

Preserving Reciprocity in the Aggregation of Fuzzy Preference Relations Using OWA Operators¹

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Abstract

In multiperson decision making (MPDM) problems, fuzzy preference relations are widely used to represent experts opinions on the set of alternatives. Fuzzy preference relations are usually assumed to be additive reciprocal. However, it is well known that reciprocity is not generally preserved after aggregation is carried out.

In this paper, we study conditions under which reciprocity property is maintained when aggregating additive reciprocal fuzzy preference relations using an OWA operator guided by a relative linguistic quantifier.

1 Introduction

We assume multiperson decision making (MPDM) problems [3] being the experts' preferences about the alternatives represented by means of the fuzzy preference relations which are additive reciprocal [6].

Usually, the solution set of alternatives is achieved in two phases [5]: *aggregation phase* and *exploitation phase*. The aggregation phase leads us to the use of an aggregation operator for getting a collective preference relation. *The OWA operator* [7] guided by *fuzzy majority* is a usual aggregation

procedure to combine the experts' fuzzy preference relations [1,4]. In the OWA operator the concept of fuzzy majority is incorporated by means of a *relative linguistic quantifier* [8,9] (e.g., such as "most of", "at least half", "as many as possible") used to compute the weighting vector [7]. In such a way, the solution set of alternatives is obtained according to a majority of experts.

The problem is that reciprocity property is not generally preserved after aggregation is carried out. Therefore, although the set of individual fuzzy preference relations are supposed to be additive reciprocal, this does not imply that the collective fuzzy preference relation is additive reciprocal.

In this contribution, we study conditions under which reciprocity is maintained when using an OWA operator guided by a relative linguistic quantifier in the aggregation phase.

The paper is organised as follows. In Section 2, we present the problem. In Section 3, we study reciprocity conditions and also give a few examples to illustrate everything. Finally, some conclusions are pointed out in Section 4.

2 Presentation of the problem

We have a set of alternatives $X = \{x_1, \dots, x_n\}$, a set of experts $E = \{e_1, \dots, e_m\}$, and a set of fuzzy preference relations $\{P^1, \dots, P^m\}$, where $P^k = (p_{ij}^k)$, and p_{ij}^k represents the preference

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degree or intensity of alternative x_i over alternative x_j for expert e_k . Fuzzy preferences are usually assumed to be additive reciprocal, i.e., $p_{ij}^k + p_{ji}^k = 1, \forall i, j, k$.

As we have said, using an OWA operator ϕ_Q guided by a linguistic quantifier Q , we derive a collective preference relation, $P^c = (p_{ij}^c)$, that indicates the global preference between every pair of alternatives according to the majority of experts' opinions, which is represented by Q . In this case,

$$p_{ij}^c = \phi_Q(p_{ij}^1, \dots, p_{ij}^m) = \sum_{k=1}^m w_k q_{ij}^k$$

where q_{ij}^k is the k -th largest value in the set $\{p_{ij}^1, \dots, p_{ij}^m\}$, Q is a relative non decreasing quantifier with membership function

$$Q(x) = \begin{cases} 0 & 0 \leq x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b < x \leq 1 \end{cases}$$

$$a, b \in [0, 1], \text{ and } w_k = Q\left(\frac{k}{m}\right) - Q\left(\frac{k-1}{m}\right), \forall k.$$

Following this methodology, the first thing we have to do is to choose the suitable relative quantifier for representing the concept of fuzzy majority that we desire to implement in our MPDM problem, what reduces to choose adequate values for parameters a and b , computing afterwards the weights of the OWA operator using the above relation. Our objective in this paper is to give values of parameters a and b that maintain reciprocity property.

3 Reciprocity of collective preference relation

The problem to solve is: What condition do parameters a and b have to verify so that $p_{ij}^c + p_{ji}^c = 1, \forall i, j$?

As we are assuming P^k additive reciprocal then $p_{ji}^k = 1 - p_{ij}^k$, and therefore if $\{q_{ij}^1, \dots, q_{ij}^m\}$ are ordered from largest to lowest, $\{q_{ji}^1, \dots, q_{ji}^m\}$, being

$q_{ji}^k = 1 - q_{ij}^k$, are ordered from lowest to largest, and in consequence we have:

$$\begin{aligned} p_{ij}^c + p_{ji}^c &= \sum_{k=1}^m w_k q_{ij}^k + \sum_{k=1}^m w_{m-k+1} q_{ji}^k = \sum_{k=1}^m w_k q_{ij}^k \\ &+ \sum_{k=1}^m w_{m-k+1} (1 - q_{ij}^k) = 1 + \sum_{k=1}^m (w_k - w_{m-k+1}) q_{ij}^k \\ &= 1 + \sum_{k=1}^m \bar{w}_k q_{ij}^k \end{aligned}$$

where

$$\bar{w}_k = \left[Q\left(\frac{k}{m}\right) - Q\left(\frac{k-1}{m}\right) \right] - \left[Q\left(\frac{m-k+1}{m}\right) - Q\left(\frac{m-k}{m}\right) \right].$$

If we denote $A(k) = Q\left(\frac{k}{m}\right) + Q\left(1 - \frac{k}{m}\right)$ then $\bar{w}_k = A(k) - A(k-1)$.

We distinguish three possible cases, according to the values of $a + b$: (A) $a + b = 1$, (B) $a + b < 1$, (C) $a + b > 1$.

CASE A: $a + b = 1$

In this case $1 - a = b$, $1 - b = a$ and we have:

$$\begin{aligned} Q(1-x) &= \begin{cases} 0 & 0 \leq 1-x < a \\ \frac{1-x-a}{b-a} & a \leq 1-x \leq b \\ 1 & b < 1-x \leq 1 \end{cases} \\ &= \begin{cases} 0 & b < x \leq 1 \\ \frac{b+a-x-a}{b-a} & a \leq x \leq b \\ 1 & 0 \leq x < a \end{cases} \end{aligned}$$

$$= \begin{cases} 1-0 & 0 \leq x < a \\ 1 - \frac{x-a}{b-a} & a \leq x \leq b \\ 1-1 & b < x \leq 1 \end{cases} = 1 - Q(x).$$

This implies that

$A(k) = Q\left(\frac{k}{m}\right) + Q\left(1 - \frac{k}{m}\right) = Q\left(\frac{k}{m}\right) + 1 - Q\left(\frac{k}{m}\right) = 1, \forall k$, and $\bar{w}_k = A(k) - A(k-1) = 0, \forall k$, and therefore $p_{ij}^c + p_{ji}^c = 1, \forall i, j$.

Summarising, we have stated the following results:

Proposition 1. If Q is a linguistic quantifier with membership function verifying

$$Q(1-x) = 1 - Q(x), \forall x,$$

then the collective fuzzy preference relation, obtained by aggregating a set of additive reciprocal fuzzy preference relations, using an OWA operator guided by Q , is additive reciprocal.

Proposition 2. If Q is a relative non decreasing linguistic quantifier with parameters a and b verifying $a+b=1$, then the OWA operator guided by Q preserves additive reciprocity.

Example 1. Suppose that we have a set of four alternatives and a set of six experts that provide their opinion using the following additive reciprocal fuzzy preference relations:

$$P^1 = \begin{pmatrix} 0.5 & 0.17 & 0.67 & 0.5 \\ 0.83 & 0.5 & 1 & 0.67 \\ 0.33 & 0 & 0.5 & 0.17 \\ 0.5 & 0.33 & 0.83 & 0.5 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.5 & 0.38 & 0.58 & 0.84 \\ 0.62 & 0.5 & 0.69 & 0.9 \\ 0.42 & 0.31 & 0.5 & 0.8 \\ 0.16 & 0.1 & 0.2 & 0.5 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.2 & 0.5 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.5 & 0.33 & 0.17 & 0.67 \\ 0.67 & 0.5 & 0.33 & 0.17 \\ 0.83 & 0.67 & 0.5 & 1 \\ 0.33 & 0.83 & 0 & 0.5 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.5 & 0.34 & 0.2 & 0.96 \\ 0.66 & 0.5 & 0.33 & 0.98 \\ 0.8 & 0.67 & 0.5 & 0.99 \\ 0.04 & 0.02 & 0.01 & 0.5 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.5 & 0.5 & 0.7 & 1 \\ 0.5 & 0.5 & 0.8 & 0.6 \\ 0.3 & 0.2 & 0.5 & 0.8 \\ 0 & 0.4 & 0.2 & 0.5 \end{pmatrix}$$

Using the linguistic quantifier with the pair of values $(0.25, 0.75)$ and the corresponding OWA operator with weight vector $(0, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}, 0)$, the collective preference relation is:

$$P^c = \begin{pmatrix} 0.5 & 0.315 & 0.538 & 0.785 \\ 0.685 & 0.5 & 0.685 & 0.64 \\ 0.462 & 0.315 & 0.5 & 0.865 \\ 0.215 & 0.36 & 0.135 & 0.5 \end{pmatrix}$$

CASE B: $a+b < 1$

In this case, we have that $1-a > b$, $1-b > a$ and as a consequence of being $a \leq b$ we have $a < 1/2$. We can assume for now that $b \geq 1/2$, what implies that $1-b \leq b$, letting for later the other case $b < 1/2$.

CASE B1. $b \geq 1/2$

Now we have that $0 \leq a < 1-b \leq b < 1-a \leq 1$, and consequently

$$Q(x) = \begin{cases} 0 & 0 \leq x < a \\ \frac{x-a}{b-a} & a \leq x < 1-b \\ \frac{x-a}{b-a} & 1-b \leq x < b \\ 1 & b \leq x < 1-a \\ 1 & 1-a \leq x \leq 1 \end{cases}$$

$$Q(1-x) = \begin{cases} 1 & 0 \leq x < a \\ 1 & a \leq x < 1-b \\ \frac{1-x-a}{b-a} & 1-b \leq x < b \\ \frac{1-x-a}{b-a} & b \leq x < 1-a \\ 0 & 1-a \leq x \leq 1 \end{cases}$$

with $x \in [0, 1]$ and

$$A(y) = \begin{cases} 1 & 0 \leq y < ma \\ \frac{y+m(b-2a)}{m(b-a)} & ma \leq y < m(1-b) \\ \frac{1-2a}{b-a} & m(1-b) \leq y < mb \\ \frac{m-y-m(b-2a)}{m(b-a)} & mb \leq y < m(1-a) \\ 1 & m(1-a) \leq y \leq m \end{cases}$$

with $y \in [0, m]$. It is clear that there exist $h_1, h_2, h_3, h_4 \in \{1, \dots, m\}$ such that

$$h_1 - 1 < ma \leq h_1, \quad h_2 - 1 < m(1-b) \leq h_2, \\ h_3 - 1 < mb \leq h_3, \quad h_4 - 1 < m(1-a) \leq h_4,$$

and in consequence:

$$\begin{aligned}
A(0) &= \dots = A(h_1 - 1) = 1 \\
A(k) &= \frac{k + m(b - 2a)}{m(b - a)}, k = h_1, \dots, h_2 - 1 \\
A(j) &= \frac{1 - 2a}{b - a}, j = h_2, \dots, h_3 - 1 \\
A(l) &= \frac{m - l - m(b - 2a)}{m(b - a)}, l = h_3, \dots, h_4 - 1 \\
A(h_4) &= \dots = A(m) = 1
\end{aligned}$$

Moreover, it is clear that $m - h_4 = h_1 - 1$, $m - h_3 = h_2 - 1$, so:

$$\begin{aligned}
\bar{w}_1 &= \dots = \bar{w}_{h_1-1} = 0, \bar{w}_{h_1} = \frac{h_1 - ma}{m(b-a)}, \bar{w}_{h_1+1} = \dots \\
&= \bar{w}_{h_2-1} = \frac{1}{m(b-a)}, \bar{w}_{h_2} = \frac{h_2 - mb}{m(b-a)}, \bar{w}_{h_2+1} = \dots \\
&= \bar{w}_{h_3-1} = 0, \bar{w}_{h_3} = \frac{mb - h_3}{m(b-a)}, \bar{w}_{h_3+1} = \dots = \\
&\bar{w}_{h_4-1} = \frac{-1}{m(b-a)}, \bar{w}_{h_4} = \frac{ma - h_4}{m(b-a)}, \\
&\bar{w}_{h_4+1} = \dots = \bar{w}_m = 0.
\end{aligned}$$

The expression for $p_{ij}^c + p_{ji}^c$ reduces to:

$$\begin{aligned}
p_{ij}^c + p_{ji}^c &= 1 + \bar{w}_{h_i} (q_{ij}^{h_i} + q_{ij}^{h_i}) + \\
&\sum_{k=h_i+1}^{h_2-1} \frac{1}{m(b-a)} (q_{ij}^k + q_{ij}^{m-k+1}) \\
&+ \bar{w}_{h_2} (q_{ij}^{h_2} + q_{ij}^{h_2}) \geq 1, \forall i, j.
\end{aligned}$$

Example 2. Suppose again the same set of additive reciprocal preference relations that in example 1. Using the linguistic quantifier "at least half" with the pair of values (0,0.5) and the corresponding OWA operator with weight vector $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0)$, then the collective preference relation is :

$$P^c = \begin{pmatrix} 0.5 & 0.4 & 0.66 & 0.94 \\ 0.8 & 0.5 & 0.87 & 0.85 \\ 0.69 & 0.55 & 0.5 & 0.96 \\ 0.38 & 0.61 & 0.41 & 0.5 \end{pmatrix}$$

CASE B2. $b < 1/2$

In this case we have that $0 \leq a \leq b < 1 - b \leq 1 - a \leq 1$, and therefore

$$Q(x) = \begin{cases} 0 & 0 \leq x < a \\ \frac{x-a}{b-a} & a \leq x < b \\ 1 & b \leq x < 1-b \\ 1 & 1-b \leq x < 1-a \\ 1 & 1-a \leq x \leq 1 \end{cases}$$

$$Q(1-x) = \begin{cases} 1 & 0 \leq x < a \\ 1 & a \leq x < b \\ 1 & b \leq x < 1-b \\ \frac{1-x-a}{b-a} & 1-b \leq x < 1-a \\ 1 & 1-a \leq x \leq 1 \end{cases}$$

$$A(y) = \begin{cases} 1 & 0 \leq y < ma \\ \frac{y+m(b-2a)}{m(b-a)} & ma \leq y < mb \\ 2 & mb \leq y < m(1-b) \\ \frac{m-y+m(b-2a)}{m(b-a)} & m(1-b) \leq y < m(1-a) \\ 1 & m(1-a) \leq y \leq m \end{cases}$$

There exist $l_1, l_2, l_3, l_4 \in \{1, \dots, m\}$ such that $l_1 - 1 < ma \leq l_1$, $l_2 - 1 < mb \leq l_2$, $l_3 - 1 < m(1-b) \leq l_3$, $l_4 - 1 < m(1-a) \leq l_4$, $m - l_4 = l_1 - 1$, $m - l_3 = l_2 - 1$. Thus,

$$\begin{aligned}
\bar{w}_1 &= \dots = \bar{w}_{l_1-1} = 0, \bar{w}_{l_1} = \frac{l_1 - ma}{m(b-a)}, \bar{w}_{l_1+1} = \dots \\
&= \bar{w}_{l_2-1} = \frac{1}{m(b-a)}, \bar{w}_{l_2} = \frac{mb - l_2 + 1}{m(b-a)}, \bar{w}_{l_2+1} = \dots \\
&= \bar{w}_{l_3-1} = 0, \bar{w}_{l_3} = \frac{l_3 - 1 - mb}{m(b-a)}, \bar{w}_{l_3+1} = \dots = \bar{w}_{l_4-1} \\
&= \frac{-1}{m(b-a)}, \bar{w}_{l_4} = \frac{ma - l_4}{m(b-a)}, \bar{w}_{l_4+1} = \dots = \bar{w}_m = 0
\end{aligned}$$

The expression for $p_{ij}^c + p_{ji}^c$ reduces to:

$$\begin{aligned}
p_{ij}^c + p_{ji}^c &= 1 + \bar{w}_{h_i} (q_{ij}^{l_1} + q_{ij}^{l_4}) + \\
&\sum_{k=l_i+1}^{l_2-1} \frac{1}{m(b-a)} (q_{ij}^k + q_{ij}^{m-k+1}) + \\
&\bar{w}_{l_2} (q_{ij}^{l_2} + q_{ij}^{l_3}) \geq 1, \forall i, j.
\end{aligned}$$

Example 3. Suppose again the same set of additive reciprocal preference relations that in

example 1. Using the linguistic quantifier with the pair of values (0.15,0.35) and the corresponding OWA operator with weight vector $(\frac{1}{3}, \frac{7}{12}, \frac{1}{12}, 0, 0, 0)$, then the collective preference relation is :

$$P^c = \begin{pmatrix} 0.5 & 0.42 & 0.53 & 0.96 \\ 0.84 & 0.5 & 0.87 & 0.91 \\ 0.78 & 0.64 & 0.5 & 0.99 \\ 0.38 & 0.66 & 0.41 & 0.5 \end{pmatrix}$$

Summarising, we have obtained the following result:

Proposition 3. Let $\{P^1, \dots, P^m\}$ be a finite set of individual additive reciprocal preference relations, and Q a relative non decreasing relative quantifier with membership function

$$Q(x) = \begin{cases} 0 & 0 \leq x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b < x \leq 1 \end{cases}$$

with $a+b < 1$. Then, the collective preference relation $P^c = (p_{ij}^c)$, $p_{ij}^c = \phi_Q(p_{ij}^1, \dots, p_{ij}^m)$, obtained using the OWA operator ϕ_Q , verifies $p_{ij}^c + p_{ji}^c \geq 1, \forall i, j$.

CASE C: $a+b > 1$

As in the previous case, we have to distinguished two sub-cases: $a < 1/2$ and $a \geq 1/2$.

CASE C1. $a < 1/2$

The expressions for $Q(x)$, $Q(1-x)$ and $A(x)$ are, respectively:

$$Q(x) = \begin{cases} 0 & 0 \leq x < 1-b \\ 0 & 1-b \leq x < a \\ \frac{x-a}{b-a} & a \leq x < 1-a \\ \frac{x-a}{b-a} & 1-a \leq x < b \\ 1 & b \leq x \leq 1 \end{cases}$$

$$Q(1-x) = \begin{cases} 1 & 0 \leq x < 1-b \\ \frac{1-x-a}{b-a} & 1-b \leq x < a \\ \frac{1-x-a}{b-a} & a \leq x < 1-a \\ 0 & 1-a \leq x < b \\ 0 & b \leq x \leq 1 \end{cases}$$

$$A(y) = \begin{cases} 1 & 0 \leq y < m(1-b) \\ \frac{m-y-ma}{m(b-a)} & m(1-b) \leq y < ma \\ \frac{1-2a}{b-a} & ma \leq y < m(1-a) \\ \frac{y-ma}{m(b-a)} & m(1-a) \leq y < mb \\ 1 & mb \leq y \leq m \end{cases}$$

There exist $r_1, r_2, r_3, r_4 \in \{1, \dots, m\}$ such that

$$\begin{aligned} r_1 - 1 &< m(1-b) \leq r_1, & r_2 - 1 &< ma \leq r_2, \\ r_3 - 1 &< m(1-a) \leq r_3, & r_4 - 1 &< mb \leq r_4, \\ m - r_4 &= r_1 - 1, & m - r_3 &= r_2 - 1, \end{aligned}$$

and therefore:

$$\begin{aligned} \bar{w}_1 = \dots = \bar{w}_{r_1-1} &= 0, \bar{w}_{r_1} = \frac{m-r_1-mb}{m(b-a)} \leq 0, \bar{w}_{r_1+1} = \dots \\ &= \bar{w}_{r_2-1} = \frac{-1}{m(b-a)}, \bar{w}_{r_2} = \frac{r_2-1-ma}{m(b-a)} \leq 0, \bar{w}_{r_2+1} = \dots \\ &= \bar{w}_{r_3-1} = 0, \bar{w}_{r_3} = -\bar{w}_{r_2}, \bar{w}_{r_3+1} = \dots = \bar{w}_{r_4-1} = \frac{1}{m(b-a)}, \\ \bar{w}_{r_4} &= -\bar{w}_{r_1}, \bar{w}_{r_4+1} = \dots = \bar{w}_m = 0. \end{aligned}$$

The expression for $p_{ij}^c + p_{ji}^c$ reduces to:

$$\begin{aligned} p_{ij}^c + p_{ji}^c &= 1 + \bar{w}_{r_1} (q_{ij}^{r_1} + q_{ij}^{r_4}) + \\ &\sum_{k=r_1+1}^{r_2-1} \frac{-1}{m(b-a)} (q_{ij}^k + q_{ij}^{m-k+1}) + \\ &\bar{w}_{r_2} (q_{ij}^{r_2} + q_{ij}^{r_3}) \leq 1, \forall i, j. \end{aligned}$$

Example 4. Using the linguistic quantifier "most of" with the pair of values (0.3,0.8) and the corresponding OWA operator with weight vector $(0, \frac{1}{15}, \frac{1}{3}, \frac{1}{3}, \frac{4}{15}, 0)$, then the collective preference relation is :

$$P^c = \begin{pmatrix} 0.5 & 0.25 & 0.49 & 0.76 \\ 0.66 & 0.5 & 0.64 & 0.59 \\ 0.42 & 0.27 & 0.5 & 0.85 \\ 0.19 & 0.31 & 0.12 & 0.5 \end{pmatrix}$$

CASE C2: $a \geq 1/2$

In this case, following a similar reasoning as in case b2., it is easily to prove again that $p_{ij}^c + p_{ji}^c \leq 1, \forall i, j$.

Example 5. Using, in this case, the linguistic quantifier "as many as possible" with the pair of values (0.5,1) and the corresponding OWA operator with weight vector $(0,0,0,\frac{1}{3},\frac{1}{3},\frac{1}{3})$, then the collective preference relation is :

$$P^c = \begin{pmatrix} 0.5 & 0.2 & 0.31 & 0.62 \\ 0.6 & 0.5 & 0.45 & 0.39 \\ 0.34 & 0.13 & 0.5 & 0.59 \\ 0.06 & 0.15 & 0.04 & 0.5 \end{pmatrix}$$

If $(a,b) = (0.7,0.9)$, the weighting vector is $(0,0,0,0,\frac{2}{3},\frac{1}{3})$ and the collective preference relation is:

$$P^c = \begin{pmatrix} 0.5 & 0.15 & 0.19 & 0.5 \\ 0.58 & 0.5 & 0.33 & 0.32 \\ 0.32 & 0.13 & 0.5 & 0.59 \\ 0.03 & 0.07 & 0.01 & 0.5 \end{pmatrix}$$

Summarising, we have obtained the following result:

Proposition 4. Let $\{P^1, \dots, P^m\}$ be a finite set of individual additive reciprocal preference relations, and Q a relative non decreasing quantifier with membership function

$$Q(x) = \begin{cases} 0 & 0 \leq x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b < x \leq 1 \end{cases}$$

with $a+b > 1$. Then, the collective preference relation $P^c = (p_{ij}^c)$, $p_{ij}^c = \phi_Q(p_{ij}^1, \dots, p_{ij}^m)$, obtained using the OWA operator ϕ_Q , verifies $p_{ij}^c + p_{ji}^c \leq 1, \forall i, j$.

4 Conclusions

We have studied necessary conditions to preserve additive reciprocity when aggregating a finite set of additive reciprocal fuzzy relations using OWA operators guided by a relative non decreasing linguistic quantifier with parameters (a,b) . We have shown that additive reciprocity is maintained

when $a+b=1$ and not when $a+b \neq 1$. Moreover, as we can see from the examples given, the bigger the value of $|a+b-1|$ the more distant the collective preference relation is from being additive reciprocal, in the sense that the bigger is $|p_{ij}^c + p_{ji}^c - 1|$.

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