A Multigranular Hierarchical Linguistic Model for Design Evaluation Based on Safety and Cost Analysis

Luis Martínez,[1,*] Jun Liu,[2,†] Jian-Bo Yang,[3,‡] Francisco Herrera[4,§]
[1]*School of Computing, University of Jaén, 23071 Jáen, Spain*
[2]*School of Computing and Mathematics, University of Ulster at Jordanstown Newtownabbey, Co. Antrim BT37 0QB, Northern Ireland, UK*
[3]*Manchester Business School (East),The University of Manchester, M601QD, Manchester, UK*
[4]*School of Computing, University of Granada, 18071 Granada, Spain*

Before implementing a design of a large engineering system different design proposals are evaluated. The information used by experts to evaluate different options may be vague and/or incomplete. Although different probabilistic tools and techniques have been used to deal with these kinds of problems, it seems better to use the fuzzy linguistic approach to model vagueness and the Dempster-Shafter theory of evidence for modeling incompleteness and ignorance. In the evaluation of alternative designs, different criteria can be considered. In this article an evaluation process is developed in terms of *Safety* and *Cost* analysis. Both criteria involve uncertainty, vagueness, and ignorance due to their nature. Therefore, we propose an evaluation process defined in a linguistic framework where both criteria will be conducted in different utility spaces, i.e., in a multigranular linguistic domain. Once the evaluation framework has been defined, we present an evaluation process based on a Multi-Expert Multi-Criteria decision model that will be able to deal with multigranular linguistic information without loss of information in order to evaluate different design options for an engineering system in a precise manner. Accordingly, we propose the use of a multigranular linguistic model based on the *Linguistic Hierarchies* presented by Herrera and Martínez ("A model based on linguistic 2-tuples for dealing with multigranularity hierarchical linguistic contexts in multi-expert decision-making." IEEE Trans Syst Man Cybern B 2001;31(2):227–234). © 2005 Wiley Periodicals, Inc.

1. INTRODUCTION

The growing technical complexity of large engineering systems such as offshore platforms or offshore support vessels, together with the public concern over their safety, has stimulated the research and development of novel safety analysis methods and safety assessment procedures.

*Author to whom all correspondence should be addressed: e-mail: martin@ujaen.es.
†e-mail: j.liu@ulster.ac.uk.
‡e-mail: Jian-bo.Yang@manchester.ac.uk.
§e-mail: herrera@decsai.ugr.es.

INTERNATIONAL JOURNAL OF INTELLIGENT SYSTEMS, VOL. 20, 1161–1194 (2005)
 Published online in Wiley InterScience (www.interscience.wiley.com). • DOI 10.1002/int.20107

The safety of a large engineering system is affected by many factors regarding its design, manufacturing, installation, commissioning, operation, and maintenance. Consequently, it may be extremely difficult to construct an accurate and complete mathematical model for the system in order to assess its safety because of inadequate knowledge about the basic failure events. This leads inevitably to problems of uncertainty in representation.[1]

In the design of traditional engineering systems, such as marine and offshore structures, the primary objective in selecting a design option is to minimize cost. All future solicitation involving source selection should be structured using safety/cost considerations.[2] Also the decision of implementing a design in a large engineering system depends on whether the design can satisfy technical and economical constraints. Multi-Criteria Decision Making (MCDM) techniques[3–7] could be applied for ranking different design options.

Information used by experts to assess design options may not be precise over different criteria. The probability theory can be a powerful tool. Indeed, traditional risk analysis is conducted primarily using probabilistic tools and techniques. However, it is not difficult to see that many aspects of uncertainties clearly have a nonprobabilistic character because they are related to imprecision and vagueness of meanings. Often the type of uncertainties encountered in engineering systems such as offshore structures do not fit the axiomatic basis of probability theory, simply because uncertainties in the systems are usually caused due to the inherent incompleteness and fuzziness of parameters rather than randomness. Traditional approaches, e.g., quantitative risk assessment (QRA) including probabilistic safety/risk analysis [such as Fault Tree Analysis (FTA) and Failure Mode, Effects, and Criticality Analysis (FMECA)], have been widely used. These methods allow us to build MCDM models, but often fail in their ability to incorporate subjective and/or vague terms as they rely heavily on supporting statistical information that may not be available for a given event.[8] Therefore, linguistic descriptors, such as *"Likely," "Impossible,"* may be used to describe an event due to the fact that they are often used by engineers and safety analysts. The linguistic terms are fuzzy judgments and not probabilistic ones. The Fuzzy Linguistic Approach[9] provides a systematic way to represent linguistic variables in a natural decision-making procedure (see Appendix). It does not require an expert to provide a precise point at which a risk factor exists. So it can be used as a powerful tool complementary to traditional methods to deal with imprecise information, especially linguistic information that is commonly used to represent risk factors in risk analysis.[1,8,10–16]

In engineering safety analysis, intrinsically vague information may coexist with conditions of "lack of specificity" originating from evidence not strong enough to completely support a hypothesis but only with degrees of belief or *credibility*. Dempster-Shafer (D-S) theory of evidence[17,18] based on the concept of *belief function* is well suited for modeling subjective credibility induced by partial evidence.[19] D-S theory enlarges the scope of traditional probability theory, and describes and handles uncertainties using the concept of the degrees of belief, which can model incompleteness and ignorance explicitly. It also provides appropriate methods for computing belief functions for combination of evidence. Besides, the D-S theory also shows great potentials in multiple attribute decision analysis

(MADA) under uncertainty, where an evidential reasoning (ER) approach for MADA under uncertainty has been developed, on the basis of a distributed assessment framework and the evidence combination rule of the D-S theory.[20–24]

Accordingly, it seems reasonable to extend the fuzzy logic framework to cover credibility uncertainty as well. The present work combines fuzzy logic and D-S models to deal with fuzziness and incompleteness in safety assessment.

The aim of this article is to develop a linguistic evaluation model that evaluates different design options for a large engineering system according to safety and cost criteria. To do so, we propose:

1. To define an evaluation framework to assess the criteria of safety and cost
   - Safety will be assessed based on fuzzy logic and the ER approach, referred to as a FUzzy Rule-Based Evidential Reasoning (FURBER) approach,[25] which is based on the RIMER approach proposed recently in Ref. 26. In this approach, a fuzzy rule-base with the belief structures was designed to capture uncertainty and nonlinear causal relationships in safety assessment. The inference process of such a rule-based system was characterized by a rule expression matrix and implemented using the ER approach.
   - The synthesis of the safety assessments for each option is expressed and implemented using a linguistic 2-tuple scheme.[27]
   - The cost assessments of each design option will be obtained from the cost assessments of each cost factor supplied directly by the experts in terms of linguistic labels.
2. To develop an evaluation model based on a Multi-Expert Multi-Criteria decision process
   - Cost and safety linguistic assessments will be synthesized from the experts knowledge regarding each design option. The assessments of each criterion are conducted in different utility spaces from each other.
   - These assessments will be the input values for a Multi-Expert Multi-Criteria Decision Making (MEMC-DM) problem defined in a multigranular linguistic domain used to evaluate the different design options in order to choose the most suitable one for the engineering system.
   - In the evaluation process the cost and safety assessments will be combined to obtain a degree of suitability for each design option. Once the suitability degrees of all options have been calculated, the best option can be chosen.

The main difficulty to solve the above evaluation problem is that the linguistic assessments for cost and safety are conducted in different utility spaces, i.e., in different linguistic term sets, because they have different scales and meaning. Therefore, we shall deal with an evaluation problem defined in a multigranular linguistic context that we shall model as an MEMC-DM process. Hence, in this article we propose the development of a multigranular linguistic decision model based on the *Linguistic Hierarchies*[28] to avoid any loss of information during the resolution of the multigranular linguistic MEMC-DM problem. Because the use of accurate models is a critical factor in the processes related to engineering systems, the linguistic hierarchies will provide us a common utility space for expressing safety and cost assessments and for ranking design options without loss of information.

This article is structured as follows. In Section 2 we present the evaluation framework used for the safety and cost modeling of large engineering systems. In Section 3 we review the concept of Linguistic Hierarchy. In Section 4 a decision model will be proposed to evaluate and choose the best design option for a large engineering system. In Section 5 an example to demonstrate this model is presented, and the paper concludes in Section 6.

2. EVALUATION FRAMEWORK FOR SAFETY AND COST MODELING IN LARGE ENGINEERING SYSTEMS

In this section we propose an evaluation framework for safety and cost modeling. We shall demonstrate how our model can be used to synthesize linguistic safety and cost assessments. Eventually, we shall construct a MEMC-DM schema to evaluate different design options for an engineering system.

2.1. Safety Evaluation Framework

A generic framework for modeling system safety estimate using the FURBER approach and for safety synthesis using the ordinal fuzzy linguistic approach is depicted in Figure 1. The framework for modeling system safety for risk analysis consists of four major steps, which include all necessary steps required for safety

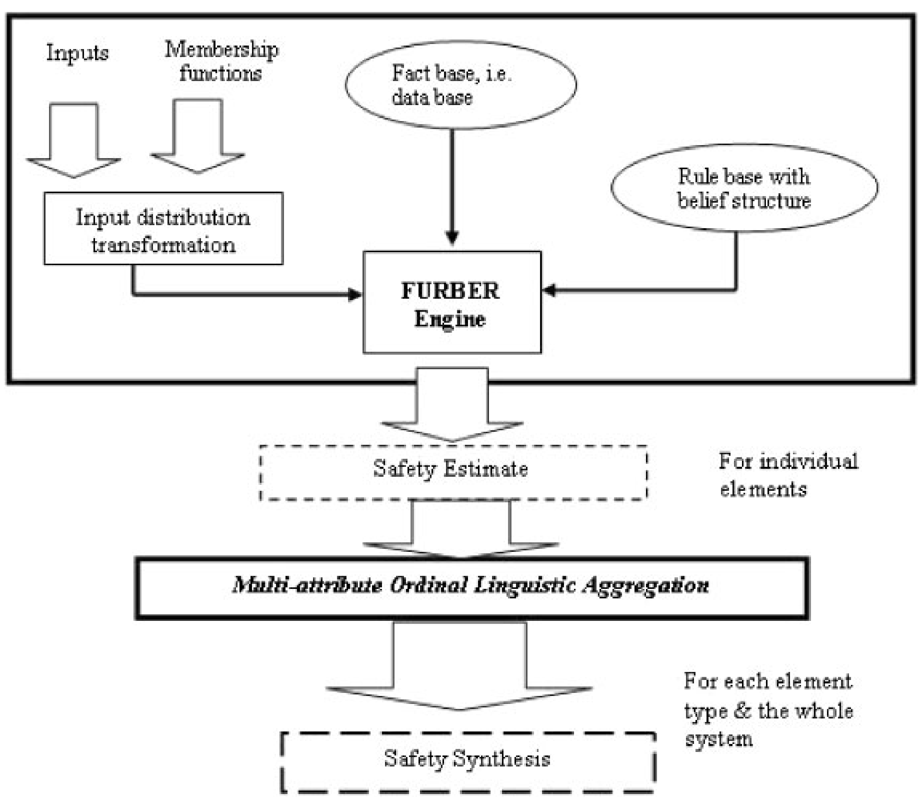

Figure 1. A generic qualitative safety assessment framework.

analysis at the bottom level of a system using the FURBER approach. The steps used are outlined as follows (more details can be found in Refs. 25 and 26):

Step #1: Identification of causes/factors: This can be done by a panel of experts during a brainstorming session at early conceptual design stages.

Step #2: Identify and definite fuzzy input and fuzzy output variables. The three parameters used to assess the safety level of an engineering system on a subjective basis are *failure rate* (**FR**), *consequence severity* (**CS**), and *failure consequence probability* (**FCP**). Subjective assessments using linguistic variables may be more appropriate for analysis using these three parameters as they are always associated with uncertainty, especially for a new system with high level of innovation. The characterization of the linguistic values used to describe **FR**, **CS**, **FCP** of a particular element are defined in Ref. 15:

- **FR** describes failure frequencies in a certain period, which directly represents the number of failures anticipated during the design life span of a particular system or an item. To estimate **FR**, one may choose to use such linguistic terms as "*very low*," "*low*," "*reasonably low*," "*average*," "*reasonably frequent*," "*frequent*," and "*highly frequent*."
- **CS** describes the magnitude of possible consequences, which is ranked according to the severity of failure effects. One may choose to use such linguistic terms as "*negligible*," "*marginal*," "*moderate*," "*critical*," and "*catastrophic*."
- **FCP** defines the probability that consequences happen given the occurrence of the even. For **FCP**, one may choose to use such linguistic terms as "*highly unlikely*," "*unlikely*," "*reasonably unlikely*," "*likely*," "*reasonably likely*," and "*definite*."

Safety estimate is the only output fuzzy variable used in this study to produce safety evaluation for a particular cause to technical failure. This variable is also described linguistically using the following linguistic term set, denoted as S_S in this article:

$$S_S = \{\text{“}Poor\text{,” “}Low\text{,” “}Average\text{,” “}High\text{,” “}Good\text{”}\}$$

which are referred to as safety expressions.

Step #3: Construct a fuzzy rule-base with belief structures. Fuzzy logic systems are knowledge-based or rule-based ones constructed from human knowledge in the form of fuzzy *IF-THEN* rules. For example, the following is a fuzzy *IF-THEN* rule for safety analysis:

IF **FR** of a hazard is *frequent AND* **CS** is *catastrophic AND* **FCP** is *likely*,
THEN ***safety estimate*** is *Poor*.

Taking into account the belief degrees of a rule, attribute weights, and rule weights, fuzzy rules for safety can be extended in the following way. In general, assume that the three antecedent parameters, $A_1 = \mathbf{FR}$, $A_2 = \mathbf{CS}$, and $A_3 = \mathbf{FCP}$ can be described with linguistic terms that belongs to $S_{J_i} = \{s_{i0}, \ldots, s_{i(J_i-1)}\}$, where $i = 1,2,3$ indicates the parameter assessed and J_i is the cardinality of the lingusitic term set used to assess the ith parameter. One consequent variable ***safety estimate*** can be described by N linguistic terms, i.e., $D_0, D_2, \ldots, D_{N-1}$. Let $s_i^k \in S_{J_i}$ be a linguistic term corresponding to the ith attribute in the kth rule, with $i = 1,2,3$. Thus the kth rule in a rule-base can be written as follows:

$$R_k\text{: IF } \mathbf{FR} \text{ is } s_1^k \text{ AND } \mathbf{CS} \text{ is } s_2^k \text{ AND } \mathbf{FCP} \text{ is } s_3^k \text{ THEN } \textbf{\textit{safety estimate}} \text{ is}$$

$$\{(D_0, \gamma_{0k}), (D_1, \gamma_{1k}), \ldots, (D_{N-1}, \gamma_{N-1k})\}, \left(\sum_{i=0}^{N-1} \gamma_{ik} \leq 1\right),$$

$$\text{with a rule weight } \theta_k \text{, and the attribute weights } \delta_1, \delta_2, \delta_3 \tag{1}$$

where γ_{ik} ($i \in \{0, \ldots, N-1\}$; $k \in \{1, \ldots, L\}$, with L being the total number of the rules in the rule-base), is a belief degree measuring the subjective uncertainty of the consequent "***safety estimate*** is D_i" drawn due to the antecedent "**FR** is s_1^k AND **CS** is s_2^k AND **FCP** is s_3^k" in the kth rule. If $\sum_{i=0}^{N-1} \gamma_{ik} = 1$, the output assessment or the kth rule is said to be complete; otherwise, it is incomplete. The rule-base in the form (1) is referred to as *a fuzzy rule-base with belief structures*.

Suppose a linguistic term set with seven grades is used for **FR** (i.e., $J_1 = 7$); five grades for **CS** (i.e., $J_2 = 5$), seven grades for **FCP** (i.e., $J_3 = 7$). In addition, suppose $D_k \in S_S = \{s_0 =$ '*Poor*,' $s_1 =$ '*Low*,' $s_2 =$ '*Average*,' $s_3 =$ '*High*,' $s_4 =$ '*Good*'$\}$ ($k = 0, \ldots, 4$), so $|S_S| = 5$. In this case, a sample of a rule base with 245 rules will be used,[25] for example:

- Rule #1: IF the **FR** is *very low* AND the **CS** is *negligible* AND the **FCP** is *highly unlikely* THEN the ***safety estimate*** is {(*good*, 1)}

  …
- Rule #100: IF the **FR** is *relatively low* AND the **CS** is *catastrophic* AND the **FCP** is *unlikely* THEN the ***safety estimate*** is {(*average*, 0.3), (*low*, 0.7)}

  …
- Rule #245: IF the **FR** is *highly frequent* AND the **CS** is *catastrophic* AND the **FCP** is *definite* THEN the ***safety estimate*** is {(*poor*, 1)}

Step #4: Fuzzy rule-base inference mechanism based on the evidential reasoning approach. Suppose a fuzzy rule-base with the belief structure is given by $R = \{R_1, \ldots, R_L\}$. The kth rule in form 1 can be represented as follows:

$$R_k\text{: IF } \boldsymbol{A} \text{ is } \boldsymbol{s}^k \text{ THEN } \textbf{\textit{safety estimate}} \text{ is } \boldsymbol{D} \text{ with belief degree } \gamma_k \tag{2}$$

where $\boldsymbol{A}$ represents the antecedent attribute vector (**FR**, **CS**, **FCP**), $\boldsymbol{s}^k$ the packet antecedents $\{s_1^k, s_2^k, s_3^k\}$. $\boldsymbol{D}$ the consequent vector $(D_0, \ldots, D_{N-1})$, γ_k the vector of the belief degrees $(\gamma_{0k}, \ldots, \gamma_{N-1k})$ and $k \in \{1, \ldots, L\}$. Each fuzzy rule with belief structure can be explained in the following way:

> The packet antecedent $\boldsymbol{A}$ of an IF-THEN rule could be considered as a global attribute, which is considered as being assessed to a linguistic term D_i (the ith possible consequent term in the kth rule) with a belief degree of γ_{ik} ($i \in \{0, \ldots, N-1\}$). This assessment can be represented by
>
> $$S(\boldsymbol{A}) = \{(D_i, \gamma_{ik}); \quad i = 0, \ldots, N-1\} \tag{3}$$

which is obviously a distributed assessment and is referred to as *a belief structure*, where γ_{ik} measures the degree to which D_i is the consequent if the input activates the packet antecedent $\boldsymbol{A}$ in the kth rule, $0 \leq \sum_{i=0}^{N-1} \gamma_{ik} \leq 1$ for all k. Here $i = 0, \ldots, N-1$; $k = 1, \ldots, L$, where L is the number of rules in the rule-base and N is the number of the possible consequent terms in the kth rule.

A fuzzy rule-base with belief structure established using rules given by Equation 3 can be summarized using the following rule expression matrix shown in Table I. In the matrix, w_k is the activation weight of $\boldsymbol{s}^k$, which measures the degree to which the kth rule is weighted and activated. w_k is generated by weighting and normalizing the individual matching degree to which the input belongs to the linguistic term of each antecedent. It could be generated using various ways depending on the nature of an antecedent attribute and the available data.

The way to determine the activation w_k is summarized as follows. For more details, see Refs. 25 and 26.

(1) Input transformation. The input is transformed into the distributed representation of linguistic values in antecedents using belief degrees. In general, we may consider a linguistic term in the antecedent as an evaluation grade, the input for an antecedent attribute A_i can be assessed to a distribution representation of the linguistic terms using belief degrees as follows:

$$S(A_i) = \{(s_{ij}, \eta_{ij}), j = 0, \ldots, J_i - 1\}, \quad i = 1,2,3 \tag{4}$$

where s_{ij} is the linguistic term of the ith attribute A_i, such that it is the jth ordinal label in $S_{J_i} = \{s_{i0}, \ldots, S_{iJ_i-1}\}$ being J_i the cardinality of the linguistic term set used to assess the ith attribute, η_{ij} is the likelihood to which the input for A_i belongs to the linguistic term s_{ij} with $\eta_{ij} \geq 0$ and $\sum_{j=0}^{J_i-1} \eta_{ij} \leq 1$ ($i = 1,2,3$), referred to as *the individual matching degree*. η_{ij} in Equation 4 could be generated using different ways depending on the nature of an antecedent attribute and the available data, which is described in the following three cases:

(a) *Matching function method*. While the input is in numerical form and the linguistic value is characterized using fuzzy membership functions (suit for both quantitative and qualitative).

Table I. Rule expression matrix for a fuzzy rule-base with belief structure.

Output / Belief / Input	D_0	D_1	…	D_i	…	D_{N-1}
$\boldsymbol{s}^1(w_1)$	γ_{01}	γ_{11}	…	γ_{i1}	…	γ_{N-11}
…	…	…	…	…	…	…
$\boldsymbol{s}^k(w_k)$	γ_{0k}	γ_{1k}	…	γ_{ik}	…	γ_{N-1k}
…	…	…	…	…	…	…
$\boldsymbol{s}^L(w_L)$	γ_{0L}	γ_{1L}	…	γ_{iL}	…	γ_{N-1L}

(b) *Rule-based or utility-based transformation methods*. While the input is in numerical forms but the fuzzy membership function is not available (only suit for the quantitative attribute).
(c) *Subjective assessment method* (suit for quantitative and qualitative attribute).

For more details about the above three cases, we refer to Refs. 25 and 26. Here we only consider Case (a). The data may be given in the following numerical form:

- a single deterministic value with 100% certainty
- a close interval defined by an equally likely range
- a triangular distribution defined by a most likely value, with lower and upper least likely values
- a trapezoidal distribution defined by a most likely value, with lower and upper least likely values.

Corresponding to the rule-base 1, the general input form corresponding to the antecedent attribute in the kth rule is given as follows:

$$(A_1^*, \varepsilon_1) \text{ AND } (A_2^*, \varepsilon_2) \text{ AND } (A_3^*, \varepsilon_3) \tag{5}$$

where ε_i expresses the degree of belief assigned by an expert to the association of A_i^* $(i = 1, \ldots, 3)$ to reflect the uncertainty of the input data, here A_i^* $(i = 1, \ldots, 3)$ can be any of the above input forms.

Finally η_{ij} in Equation 4 could be formulated in the following way:

$$\eta_{ij} = \frac{\tau(A_i^*, s_{ij}) \cdot \varepsilon_i}{\sum_{j=0}^{J_i-1} [\tau(A_i^*, s_{ij})]}, \quad i = 1,2,3 \tag{6}$$

here (A_i^*, ε_i) is the actual input corresponding to the ith antecedent, τ is a matching function, $\tau(A_i^*, s_{ij}) = \tau_{ij}$ is a matching degree to which A_i^* belong to s_{ij}. One possible matching function τ is given as follows and is used in our case study in Section 4:

$$\tau(A_i^*, s_{ij}) = \max_x [\min(A_i^*(x) \wedge s_{ij}(x))] \tag{7}$$

(2) Activation weight for a packet antecedent. Considering an input given by Equation 5 corresponding to the kth rule defined as in form 1,

$$\textbf{FR} \text{ is } (s_1^k, \eta_1^k) \text{ AND } \textbf{CS} \text{ is } (s_1^k, \eta_2^k) \text{ AND } \textbf{FCP} \text{ is } (s_1^k, \eta_3^k) \tag{8}$$

where η_i^k is the η_{ij} that represents the individual matching belief degree that belongs to $s_i^k \in S_{J_i} = \{s_{i0}, \ldots, s_{J_i-1}\}$ of the individual antecedent A_i appearing in the kth rule.

The activation weight w_k of the packet antecedent $\boldsymbol{A}$ in the kth rule is generated by weighting and normalizing the η_k given by Equation 9 as follows:

$$w_k = (\theta_k \cdot \eta_k) \Big/ \left(\sum_{i=1}^{L} \theta_i \eta_i \right) \tag{9}$$

where

θ_k is the relative weight of the kth rule
δ_i $(i = 1,2,3)$ is the weight of the ith antecedent attribute
$\eta_k \prod_{i=1}^{3} (\eta_i^k)^{\bar{\delta}_i}$ is the global matching degree, $\bar{\delta}_i = \delta_i / (\max_{i=1,2,3}\{\delta_i\})$
L is the number of rules in the rule-base.

Note that the "AND" connective is used for three antecedents in a rule. In other words, the consequent of a rule is not believed to be true unless all the antecedents of the rule are activated. In such cases, the simple multiplicative aggregation function is used to calculate η_k. Note that $0 \le w_k \le 1 (k = 1, \ldots, L)$ and $\sum_{k=1}^{L} w_k = 1$.

Having represented each rule-base using rule expression matrix, the ER approach,[22–24] which is developed on the basis of a distributed assessment framework and the evidence combination rule of the D-S theory, can be used to combine rules and generate final conclusions, the detailed algorithm can be found in Refs. 25 and 26.

The aggregation of consequents, i.e., the ***safety estimate*** $S(e_i(a_l))$ across the rules is expressed as follows for the assessment done by the ith expert on the lth potential cause to a technical failure:

$$S(e_i(a_l)) = \{(Poor, \vartheta_{0i}^l), (Low, \vartheta_{1i}^l), (Average, \vartheta_{2i}^l), (High, \vartheta_{3i}^l), (Good, \vartheta_{4i}^l)\} \tag{10}$$

where e_i represents the ith expert $(i = 1, \ldots, p)$ and a_l represents the lth $(l = 1, \ldots, q)$ potential cause to a technical failure. ϑ_{ti}^l represents the belief degree to which the safety of a_l is believed to be assessed to $D_t \in S_S$ by the expert e_i. The inference procedure is based on the fuzzy rule-base and evidential reasoning approach, referred to as a FURBER approach.[25] The final result is still a belief distribution on safety expression, which gives a panoramic view about the safety level for a given input.

2.2. Safety Synthesis Framework

Considering that safety level is expressed as a linguistic variable in a qualitative format, it is difficult to establish their membership functions. The ordinal fuzzy linguistic approach (symbolic approach) is considered here by the direct computation on linguistic values[29] instead of the semantic approach by using the associated membership function.[30] The 2-tuple aggregation approach (see the Appendix) can be applied to synthesize the safety estimate. The 2-tuple linguistic representation model has been presented in Ref. 27 that presents different advantages to manage linguistic information over semantic and symbolic models,[31] some concepts and properties are detailed in the Appendix of this article.

In this phase for the synthesis purpose, we transform the safety estimate into a linguistic 2-tuple, i.e., transform the distribution assessment $S(e_i(a_l))$ in Equation 10 on the S_S into linguistic 2-tuples over the S_S. A function χ_i^l is introduced that transforms a distribution assessment in a linguistic term set S_S into a numerical value in the interval of granularity of S_S, $[0, T-1]$, where T is the cardinality of S_S:

$$\chi_i^l : S(e_i(a_l)) \rightarrow [0, T-1]$$

$$\chi_i^l(\{(s_t; \vartheta_{ti}^l), t = 0, \ldots, T-1\}) = \frac{\sum_{t=0}^{T-1} t\vartheta_{ti}^l}{\sum_{t=0}^{T-1} \vartheta_{ti}^l} = \beta_i^l \tag{11}$$

Here $S_S = \{s_0 =$ '*Poor*,' $s_1 =$ '*Low*,' $s_2 =$ '*Average*,' $s_3 =$ '*High*,' $s_4 =$ '*Good*'$\}$ and $T = |S_S| = \mathbf{5}$. $\vartheta_{ti}^l(t = 0, \ldots, |S_S| - 1)$ are obtained from Equation 10. Therefore, applying the Δ function (Definition A.2.2, Appendix) to β_i^l $(i = 1, \ldots, p;\ l = 1, \ldots, q)$ we shall obtain a safety estimate whose values are linguistic 2-tuples (by the ith expert on the lth potential cause to a technical failure), e.g., if $\beta_i^l = 1.2$, then its equivalent linguistic 2-tuple representation is:

$$\Delta(1{,}2) = (Low, 0.2)$$

2.3. Cost Modeling

Cost and safety are two of the most important criteria in design of complex engineering systems, but usually they are in conflict because higher safety normally leads to higher costs. The cost incurred for safety improvement associated with a design option is usually affected by many factors,[16] for example,

- cost for provision of redundancies of critical components
- provision of protection systems
- alarm systems
- use of more reliable components
- cost for redesign of the system

These factors can be different in each engineering system and often include uncertainties. Therefore, it may be more appropriate to model cost incurred in safety improvement associated with design options on a subjective basis.

In the literature[1,16] cost was estimated and described using fuzzy sets over the linguistic variables belonging to the linguistic term set, for example:

{*Very Low*, *Low*, *Moderately Low*, *Average*, *Moderately High*, *High*, *Very High*}

In this article we develop a cost and safety model dealing with multigranular linguistic information without loss of information. To do so, we shall use linguistic hierarchies that are linguistic structures that will be described in Section 3. These structures follow different rules. Because of this, according to the granularity of safety assessments, we propose to express the cost assessments for the different factors in the linguistic term set, $\boldsymbol{S_C}$, with nine labels (triangular shaped and symmetrically distributed) (see Figure 2).

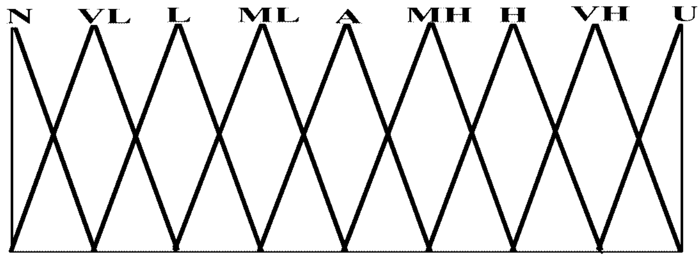

Figure 2. Linguistic assessments for cost factors, S_C.

$$S_C = \{None, Very\ Low, Low, Moderately\ Low, Average, Moderately\ High, High, Very\ High, Unacceptable\}$$

The use of S_C will prevent the loss of linguistic information in linguistic computations of our evaluation model. In our approach, the jth expert, e_j, provides an assessment for the kth cost factor f_k, of the ith design, O_i, $j = 1, \ldots, p$; $k = 1, \ldots, n$; $i = 1, \ldots, m$, expressed by means of labels $c_{ij}^k \in S_C$ (see Table II).

To obtain a cost assessment for a design from each expert, we shall aggregate the assessments for each factor using the weighted average 2-tuple operator (see Appendix, Definition A.2.4). With this operator we can give different importance to each cost factor. So, the cost for the ith design option by the jth expert is obtained as follows (see Table III):

$$\Delta(W\_AM * ((c_{1j}^1, 0), \ldots, (c_{1j}^n, 0))) = \Delta\left(\sum_{k=1}^{n} \beta_{ij}^k \cdot w_k \middle/ \sum_{i=1}^{n} w_k\right) = (c_{ij}, \alpha_{ij}) \tag{12}$$

where

$(c_{ij}^k, 0)$ is the equivalent linguistic 2-tuple (Remark A.2.1) of the linguistic cost assessment provided by the jth expert to the kth cost factor f_k (alarm systems, cost for redesign, ...) of the ith design option O_i
β_{ij}^k is its equivalent numerical value (Proposition A.2.1)
w_k is the importance of the factor f_k
(c_{ij}, α_{ij}) is the overall cost linguistic assessment for the ith design option synthesized from the jth expert e_j.

Table II. Expert's j linguistic assessments for cost factors.

e_j	Designs / Factors	O_1	...	O_m
	f_1	c_{1j}^1	...	c_{mj}^1
	...		...	
	f_n	c_{1j}^n	...	c_{mj}^n

Table III. Expert's *j overall* cost assessment for all the design options.

e_j	O_1	…	O_m
	(c_{1j}, α_{1j})	…	(c_{mj}, α_{mj})

Remark 1. Cost assessments have a different interpretation of suitability for a design option regarding safety assessment. In other words, high cost assessments indicate the low suitability of a design option. Therefore, to calculate a value for the suitability of a design option, we shall take into account this feature.

So far, cost and safety assessments have been assessed by means of linguistic values but in different linguistic utility spaces.

2.4. Multi-Expert Multi-Criteria Decision Making Schema

We shall model our evaluation problem as a MEMC-DM problem where all the experts provide their assessments for different cost factors and his/her opinions for **FR**, **CS**, and **FRP** that will be synthesized to obtain the safety assessments. The cost and safety assessments used as inputs for the DM problem are summarized in Table IV: where $(s_{ij}, \alpha_{ij}^S)(i = 1, \ldots m; j = 1, \ldots, p; \alpha_{ij}^S \in [-0.5, 0.5))$ are the safety assessments synthesized from the opinions of the expert e_j for the design option O_i, i.e., estimated based on the fuzzy rule-based system produced at lower levels, and then synthesized to obtain the safety assessment of the system by means of linguistic 2-tuples in the linguistic term set S_S. While $(c_{ij}, \alpha_{ij}^C)(i = 1, \ldots, m; j = 1, \ldots, p; \alpha_{ij}^C \in [-0.5, 0.5))$ are the overall cost assessment obtained by aggregating the cost of the different cost factors, provided by the expert e_j for the design option O_i, assessed in the linguistic term set S_C.

Note that S_S and S_C are linguistic term sets with different granularity and semantics. Therefore our MEMC-DM problem is defined over a multigranular linguistic domain.

A solution for a MEMC-DM problem is derived from the individual preference problems following a common resolution scheme composed by two phases:[32]

Table IV. Safety and cost assessments synthesized from e_j.

	Criteria	
Design options	Safety	Cost
O_1	(s_{1j}, α_{1j}^S)	(c_{1j}, α_{1j}^C)
⋮	⋮	⋮
O_m	(s_{mj}, α_{mj}^S)	(c_{mj}, α_{mj}^C)

- *aggregation phase*: that combines the expert preferences, and
- *exploitation one*: that obtains a solution set of alternatives from a preference relation

The main difficulty for managing MEMC-DM problems defined in multigranular linguistic information contexts is how to aggregate this type of information.

The 2-tuple fuzzy linguistic representation model presented in Ref. 27 has shown itself as a good choice to manage nonhomogeneous information in aggregation processes.[28,33] This representation model together with the structure of the Linguistic Hierarchies[28] provide a preference modeling and a computational model able to manage multigranular linguistic information in a precise way. In this article, we present in Section 4 an evaluation model that uses the linguistic hierarchies to obtain the most suitable design option for a large engineering system.

3. LINGUISTIC HIERARCHIES

The hierarchical linguistic contexts were introduced in Ref. 28 to improve the precision of the processes of *Computing with Words* in multigranular linguistic contexts, which is the aim of this article.

A Linguistic Hierarchy is a set of levels, where each level represents a linguistic term set with different granularity to the remaining levels. Each level is denoted as $\boldsymbol{l(t, n(t))}$:

- t is a number that indicates the level of the hierarchy.
- $n(t)$ is the granularity of the term set of the level t.

We assume that levels containing linguistic terms are triangular shaped, symmetrical, and uniformly distributed. In addition, the linguistic term sets have an odd number of linguistic terms being the middle one the value of *indifference*.

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for two consecutive levels t and $t+1$, $n(t+1) > n(t)$. Therefore, the level $t+1$ is a refinement of the previous level t.

From the above concepts, we define a linguistic hierarchy (LH) as the union of all levels t:

$$LH = \bigcup_t l(t, n(t))$$

Given an LH, we denote as $S^{n(t)}$ the linguistic term set of LH corresponding to the level t of LH characterized by a granularity of uncertainty $n(t)$:

$$S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$$

Generically, we can say that the linguistic term set of level $t+1$ is obtained from its predecessor as

$$l(t, n(t)) \to l(t+1, 2 \cdot n(t) - 1)$$

A graphical example of a linguistic hierarchy can be seen in Figure 3.

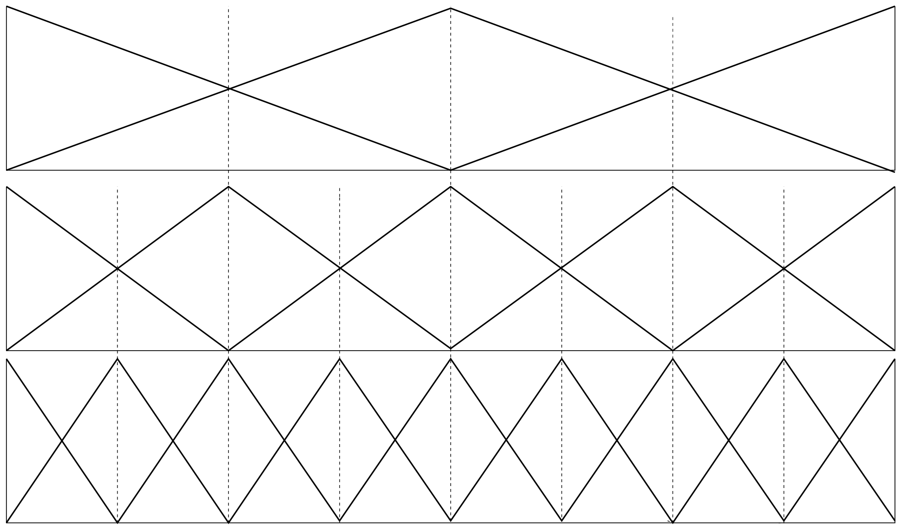

Figure 3. Linguistic hierarchy.

In Ref. 28 different transformation functions between labels of different levels were developed without loss of information. To understand how these functions are working, there were defined transformation functions between two consecutive levels and afterward between any levels of the hierarchy. These transformation functions use the linguistic 2-tuple computational model.

DEFINITION 1. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \dots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function between a label from level t and a label belonging to level $t+1$, satisfying the linguistic hierarchy basic rules, is defined as*

$$TF_{t+1}^{t} : l(t, n(t)) \to l(t+1, n(t+1))$$

$$TF_{t+1}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta\left(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t+1) - 1)}{n(t) - 1}\right)$$

DEFINITION 2. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \dots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function between a label from level t and a label belonging to level $t-1$, satisfying the linguistic hierarchy basic rules, is defined as*

$$TF_{t-1}^{t} : l(t, n(t)) \to l(t-1, n(t-1))$$

$$TF_{t-1}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta\left(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t-1) - 1)}{n(t) - 1}\right)$$

From Definitions 1 and 2 we can generalize these transformation functions to transform linguistic labels between any two linguistic levels of the hierarchy. This generalization can be carried out in a recursive way or in a nonrecursive one.

DEFINITION 3. *Let $L = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $\{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$. The recursive transformation function between a label belonging to a level t and a level $t' = t + a$, with $a \in Z$, is defined as*

$$TF_{t'}^{t} : l(t, n(t)) \rightarrow l(t', n(t'))$$

if $|a| > 1$ *then*

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = TF_{t'}^{t+[(t-t')/|t-t'|]}(TF_{t+[(t-t')/|t-t'|]}^{t}(s_i^{n(t)}, \alpha^{n(t)}))$$

if $|a| = 1$ *then*

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = TF_{t+[(t-t')/|t-t'|]}^{t}(s_i^{n(t)}, \alpha^{n(t)})$$

This recursive transformation function can be easily defined in a nonrecursive way, as can be seen in the following definition:

DEFINITION 4. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level t′ is defined as*

$$TF_{t'}^{t} : l(t, n(t)) \rightarrow l(t', n(t'))$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta_{n(t')}\left(\frac{\Delta_{n(t)}^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1}\right)$$

PROPOSITION 1. *The transformation function between linguistic terms in different levels of the linguistic hierarchy is bijective:*

$$TF_{t}^{t'}(TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)})) = (s_i^{n(t)}, \alpha^{n(t)})$$

4. DECISION MODEL: EVALUATING THE DESIGN OPTIONS

The aim of this article is the development of an evaluation model to choose the best design option among several ones for a large engineering system taking into account the features of safety and cost despite both of them are conducted in different utility spaces. To achieve our objective in Section 2, we presented an MEMC-DM schema defined in a multigranular linguistic framework to evaluate the different designs. Therefore, to evaluate and rank the different options we present a multigranular linguistic decision model that uses a linguistic hierarchy, LH^*, to model and manage the multigranular linguistic information without loss of information. This decision model consists of the following phases (graphically Figure 4):

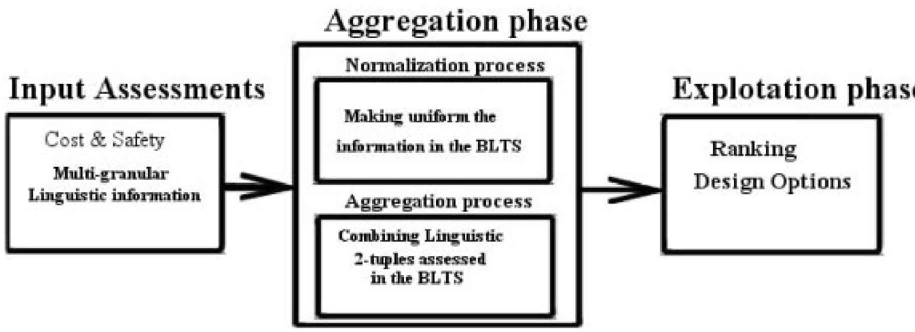

Figure 4. Multigranular linguistic decision model.

- *Aggregation phase*: It combines the safety and cost assessments provided by the experts to obtain an overall suitability assessment for each design option. In our case as the input information is assessed in a multigranular linguistic domain, this phase consists of two steps:
  (1) Normalization process: Safety and cost assessments are conducted in two different utility spaces, S_S and S_C, contained in LH^*. To operate over these assessments, this process unifies both criteria assessments into a sole utility space called *Basic Linguistic Term Set* (BLTS). This unification process is carried out in a precise way without loss of information.
  (2) Aggregation process: It combines the unified information to obtain an overall value of suitability for each design option.
- *Exploitation phase*: It ranks the different design options according to assessments obtained in the aggregation phase by means of a choice degree.

We propose this decision-making model for evaluating the different design options and choose the best one for an engineering system.

In the next subsections, we present each phase of the linguistic decision model, but first of all we shall choose a linguistic hierarchy for modeling the cost and safety assessments we shall use in our problem.

4.1. Safety and Cost Problem Modeled by Means of Linguistic Hierarchies

In Sections 2.2 and 2.3, we have modeled the safety and the cost assessments of each design option expressed by means of linguistic values assessed in different linguistic utility spaces, S_S and S_C, with five and nine labels, respectively. Therefore, we have to choose a linguistic hierarchy, LH^*, which contains levels with these cardinalities for modeling our problem. In our case we shall choose as LH^*, the linguistic hierarchy showed in the Figure 5.

4.2. Decision Process

Once we have chosen the Linguistic Hierarchy, LH^*, to model the safety and cost assessments. Here we shall describe the decision model used to solve our

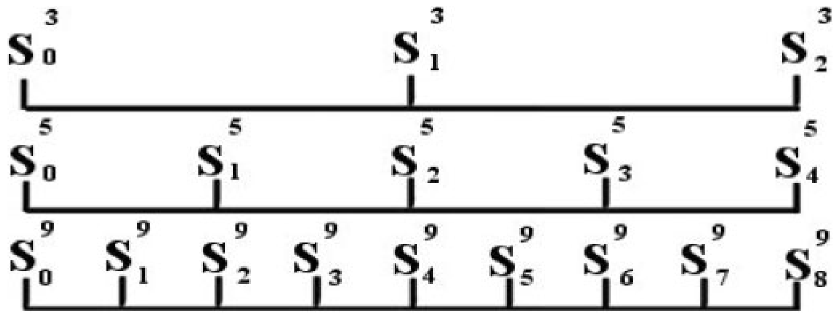

Figure 5. Linguistic hierarchy 3, 5, and 9 labels.

evaluation problem that we have modeled as a Multi-Expert Multi-Criteria decision-making process, in which each expert e_j provides his/her opinions about the different safety parameters and cost factors. And such as it has been indicated in Sections 2.2 and 2.3, the cost and safety assessments are synthesized. So the inputs provided by the expert e_j for our evaluation problem have been summarized in the Table IV.

In the following subsections the two phases involved in the resolution process of the decision model are described in detail that evaluates the different design options for an engineering system: (i) aggregation phase and (ii) exploitation one.

4.2.1. Aggregation Phase

In this phase the multigranular linguistic information synthesized from the opinions provided by the experts (Table IV) is combined to obtain collective evaluation value for all the design options. This phase combines the multigranular linguistic information in two steps.

4.2.1.a. Normalization Process. Due to the fact that the cost and safety assessments are assessed in a multigranular linguistic framework to combine both types of assessments firstly they must be conducted into a common utility space called the Basic Linguistic Term Set (BLTS), represented by S_T. In this problem we could choose as BLTS any linguistic level of LH^*, but in our case we have decided to choose as BLTS to unify the input assessments a linguistic term set with five linguistic labels that corresponds to the second level, S^5, of LH^* (Figure 5).

$$S_T = S^5 = \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\}$$

We have chosen this term set as BLTS because of minimizing the number of operations. On the one hand, safety assessments are expressed in this term set so we do not have to transform these assessments to unify them, and on the other hand the utility space we shall use in our problem the *suitability utility* of each design option is represented using a five labels linguistic term set with the following syntax:

$$S_E = \{Slightly\ Preferred,\ Moderate\ Preferred,\ Average,\ Preferred,\ Greatly\ Preferred\}$$

but with the same semantics as S_T. So the results obtained in S_T are directly translated to S_E.

Remark 2: We need S_T and S_E because during the aggregation process we cannot use S_E due to its syntax might lead to misunderstandings because cost is in conflict with safety and this syntax does not reflect the conflict. Hence during the aggregation computations we shall use S_T with the notation s_i^5 to refer to the aggregated values and when we obtain the suitability utility values then they will be expressed by means of labels in S_E.

The multigranular information is unified by means of the transformation function between the levels of the hierarchy (Definition 4):

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta_{n(t')}\left(\frac{\Delta_{n(t)}^{-1}(s_i^{n(t)}, \alpha^{n(t)})\cdot(n(t') - 1)}{n(t) - 1}\right)$$

Once we have chosen the common utility space to express the preferred design options, we make uniform all the safety and cost assessments into the BLTS. The safety assessments are directly transformed into the BLTS because they are expressed in a term set with the same semantics, while the cost assessments will be unified by means of the transformation function $TF_5^9(c_{ij}, \alpha), c_{ij} \in S_C$. For instance:

$$TF_5^9(ModerateLow, 0) = (s_2^5, -0.5)$$

After this transformation process expert, safety and cost assessments provided by the expert e_j are expressed by means of linguistic 2-tuples in the common utility space, BLTS. Therefore, from Table IV the normalization process will obtain (see Table V). Here $s_{ij}^5 \in S_T = S^5 = \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\}$, $\alpha_{ij}^S \in [-0.5, 0.5)$, $\alpha_{ij}^C \in [-0.5, 0.5)$, $i = 1, \ldots, m; j = 1, \ldots, p$. This process is applied to all the expert opinions.

Table V. Safety and cost unified assessments by e_j expressed in S_T.

	Criteria	
Design options	Safety	Cost
$\boldsymbol{O_1}$	$(s_{1j}^5, \alpha_{1j}^S)$	$(s_{1j}^5, \alpha_{1j}^C)$
:	:	:
$\boldsymbol{O_m}$	$(s_{mj}^5, \alpha_{mj}^S)$	$(s_{mj}^5, \alpha_{mj}^C)$

Table VI. Unified assessments in S_T for safety and cost by e_j.

	Criteria	
Design options	Safety	Cost
$\boldsymbol{O_1}$	$(s_1^5, -0.3)$	$(s_3^5, -0.12)$
:	:	:
$\boldsymbol{O_m}$	$(s_0^5, 0.34)$	$(s_3^5, 0.23)$

4.2.1.b. Aggregation Process. Let suppose that the Table VI contains the unified safety and cost assessments by the expert e_j. This process combines the unified cost and safety assessments of all the experts to obtain a global evaluation value for each design option in a two-step process.

1. *Individual evaluation value*, $\boldsymbol{I_{ij}}$: In this step an evaluation value I_{ij} is computed by the expert e_j for the ith design options O_i according to its synthesized and unified cost and safety assessments. To compute this evaluation value, we propose the use of the weighted aggregation operator (Definition A.2.4). We have a set of pairs of assessments $\{(s_i, \alpha^S), (c_i, \alpha^C)\}$ expressed in S_T for each design option. Taking into account Remark 2, since the cost assessments have a decreasing interpretation for the suitability, the individual evaluation I_{ij} for the ith design option by e_j is obtained using the following expression:

$$
\begin{aligned}
I_{ij} &= W\_AM^*((s_{ij}, \alpha^S), (c_{ij}\alpha^C)) \\
&= \Delta(\Delta^{-1}(s_{ij}, \alpha^S)\cdot\omega + \Delta^{-1}(Neg(c_{ij}, \alpha^C))\cdot(1-\omega)) = (s_i^5, \alpha)
\end{aligned}
$$

   where $Neg(c_{ij}, \alpha)$ (see the Appendix for the 2-tuple Neg operator) is the assessment for the cost of the ith design option synthesized from e_j taking into account its decreasing interpretation and (s_{ij}, α^S) is the assessment for the safety of the ith design option. And ω is the weight for the safety assessment and $1-\omega$ the weight for the cost assessment. This individual evaluation value is initially expressed in S_T but we want to express it in S_E so we have to translate the results from S_T to S_E just using the S_E syntax. Let us suppose a value of $\omega = 0.6$. We obtain the individual evaluations from Table VII. These individual values are expressed in terms assessed in S_E as in Table VIII.
2. *Global evaluation value*, G_j: So far, we have an individual suitability value, I_{ij} ($i = 1,\ldots,m; j = 1,\ldots,p$), of the ith design option expressed by means of a linguistic 2-tuple in S_T and S_E by the expert e_j. To obtain a global suitability assessment for each design option, we shall apply another aggregation operator to the individual assessments of all

Table VII. Individual evaluation value for design options in S_T by e_j.

Options	Utility
$\boldsymbol{O_1}$	$(s_1^5, -0.13)$
:	:
$\boldsymbol{O_m}$	$(s_1^5, -0.49)$

Table VIII. Individual evaluation value for design options in S_E by e_j.

Options	Utility
$\boldsymbol{O_1}$	$(\textit{Moderated Preferred}, -0.13)$
:	:
$\boldsymbol{O_m}$	$(\textit{Moderated Preferred}, -0.49)$

experts. Now, we could consider that all the experts are equally important (arithmetic mean) or we could assign different weights to each expert (weighted average). Let us suppose we consider all the experts equally important; the global evaluation value G_i is given as

$$G_i = AM^*((I_{ij})) = \Delta\left(\sum_{j=1}^{p} \frac{1}{p}\,\Delta^{-1}(I_{ij})\right), \quad (i = 1,\ldots,m; j = 1,\ldots,p)$$

Initially this value is obtained in S_T but afterward will be expressed in S_E.

4.2.2. Exploitation Phase

Finally the decision process applies a choice degree to obtain a selection set of alternatives. Different choice functions have been proposed in the choice theory literature.[34–36] The choice functions rank the alternatives according to different possibilities and from the ranking the best ones are obtained.

In our problem the information is expressed by means of the linguistic 2-tuple representation model that has defined a total order over itself. Then we can rank the results using this order.

5. EXAMPLE

In this section we shall develop an example for evaluating different designs for a floating production storage offloading (FPSO) systems[4] according to the safety and cost analysis of those designs. It is essential that anticipated hazards due to technical factors be identified, risk control options proposed, and risk reduction or control measures taken to reduce the risk to as low as reasonably practical (ALARP). Scenarios involving potential major hazards that might threaten an FPSO or loss of operational control are assessed at an early stage in the design of new facilities to optimise technical and operational solutions.[37]

The safety assessment is carried out on risk introduced by the collision of FPSO and shuttle tanker during tandem offloading operation. Only technical failures caused risk is assessed here, though the operational failure has been also recognized as one of the major causes of collision.

According to the literature survey, the technical failures that might cause collisions between an FPSO and a shuttle tanker during tandem offloading operations are malfunction of propulsion systems.[4] The four major causes to these technical failures are:

(1) controllable pitch propeller (CPP) failure
(2) thruster failure
(3) position reference system (PRS) failure
(4) dynamics positioning (DP) system failure

The cost factors evaluated in this case are:

(1) cost for provision of redundancies of critical components (f_1)
(2) provision of protection systems (f_2)
(3) alarm systems (f_3)
(4) use of more reliable components (f_4)

For the purpose of safety modeling, it is assumed that each input parameter (i.e., **FR**, **CS**, and **FCP**) will be fed in term of any one of the four input forms described in Subsection 2.1: numerical values, interval values, parametrical fuzzy numbers. Moreover, seven levels of linguistic variable may be used for **FC**, five levels for **CS**, and seven levels for **FCP**.

Suppose a panel of experts from different disciplines participated in the analyses of three designs. Each one focuses on the above identified four causes to technical failures, which result in collision between an FPSO and a shuttle tanker. They will use different input assessments to describe the collision risk scenarios in terms of **FR**, **CS**, and **FCP**. And the linguistic term set S_C to express the cost incurred for the safety improvement in the different designs. In the following, we consider design option O_1 in detail for the illustration purpose of safety and cost assessment and synthesis. The final results only will be shown for the other two designs.

5.1. Safety Assessments

For each design, the safety estimate of each technical failure is assessed by five experts separately. Considering design option O_1, the assessment made by the four experts in terms of **FR**, **CS**, and **FCP** is depicted in Table IXa for collision between an FPSO and a shuttle tanker during tandem offloading operation due to a CPP caused technical failure. Other three types of assessments are depicted in Table IXb, Table IXc, and Table IXd, respectively.

The expert e_1 used the triangular distribution input form to address the inherent uncertainty associated with the data and information available while carrying out an assessment on the three input parameters. The **FR** is described triangularly as (6.5, 8.0, 9.5), the **CS** as (7.5, 8.5, 9.5), and the **FCP** as (5.5, 7.0, 8.5).

In the rule base, 245 rules with belief structure have been established.[25] The evaluation of ***safety estimate*** for design O_1 made by the experts for collision risk between FPSO and shuttle tanker due to CPP caused technical failure are performed separately according to the general safety modeling framework in Section 2.1 using the FURBER approach.

The Window-based and graphically designed intelligent decision system (IDS),[38] which has been developed based on the evidential reasoning approach, is used to implement the combination of the rules and generate safety estimates. The final assessment result for CPP by expert e_1 is obtained generated as follows:

$$\{(Good, 0), (High, 0), (Average, 0.0057), (Fair, 0.3735), (Poor, 0.6208)\}$$

Table IXa. Expert judgment for technical failure caused by malfunction of the CPP.

Expert	**FR**	**CS**	**FCP**
e_1	(6.5, 8, 9.5)	(7.5, 8.5, 9.5)	(5.5, 7, 8.5)
e_2	(5.5, 7.5, 9)	(7, 8.5, 10)	(5, 7.5, 9.5)
e_3	[6, 8]	[7, 9]	[6.5, 9]
e_4	{5.5, 6.5, 9, 10}	{5.5, 7, 8, 10}	{5, 7, 8, 8.5}
e_5	7.75	8.25	7.6

Table IXb. Expert judgment for technical failure caused by malfunction of the thruster.

Expert	**FR**	**CS**	**FCP**
e_1	(6, 7, 7.5)	(6.5, 7, 8)	(4.5, 5.5, 6)
e_2	(6, 6.5, 8)	(7, 8, 9)	(6, 7.5, 8)
e_3	[5.5, 7.5]	[6, 8]	[6, 8]
e_4	{5, 6, 7, 8}	{5, 7, 8, 9}	{5, 6, 7, 9}
e_5	7.15	7.95	7.25

Table IXc. Expert judgment for technical failure caused by malfunction of the PRS.

Expert	**FR**	**CS**	**FCP**
e_1	(6.5, 7, 7.5)	(8, 8.5, 9)	(5.5, 7, 8)
e_2	(6, 7.5, 8)	(7.5, 8, 9.5)	(5, 6, 7)
e_3	[6.5, 8]	[7, 7.5]	[6.5, 7.5]
e_4	{6, 7, 8, 9}	{5, 7, 8, 8.5}	{6, 7, 8, 9}
e_5	7.5	7.2	7.1

Table IXd. Expert judgment for technical failure caused by malfunction of the DP.

Expert	**FR**	**CS**	**FCP**
e_1	(7, 7.5, 8)	(7.5, 8.5, 9)	(6, 7, 7.5)
e_2	(6.5, 7, 8)	(6.5, 7, 8.5)	(5.5, 6, 7)
e_3	[7, 9]	[7.5, 9.5]	[7, 8]
e_4	{6.5, 7, 7.5, 8}	{6, 6.5, 7, 8}	{6.5, 7, 7.5, 9}
e_5	7.95	8.25	7.9

This result can be interpreted in such a way that the safety estimate of CPP to technical failure is "*Average*" with a belief degree of 0.0057, "*Fair*" with a belief degree of 0.3735, and "*Poor*" with a belief degree of 0.6208. The similar computations are performed for the other four experts for CPP, and for the other three potential causes (the Thruster, PRS, and DP) to technical failure. The safety estimate results estimates attained generated for CPP, thrusters, PRS, and DP caused technical failure by five experts are summarized in Tables Xa–d, respectively.

Furthermore, these safety assessment results are transformed into linguistic 2-tuple values using Equation 11, for example, the linguistic 2-tuple value of assessment by the expert e_1 for CPP caused technical failure of design O_1 is

$$\chi_1^1(\{(s_t, \vartheta_{t1}^1), t = 0, \ldots, 4\}) = \frac{\sum_{t=0}^{4} t\vartheta_{t1}^1}{\sum_{t=0}^{4} \vartheta_{t1}^l} = 0.3849 = (Poor, 0.3849)$$

The similar computation is performed for safety assessment of design O_1 by the other four experts using the FURBER approach for the CPP caused technical failure and the other three potential causes to technical failure, and the results attained for thrusters, PRS, and DP caused technical failure by five experts are shown in Table XI.

From these values we obtain a safety value for the design O_1 for each expert using the arithmetic mean for linguistic 2-tuples as shown in Table XII. The similar computations are performed for safety assessment for other two design options. The results are summarized in Table XIII.

5.2. Cost Assessments

Let us suppose the linguistic cost assessments are provided by the experts in the linguistic term set S_C for the corresponding factors of the safety improvement in design option O_1. These values are shown in Table XIV.

To synthesize a cost value for the design option O_1, we aggregate these assessments with arithmetic mean for linguistic 2-tuples, leading to the results in Table XV.

The similar computations are performed for cost assessment for the other two design options. The results are summarized in Table XVI.

5.3. Decision Process

Now we apply the decision model presented in Section 4 using as input the cost and safety assessments synthesized in the Tables XV and XVI.

5.3.1. Aggregation Phase

5.3.1.a. Normalization Process. The assessments for the safety and cost are transformed into linguistic 2-tuples assessed in S_T (shown in the Tables XVII and XVIII).

Table Xa. Safety estimate by each expert on collision risk between FPSO and shutter tanker due to CPP caused technical failure.

				Safety estimate				
E	**FR**	**CS**	**FCP**	*Good*	*High*	*Average*	*Low*	*Poor*
e_1	(6.5, 8, 9.5)	(7.5, 8.5,9.5)	(5.5, 7, 8.5)	0	0	0.0057	0.3735	0.6208
e_2	(5.5, 7.5, 9.0)	(7, 8.5, 10)	(5, 7.5, 9.5)	0	0	0.0075	0.3750	0.6175
e_3	[6, 8]	[7, 9]	[6.5, 9]	0	0	0.0033	0.3090	0.6876
e_4	{5.5,6.5,9,10}	{5.5,7,8, 10}	{5, 7, 8, 8.5}	0	0	0.0233	0.4751	0.5016
e_5	7.75	8.25	7.6	0	0	0.0123	0.3641	0.6236

Table Xb. Safety estimate by each expert on collision risk between FPSO and shutter tanker due to the thruster caused technical failure.

				Safety estimate				
E	**FR**	**CS**	**FCP**	*Good*	*High*	*Average*	*Low*	*Poor*
e_1	(6, 7, 7.5)	(6.5, 7, 8)	(4.5, 5.5, 6)	0	0	0.0373	0.7802	0.1825
e_2	(6, 6.5, 8)	(7, 8, 9)	(6, 7.5, 8)	0	0	0.0640	0.4165	0.5195
e_3	[5.5, 5.5, 7.5, 7.5]	[6, 6, 8, 8]	[6, 6, 8, 8]	0	0	0.0375	0.6503	0.3122
e_4	{5, 6, 7, 8}	{5, 7, 8, 9}	{5, 6, 7, 9}	0	0	0.0274	0.5540	0.4186
e_5	7.15	7.95	7.25	0	0	0.0013	0.4179	0.5808

Table Xc. Safety estimate by each expert on collision risk between FPSO and shutter tanker due to the PRS caused technical failure.

				Safety estimate				
E	**FR**	**CS**	**FCP**	*Good*	*High*	*Average*	*Low*	*Poor*
e_1	(6.5, 7, 7.5)	(8, 8.5, 9)	(5.5, 7, 8)	0	0	0.0047	0.6151	0.3802
e_2	(6, 7.5, 8)	(7.5, 8, 9.5)	(5, 6, 7)	0	0	0.0041	0.6142	0.3817
e_3	[6.5,6.5,8,8]	[7,7,7.5, 7.5]	[6.5,6.5,7.5,7.5]	0	0	0.0055	0.3845	0.6100
e_4	{6, 7, 8, 9}	{5, 7, 8,8.5}	{6, 7, 8, 9}	0	0	0.0204	0.5111	0.4685
e_5	7.5	7.2	7.1	0	0	0.0080	0.3694	0.6226

Table Xd. Safety estimate by each expert on collision risk between FPSO and shutter tanker due to the DP system caused technical failure.

				Safety estimate				
E	**FR**	**CS**	**FCP**	*Good*	*High*	*Average*	*Low*	*Poor*
e_1	(7, 7.5, 8)	(7.5, 8.5, 9)	(6, 7, 7.5)	0	0	0.0102	0.3595	0.6303
e_2	(6.5, 7, 8)	(6.5, 7, 8.5)	(5.5, 6, 7)	0	0	0.0097	0.6926	0.2977
e_3	[7, 7, 9, 9]	[7.5,7.5,9.5,9.5]	[7, 7, 8, 8]	0	0	0.0097	0.3930	0.5973
e_4	{6.5,7,7.5,8}	{6, 6.5, 7, 8}	{6.5,7,7.5,9}	0	0	0.0200	0.5733	0.4067
e_5	7.95	8.25	7.9	0	0	0.0256	0.2688	0.7056

Table XI. Safety estimate by each expert on collision risk between FPSO and shutter tanker.

	e_1	e_2	e_3	e_4	e_5
CPP	(Poor, 0.3849)	(Poor, 0.39)	(Poor, 0.3156)	(Low, −0.4783)	(Poor, 0.3887)
Thruster	(Low,−0.1452)	(Low, −0.4555)	(Low, −0.2747)	(Low, −0.3912)	(Poor, 0.4205)
PRS	(Low, 0.3755)	(Low, −0.3776)	(Poor, 0.3955)	(Low, −0.4481)	(Poor, 0.3854)
DP	(Poor, 0.3799)	(Low, −0.288)	(Poor, 0.4124)	(Low, −0.3867)	(Poor, 0.32)

Table XII. Safety estimate on collision risk between FPSO and shutter tanker for design O_1.

	e_1	e_2	e_3	e_4	e_5
O_1	(Low, −0.2523)	(Low, −0.4301)	(Poor, 0.4622)	(Low, −0.4261)	(Poor, 0.37865)

Table XIII. Safety estimate for all the design options.

	e_1	e_2	e_3	e_4	e_5
O_1	(Low, −0.2523)	(Low,− 0.4301)	(Poor, 0.4622)	(Low, −0.4261)	(Poor, 0.3786)
O_2	(Poor, 0.4599)	(Low, −1248)	(Poor, 0.3823)	(Low, −0.2857)	(Poor, 0.4132)
O_3	(Low, 0.0755)	(Low, −0.4761)	(Poor, 0.2953)	(Low, −0.4112)	(Poor, 0.4351)

Table XIV. Cost estimated by each expert.

	e_1	e_2	e_3	e_4	e_5
f_1	High	Moderately high	Average	High	Very high
f_2	High	Very high	Very high	Very high	High
f_3	Average	Very high	High	Very high	Very high
f_4	Very high	Moderately high	Average	High	Very high

Table XV. Cost assessments for design O_1 by each expert.

	e_1	e_2	e_3	e_4	e_5
O_1	(High, −0.25)	(High, 0)	(Moderately high, 0.25)	(High, −0.5)	(Very high, −0.25)

Table XVI. Cost assessments for all the design options by each expert.

	e_1	e_2	e_3	e_4	e_5
O_1	(High, −0.25)	(High, 0)	(Moderately high, 0.25)	(High, −0.5)	(Very high, −0.25)
O_2	(Moderately high, −0.25)	(High, 0.25)	(High, 0)	(Very high, 0)	(High, −0.25)
O_3	(Very high, 0.25)	(High,-0.25)	(High, −0.25)	(High, 0.25)	(High, 0.25)

Table XVII. Safety estimate on collision risk for the design options in S_T.

	e_1	e_2	e_3	e_4	e_5
O_1	$(s_1^5, -0.2523)$	$(s_1^5, -0.4301)$	$(s_0^5, 0.4622)$	$(s_1^5, -0.4261)$	$(s_0^5, 0.3786)$
O_2	$(s_0^5, 0.4599)$	$(s_1^5, -0.1248)$	$(s_0^5, 0.3823)$	$(s_1^5, -0.2857)$	$(s_0^5, 0.4132)$
O_3	$(s_1^5, 0.0755)$	$(s_1^5, -0.4761)$	$(s_0^5, 0.2953)$	$(s_1^5, -0.4112)$	$(s_0^5, 0.4351)$

Table XVIII. Cost assessments for the design options in S_T.

	e_1	e_2	e_3	e_4	e_5
O_1	$(s_3^5, -0.125)$	$(s_3^5, 0)$	$(s_3^5, -0.375)$	$(s_3^5, -0.25)$	$(s_3^5, 0.375)$
O_2	$(s_2^5, 0.375)$	$(s_3^5, 0.125)$	$(s_3^5, 0)$	$(s_4^5, -0.5)$	$(s_3^5, -0.125)$
O_3	$(s_4^5, -0.375)$	$(s_3^5, -0.125)$	$(s_3^5, -0.125)$	$(s_3^5, -0.125)$	$(s_3^5, 0.125)$

5.3.1.b. Aggregation Process. In this process, first we shall obtain an individual value I_{ij} for each expert and each design according his cost and safety assessments. To do so, it is used the following aggregation expression:

$$I_{ij} = W\_AM^*((s_{ij}, \alpha^S), (c_{ij}, \alpha^C))$$

$$= \Delta(\Delta^{-1}(s_{ij}, \alpha^S)\cdot\omega + \Delta^{-1}(Neg(c_{ij}, \alpha^C))\cdot(1-\omega))$$

In this case we use $\omega = 0.6$. Hence, the individual values I_{ij} for each design is summarized in Table XIX.

Now we use the arithmetic mean for 2-tuple is used again to get the global evaluation values for each design, which are shown in Table XX. These values are computed in terms of S_T but finally expressed by means of terms of S_E that it is a simple translation.

Table XIX. The individual utility assessments in S_T for each design by each expert.

	e_1	e_2	e_3	e_4	e_5
O_1	$(s_1^5, -0.1020)$	$(s_1^5, -0.2580)$	$(s_1^5, -0.1726)$	$(s_1^5, -0.1557)$	$(s_0^5, 0.4771)$
O_2	$(s_1^5, -0.0741)$	$(s_1^5, 0.0500)$	$(s_1^5, -0.3706)$	$(s_1^5, -0.3715)$	$(s_1^5, -0.3021)$
O_3	$(s_1^5, -0.2047)$	$(s_1^5, -0.2360)$	$(s_1^5, -0.3729)$	$(s_1^5, -0.2968)$	$(s_1^5, -0.3890)$

Table XX. The global utility assessments in S_E for each design.

O_1	O_2	O_3
(Moderately preferred, -0.2423)	(Moderately preferred, -0.2137)	(Moderately preferred, -0.2999)

5.3.2. Exploitation Phase

In our example the exploitation phase chooses the design option with the highest value; in our case the best design option is

O_2	(Moderately preferred, -0.2137)

6. CONCLUDING REMARKS

In this article we have presented an evaluation approach for design assessment of complex engineering systems based on safety and cost analysis. To develop this evaluation process and due to the vagueness and ignorance that is involved in the safety and cost criteria, we have used fuzzy rule-based evidential reasoning approach for safety assessment and the fuzzy linguistic approach for the cost assessments; finally a multigranular linguistic framework is used to synthesize the safety assessments and cost assessments. The use of a linguistic model facilitates the process of dealing with vagueness and uncertainty more than the use of traditional probabilistic models and tools. The linguistic approach provides a useful and natural way to support the solution of such complex decision problems.

Our proposal for evaluating design options before its implementation for a large engineering system is based on a cost and safety analysis that defines a multigranular linguistic framework where each criterion is conducted in different expression domains.

Once it has been defined as the evaluation framework that will be used by our evaluation problem, we have modeled it as an MEMC decision model that is able to deal with multigranular linguistic assessments without loss of information in order to evaluate and rank the different options.

Finally we have demonstrated the application of the new approach using an example of assessing three different possible designs in a large engineering system.

The proposed framework offered great potential in safety assessment of engineering systems, especially in the initial concept design stages or a system with a high level of innovation where the related safety information is scanty or with various types of uncertainty involved.

Acknowledgments

This work has been partially supported by the Research Project TIC 2002-03348, by the "Secretaría de Estado de Educación y Universidades" through the "Programa Nacional de ayudas para la movilidad de profesores de universidad e investigadores españoles y extranjeros" and by the U.K. Engineering and Physical Sciences Research Council (EPSRC) under Grant No. GR/S85498/01 and Grant No. GR/S85504/01.

References

1. Wang J, Yang JB, Sen P. Safety analysis and synthesis using fuzzy sets and evidential reasoning. Reliab Eng Syst Safety 1995;41:103–118.
2. Sii HS, Wang J. A subjective design for safety framework for offshore engineering products. Workshops on Reliability and Risk Based Inspection Planning and ESRA Technical Committee on Offshore Safety, December 14–15. Zurich, Switzerland: Swiss Federal Institute of Technology; 2000.
3. Adamopoulos GP Pappis GP. A fuzzy linguistic approach to a multicriteria sequencing problem. Eur J Oper Res 1996;92:628–636.
4. Cheng C-H, Lin Y. Evaluating the best main battle tank using fuzzy decision theory with linguistic criteria evaluation. Eur J Op Res 2002;142:174–186.
5. Liang GS, Wang MJJ. Personnel selection using fuzzy MCDM algorithm. Eur J Op Res 1994;78:22–33.
6. Liu CM, Wang MJ, Pang YS. A multiple criteria linguistic decision model (MCLDM) for human decision making. Eur J Op Res 1994;76:466–485.
7. Yager RR. Non-numeric multi criteria multi-person decision making. Group Decision and Negotiation 1993;2:81–93.
8. Wang J, Ruxton T. Design for safety. J Am Soc Safety Eng 1997;January:23–29.
9. Zadeh LA. The concept of a linguistic variable and its applications to approximate reasoning. Part I: Inform Sci 1975;8:199–249. Part II: Inform Sci 1975;8:301–357. Part III: Inform Sci 1975;9:43–80.
10. An M, Wang J, Ruxton T. The development of fuzzy linguistic risk levels for risk analysis of offshore engineering products using approximate reasoning approach. Proc OMAE 2000, 19th Int Conf on Offshore Mechanics and Arctic Engineering, New Orleans, American Society for Mechanical Engineering; 2000.
11. Bell PM, Badiru AB. Fuzzy modelling and analytic hierarchy processing to quantify risk levels associated with occupational injuries—Part I: The development of fuzzy-linguistic risk levels. IEEE Trans Fuzzy Syst 1996;4(2):124–131.
12. Bowles JB, Pelaez CD. Fuzzy logic prioritisation of failures in a system failure mode effects and criticality analysis. Reliab Eng Syst Safety 1995;50:203–213.
13. Duckstein L. Elements of fuzzy set analysis and fuzzy risk. In: Nachtnebel HP, editor. Decision support systems in water resources management. Paris: UNESCO Press; 1994. pp 410–430.
14. Karwowski W, Mital A. Potential applications of fuzzy sets in industrial safety engineering. Fuzzy Sets Syst 1986;19:105–120.
15. Sii HS, Wang J, Ruxton T. A fuzzy-logic-based approach to subjective safety modelling for maritime products. J UK Safety Reliab Soc 2001;21(2):65–79.
16. Wang J, Yang JB, Sen P. Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analyses. Reliab Eng Syst Safety 1996;52(2):113–129.
17. Dempster AP. A generalization of Bayesian inference. J Roy Stat Soc B 1968;30:205–247.
18. Shafer G. A mathematical theory of evidence. Princeton, NJ: Princeton University Press; 1976.
19. Smets P. Belief functions in non-standard logics for automated reasoning. In: Smet P, Mamdani EH, Dubois D, Prade H, editors. London: Academic Press; 1988. pp 253–277.
20. Yang JB, Singh MG. An evidential reasoning approach for multiple attribute decision making with uncertainty. IEEE Trans Syst Man Cybern 1994;24(1):1–18.
21. Yang JB, Sen P. A general multi-level evaluation process for hybrid MADM with uncertainty. IEEE Trans Syst Man Cybern 1994;24(10):1458–1473.
22. Yang JB. Rule and utility based evidential reasoning approach for multi-attribute decision analysis under uncertainties. Eur J Op Res 2001;131:31–61.
23. Yang JB, Xu DL. On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty. IEEE Trans Syst Man Cybern A 2002;32(3):289–304.

24. Yang JB, Xu DL. Nonlinear information aggregation via evidential reasoning in multiattribute decision analysis under uncertainty. IEEE Trans Syst Man Cybern A 2002;32(3): 376–393.
25. Liu J, Yang JB, Wang J, Sii HS, Wang YW. Fuzzy rule-based evidential reasoning approach for safety analysis. Int J Gen Syst 2004;33:183–204.
26. Yang JB, Liu J, Wang J, Sii HS, Wang HW. A belief rule-base inference methodology using the evidential reasoning approach—RIMER. IEEE Trans Syst Man Cybern A 2005. In press.
27. Herrera F, Martínez L. A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Trans Fuzzy Syst 2000;8(6):746–752.
28. Herrera F, Martínez L. A model based on linguistic 2-tuples for dealing with multigranularity hierarchical linguistic contexts in multi-expert decision-making. IEEE Trans Syst Man Cybern B 2001;31(2):227–234.
29. Delgado M, Verdegav JL, Vila MA. Linguistic decision making models. Int J Intell Syst 1992;7:479–492.
30. Degam R, Bortolan G. The problem of linguistic approximation in clinical decision making. Int J Approx Reason 1988;2:143–162.
31. Herrera F, Martínez L. The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistenty. Int J Uncertain Fuzziness Knowl Based Syst 2001;9:33–48.
32. Roubens M. Fuzzy sets and decision analysis. Fuzzy Sets Syst 1997;90:199–206.
33. Herrera F, Martínez L. An approach for combining numerical and linguistic information based on the 2-tuple fuzzy linguistic representation model in decision making. Int J Uncertain Fuzziness Knowl Based Syst 2000;8(5):539–562.
34. Arrow KJ. Social choice and individual values. New Haven, CT: Yale University Press; 1963.
35. Orlovsky SA. Decision making with a fuzzy preference relation. Fuzzy Sets Syst 1978;1: 155–167.
36. Roubens M. Some properties of choice functions based on valued binary relations. Eur J Op Res 1989;40:309–321.
37. Sii HS, Wang J. Safety assessment of FPSOs—The process of modelling system safety and case studies. Report of the Project—The Application of Approximate Reasoning Methodologies to Offshore Engineering Design (EPSRC GR/R30624 and GR/R32413). Liverpool, UK: Liverpool John Moores University; 2002.
38. Yang JB, Xu DL. Intelligent decision system via evidential reasoning, Ver. 1.1. UK: IDSL; 1999.
39. Levrat E, Voisin A, Bombardier S, Bremont J. Subjective evaluation of car seat comfort with fuzzy set techniques. Int J Intell Syst 1997;12:891–913.
40. Bordogna G, Pasi G. A fuzzy linguistic approach generalizing boolean information retrieval: A model and its evaluation. J Am Soc Inform Sci 1993;44:70–82.
41. Herrera-Viedma E. Modelling the retrieval process of an information retrieval system using an ordinal fuzzy linguistic approach. J Am Soc Inform Sci Technol 2001;52(6): 460–475.
42. Yager RR, Goldstein LS, Mendels E. FUZMAR: An approach to aggregating market research data based on fuzzy reasoning. Fuzzy Sets Syst 1994;68:1–11.
43. Lee HM. Applying fuzzy set theory to evaluate the rate of aggregative risk in software development. Fuzzy Sets Syst 1996;80:323–330.
44. Chang P, Chen Y. A fuzzy multicriteria decision making method for technology transfer strategy selection in biotechnology. Fuzzy Sets Syst 1994;63:131–139.
45. Law CK. Using fuzzy numbers in educational grading system. Fuzzy Sets Syst 1996;83: 311–323.
46. Bordogna G, Pasi G. A linguistic modelling of consensus in group decision making based on OWA operators. IEEE Trans Syst Man Cybern A 1997;27:126–132.
47. Herrera-Viedma E, Herrera F, Chiclana F. A consensus model for multi-person decision making with different preference structures. IEEE Trans Syst Man Cybern A 2002;32(3): 394–402.

48. Chen SM. A new method for tool steel materials selection under fuzzy environment. Fuzzy Sets Syst 1997;92:265–274.
49. Herrera F, Herrera-Viedma E, Verdegay JL. A sequential selection process in group decision making with linguistic assessment. Inform Sci 1995;85:223–239.
50. Yager RR. An approach to ordinal decision making. Int J Approx Reason 1995;12:237–261.
51. Bonissone PP, Decker KS. Selecting uncertainty calculi and granularity: An experiment in trading-off precision and complexity. In: Kanal LH, Lemmer JF, editors. Uncertainty in artificial intelligence. Amsterdam: North-Holland; 1986. pp 217–247.
52. Miller GA. The magical number seven or minus two: Some limits on our capacity of processing information. Psychol Rev 1956;63:81–97.

APPENDIX A: LINGUISTIC BACKGROUND

In this section we shall review some core concepts about linguistic information that we shall use in this article. First, we will briefly review the *Fuzzy Linguistic Approach*, and afterward we shall revise the *Linguistic 2-tuple representation model* and its computational model as well as the *Linguistic Hierarchies*.

A.1. Fuzzy Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables.[9] This approach is adequate in some situations; for example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language. This may arise for different reasons. There are some situations where the information may be unquantifiable due to its nature, and thus it may be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car, terms like "bad," "poor," "tolerable," "average," and "good" can be used[39]). In other cases, precise quantitative information may not be stated because either it is not available or the cost of its computation is too high, then an "approximate value" may be tolerated (e.g., when evaluating the speed of a car, linguistic terms like "fast," "very fast," and "slow" are used instead of numerical values).

The fuzzy linguistic approach has been applied with very good results to different problems, such as "information retrieval,"[40,41] "clinical diagnosis,"[30] "marketing,"[42] "risk in software development,"[43] "technology transfer strategy selection,"[44] "educational grading systems,"[45] "scheduling,"[3] "consensus,"[46,47] "materials selection,"[48] "decision-making,"[29,49,50] etc.

We have to choose the appropriate linguistic descriptors for the term set and their semantics. To accomplish this objective, an important aspect to analyze is the "granularity of uncertainty," i.e., the level of discrimination among different counts of uncertainty. The universe of the discourse over which the term set is defined can be arbitrary, usually linguistic term sets are defined in the interval $[0, 1]$. In

Ref. 51 the use of term sets with an odd cardinal was studied, representing the midterm by an assessment of "approximately 0.5," with the rest of the terms being placed symmetrically around it and with typical values of cardinality, such as 7 or 9. These classical cardinality values seem to satisfy Miller's observation that human beings can reasonably manage to bear in mind seven or so items.[52]

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined.[49,50] For example, a set of seven terms S could be given as follows:

$$S = \{s_0\text{: None; } s_1\text{: Very Low; } s_2\text{: Low; } s_3\text{: Medium;}\\ s_4\text{: High; } s_5\text{: Very High; } s_6\text{: Perfect}\}$$

In these cases, it is usually required that there exist

- a negation operator $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality)
- a minimization and a maximization operator in the linguistic term set: $s_i \Leftarrow s_j \Leftrightarrow i \Leftarrow j$.

The semantics of the terms is given by fuzzy numbers defined in the [0,1] interval, which are described by membership functions. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function.[51] Since the linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible and unnecessary to obtain more accurate values.[29]

This parametric representation is achieved by the 4-tuple (**a**, **b**, **d**, **c**), where **b** and **d** indicate the interval in which the membership value is 1, with **a** and **c** indicating the left and right limits of the definition domain of the trapezoidal membership function.[51] A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., b = d, so we represent this type of membership function by a 3-tuple (a; b; c). For example, we may assign the following semantics to the set of seven terms:

$$\begin{aligned} \mathrm{P} &= \text{Perfect} = (0.83; 1; 1)\\ \mathrm{VH} &= \text{Very High} = (0.67; 0.83; 1)\\ \mathrm{H} &= \text{High} = (0.5; 0.67; 0.83)\\ \mathrm{M} &= \text{Medium} = (0.33; 0.5; 0.67)\\ \mathrm{L} &= \text{Low} = (0.17; 0.33; 0.5)\\ \mathrm{VL} &= \text{Very Low} = (0; 0.17; 0.33)\\ \mathrm{N} &= \text{None} = (0; 0; 0.17) \end{aligned}$$

which is graphically shown in Figure A1. Other authors use a nontrapezoidal representation, e.g., Gaussian functions.[40]

The most often used models for dealing with linguistic information are (i) the semantic model[30] that uses the linguistic terms just as labels for fuzzy numbers, while the computations over them are done directly over those fuzzy numbers; (ii) the second one is the symbolic model[29] that uses the order index of the linguistic

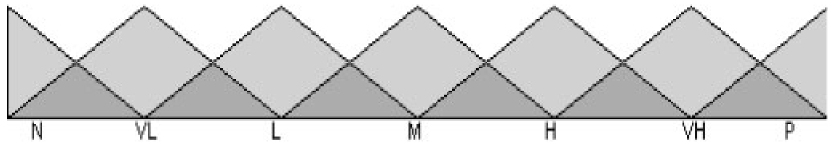

Figure A1. A set of seven terms and their semantics.

terms to make direct computations on labels. However, our proposal for computing with cost and safety linguistic assessments assessed in different utility spaces takes as representation base the linguistic 2-tuple representation model presented in Ref. 27 that has shown itself as a good choice to manage nonhomogeneous information.[28,33] In the following subsection we review this representation model.

A.2. The 2-Tuple Linguistic Representation Model

This model was presented in Ref. 27 for overcoming the drawback of the loss of information presented by the classical linguistic computational models:[33] (i) the semantic model and (ii) the symbolic one. The 2-tuple fuzzy linguistic representation model is based on the symbolic method and takes as the base of its representation the concept of Symbolic Translation.

DEFINITION A.2.1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, \ldots, s_g\}$ is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between an amount of information $\beta \in [0, g]$ and the closest value in $\{0, \ldots, g\}$ that indicates the index of the closest linguistic term in S (s_i), being $[0, g]$ the interval of granularity of S.*

From this concept a new linguistic representation model is developed, which represents the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$.

This model defines a set of functions between linguistic 2-tuples and numerical values.

DEFINITION A.2.2. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \rightarrow S \times (-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \textit{ with } \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0,5) \end{cases}$$

where round(·) is the usual round operation, s_i has the closest index label to "β," and "α" is the value of the symbolic translation.

PROPOSITION A.2.1. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α_i) be a linguistic 2-tuple. There is always a Δ^{-1} function, such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$ in the interval of granularity of S.*

Proof. It is trivial; we consider the following function:

$$\Delta^{-1} : S \times [-0,5,0.5) \rightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$ ■

Remark A.2.1. From Definitions 1 and 2 and Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \Rightarrow (s_i, 0)$$

This model has a computational technique based on the 2-tuples that were presented in Ref. 27.

1. *Aggregation of 2-tuples.* The aggregation of linguistic 2-tuples consist of obtaining a value that summarizes a set of values; therefore, the result of the aggregation of a set of 2-tuples must be a linguistic 2-tuple. In Ref. 27 we can find several 2-tuple aggregation operators based on classical aggregation operators as the arithmetic mean and weighted mean operators:

DEFINITION A.2.3. *Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of 2-tuples; the extended Arithmetic Mean AM^* using the linguistic 2-tuples is computed as*

$$AM^*((r_1, \alpha_1), \ldots, (r_n, \alpha_n)) = \Delta\left(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n} \sum_{i=1}^{n} \beta_i\right)$$

DEFINITION A.2.4. *Let $\{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of linguistic 2-tuples and $W = \{w_1, \ldots, w_n\}$ his associated weights. The 2-tuple weighted mean, W_AM^*, is computed as*

$$W\_AM^*((r_1, \alpha_1), \ldots, (r_n, \alpha_n)) = \Delta\left(\frac{\sum_{i=1}^{n} \Delta^{-1}(r_i, \alpha_i) \cdot w_i}{\sum_{i=1}^{n} w_i}\right) = \Delta\left(\frac{\sum_{i=1}^{n} \beta_i \cdot w_i}{\sum_{i=1}^{n} w_i}\right)$$

2. *Comparison of 2-tuples.* The comparison of information represented by 2-tuples is carried out according to an ordinary lexico-graphic order. Let (s_k, α_1) and (s_l, α_2) be two 2-tuples represented two assessments:

- If $k < l$ then (s_k, α_1) is smaller than (s_l, α_2).
- If $k = l$ then
  (1) If $\alpha_1 = \alpha_2$ then (s_k, α_1) and (s_l, α_2) represent the same value.
  (2) If $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2).
  (3) If $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2).

3. *Negation operator of a 2-tuple*. The negation operator over 2-tuples is defined as

$$Neg(s_i, \alpha) = \Delta(g - \Delta^{-1}(s_i, \alpha))$$

where $g + 1$ is the cardinality of $\boldsymbol{S}$, $s_i \in S = \{s_0, \ldots, s_g\}$.