *Ordinal model.* Its linguistic description is HIGH because it expresses the results in the initial linguistic term set and also it has a HIGH value for consistency due to from the same input always obtains the same results, while its accuracy is LOW because its computation processes have loss of information.
- *2-tuple model.* This model has all the scores HIGH because its results are expressed in the initial linguistic domain, from the same inputs always obtains the same results and its computations have not loss of information.

5 Concluding Remarks

The above analysis shows that the 2-tuple computational model is always so good as the other linguistic computational models. In linguistic description, accuracy and consistency to be used as the base of representation for expressing aggregation operators. Therefore, we can say the 2-tuple linguistic computational model is the most adequated for dealing with linguistic information in processes of computing with words.

We have just seen that the linguistic 2-tuple model is very adequated for computing with words processes, besides this model provides tools for dealing with multigranularity linguistic contexts [15], linguistic and numerical contexts [16] easily.

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