Group Decision Making With Incomplete Information

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Resumen

In Group Decision Making problems experts have to provide their preferences about some alternatives to solve a particular problem. Fuzzy Preference Relations are a very widely preference representation format that experts can use to express their opinions about the alternatives. But due to their own personal background and abilities experts are not often capable of provide complete and consistent preference relations.

In this work we present two different tools that can be used to solve GDM problems where the experts give their preferences by means of incomplete fuzzy preference relations. These tools are an iterative procedure capable of estimating missing information on the incomplete fuzzy preference relations and the AC-IOWA operator, capable of aggregating the information provided by the experts into a global fuzzy preference relations that summarizes the experts opinions and that takes into account the consistency expressed by each expert to give more importance to the most consistent ones.

Keywords: Group Decision Making, Incomplete Information, Fuzzy Preference Relations, Additive Consistency.

1 Introduction

Decision-making procedures are increasingly being used in various different fields for evaluation, selection and prioritisation purposes, that is, making preference decisions about a set of different choices. Furthermore, it is also obvious that the comparison of different alternative actions according to their desirability in decision problems, in many cases, cannot be done using a single criterion or one person. Indeed, in the majority of decision making problems, procedures have been established to combine opinions about alternatives related to different points of view. These procedures are based on pair comparisons, in the sense that processes are linked to some degree of credibility of preference of one alternative over another. Many different representation formats can be used to express preferences. *Fuzzy preference relation* is one of these formats, and it is usually used by an expert to provide his/her preference degrees when comparing pairs of alternatives [1, 3, 6, 8].

Since each expert is characterised by their own personal background and experience of the problem to be solved, experts' opinions may differ substantially (there are plenty of educational and cultural factors that influence an expert's preferences). This diversity of experts could lead to situations where some of them would not be able to efficiently express any kind of preference degree between two or more of the available options. Indeed, this may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. In these situations such an expert is forced to provide an *incomplete fuzzy preference relation* [10].

Usual procedures for multi-person decision-making problems correct this lack of knowledge of a particular expert using the information provided by the rest of the experts together with aggregation procedures [7]. These approaches have several disadvantages. Among them we can cite the requirement of multiple experts in order to learn the missing value of a particular one. Another drawback is that these procedures normally do not take into account the differences between experts' preferences, which could lead to the estimation of a missing value that would not naturally be compatible with the rest of the preference values given by that expert. Finally, some of these missing information-retrieval procedures are interactive, that is, they need experts to collaborate in "real time", an option which is not always possible.

Our proposal is quite different to the above procedures. We put forward an iterative procedure which attempts to estimate the missing information in an expert's incomplete fuzzy preference relation, using only the preference values provided by that particular expert. By doing this, we assure that the reconstruction of the incomplete fuzzy preference relation is compatible with the rest of the information provided by that expert. In fact, the procedure we propose in this paper is guided by the expert's consistency which is measured taking into account only the provided preference values. Thus, an important objective in the design of our procedure is to maintain experts' consistency levels. In particular, in this paper we use the additive consistency property [5] to define a consistency measure of the expert's information.

Once the missing information in a fuzzy preference relation is estimated we can find the solution to our GDM problem using a particular Decision Model. To do so we have to carry out different actions, and one of the most important ones is aggregating the information from every expert in order to achieve a global fuzzy preference relation which summarizes all the information provided by the different experts. We have also developed an aggregation operator, the Additive-Consistency Induced Ordered Weighted Averaging (AC-IOWA) which is capable of aggregating the fuzzy preference relations giving more importance to the oppinions of the most consistent experts (we asume that consistent information is more valuable than inconsistent one).

In order to do this, the paper is set out as follows: In *Section 2* we present our preliminaries: the incomplete fuzzy preference relation definition and the additive consistency property. In *Section 3* we present our iterative procedure to estimate missing information on incomplete fuzzy preference relations. *Section 4* presents the AC-IOWA operator and how can both the estimation procedure and the AC-IOWA operator be integrated into a complete decision model. Finally, in *Section 5* we point out some conclusions and briefly describe our current developments in this line of work.

2 Preliminaries

In this section we will present the preliminary concepts that will be used in the following sections. First of all we will define what an incomplete fuzzy preference relation is and laterly we will describe the additive consistency property, on which our iterative estimation procedure is based.

2.1 GDM with Incomplete Fuzzy Preference Relations

The problem we deal with is that of choosing the best alternative(s) among a finite set, $X = \{x_1, ..., x_n\}, (n \geq 2)$. The alternatives will be classified from best to worst, using the information known according to a set of experts, i.e., $E = \{e_1, ..., e_m\} (m \geq 2)$.

Each expert $e_k \in E$, will provide his preferences by means of a *fuzzy preference relation* which is one of the most common representation formats of information used in decision-making problems due to their effectiveness as a tool for modelling decision processes and, above all, their utility and easiness of use when we want to aggregate experts' preferences into group preferences [3, 5, 6, 9]. In particular, they have been used in the development of many important decision-making procedures.

Definition [6, 8]. A fuzzy preference relation P on a set of alternatives X is a fuzzy set on the product set $X \times X$, that is, is characterized by a membership function

$$\mu_P : X \times X \longrightarrow [0, 1]$$

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ij})$ being $p_{ij} = \mu_P(x_i, x_j) \quad \forall i, j \in \{1, \ldots, n\}$ interpreted as the preference degree or intensity of the alternative x_i over x_j: $p_{ij} = 1/2$ indicates indifference between x_i and x_j $(x_i \sim x_j)$, $p_{ij} = 1$ indicates that x_i is absolutely preferred to x_j, and $p_{ij} > 1/2$ indicates that x_i is preferred to x_j $(x_i \succ x_j)$. Based on this interpretation we have that $p_{ii} = 1/2 \quad \forall i \in \{1, \ldots, n\}$ $(x_i \sim x_i)$.

As we have mentioned, due the fact that every expert has his own experience on the problem being studied, a general drawback is the lack of knowledge that experts may have. They should have the tools to be able to express this lack of knowledge in the fuzzy preference relations they provide. It is important to remark the fact that an expert not being able to express every p_{ij} value, because he doesn't have a clear idea of how better is the alternative x_i over the alternative x_j, does not mean that he thinks that both options are equally preferred to be chosen ($x_i \sim x_j$ or $p_{ij}, p_{ji} = 1/2$).

To solve this issue we introduce the new concept of *incomplete fuzzy preference relation*, which allow to express this kind of situations where some p_{ij} values are missing:

Definition. A function $f : X \longrightarrow Y$ is *partial* when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a *total* function.

Definition. An *incomplete fuzzy preference relation* P on a set of alternatives X is a fuzzy set on the product set $X \times X$ that is characterized by a *partial* membership function.

Following this last definition, we call a fuzzy preference relation complete when its membership function is a total one. Clearly, the usual definition of a fuzzy preference relation includes both definitions of complete an incomplete fuzzy preference relations. However, as there is no risk of confusion between a complete and an incomplete fuzzy preference relation, in this paper we will refer to the first type as simply fuzzy preference relations.

In the case of an incomplete fuzzy preference relation there exists at least a pair of alternatives (x_i, x_j) for which p_{ij} is unknown. We will introduce and use throughout this paper the letter x to represent these unknown preference values, that is $p_{ij} = x$.

2.2 Additive Consistency

The definition of a fuzzy preference relation does not imply any kind of consistency. In fact, preferences expressed in the fuzzy preference relation can be contradictory. As studied in [5], for making a rational choice, a set of properties to be satisfied by such fuzzy preference relations have been suggested. Transitivity is one of the most important properties concerning preferences, and it represents the idea that the preference value obtained by comparing directly two alternatives should be equal to or greater than the preference value between those two alternatives obtained using an indirect chain of alternatives. One of these properties is the *additive transitivity* [9]:

$$(p_{ij}-0.5)+(p_{jk}-0.5) = (p_{ik}-0.5) \;\; \forall i, j, k \in \{1, \ldots, n\} \quad (1)$$

This kind of transitivity has the following interpretation: suppose we do want to establish a ranking between three alternatives x_i, x_j and x_k. If we do not have any information about these alternatives it is natural to start assuming that we are in an indifference situation, that is, $x_i \sim x_j \sim x_k$, and therefore when giving preferences this situation is represented by $p_{ij} = p_{jk} = p_{ki} = 0.5$. Suppose now that we have a piece of information that says alternative $x_i \prec x_j$, that is $p_{ij} < 0.5$. It is clear that p_{jk} or p_{ki} have to change, otherwise there would be a contradiction, because we would have $x_i \prec x_j \sim x_k \sim x_i$. If we suppose that $p_{jk} = 0.5$ then we have the situation: x_j is preferred to x_i and there is no difference in preferring x_j to x_k. We must then conclude that x_k has to be preferred to x_i. Furthermore, as $x_j \sim x_k$ then $p_{ji} = p_{ki}$, and so $p_{ij}+p_{jk}+p_{ki} = p_{ij}+p_{jk}+p_{ji} = 1+0.5 = 1.5$. We have the same conclusion if $p_{ki} = 0.5$. In the case of being $p_{jk} < 0.5$, then we have that x_k is preferred to x_j and this to x_i, so x_k should be preferred to x_i. On the other hand, the value p_{ki} has to be equal or greater than p_{ji}, being equal only in the case of $p_{jk} = 0.5$ as we have seen. Interpreting the value $p_{ji} - 0.5$ as the intensity of preference of alternative x_j over x_i, then it seems reasonable to suppose that the intensity of preference of x_k over x_i should be equal to the sum of the intensities of preferences when using and intermediate alternative x_j, that is, $p_{ki} - 0.5 = (p_{kj} - 0.5) + (p_{ji} - 0.5)$. The same reasoning can be applied in the case of $p_{jk} > 0.5$.

From the previous equations we obtain the following expression:

$$p_{ij} + p_{jk} - 0.5 = p_{ik} \;\; \forall i, j, k \in \{1, \ldots, n\} \quad (2)$$

In this paper, we will consider a fuzzy preference relation to be be additive consistent when for every three options in the problem $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik} fulfil *Expression 2*. An additive consistent fuzzy preference relation will be referred as consistent throughout the paper, as this is the only transitivity property we are considering.

3 Estimating Missing Values in Incomplete FPR

In this section we present an iterative estimation procedure capable of reconstructing, under some circunstances, the missing values on incomplete fuzzy preference relations. We will also provide sufficient conditions which assure that the procedure can succesfully estimate all missing information, and will finally point out some considerations about using the additive reciprocity property to design a more powerful procedure, capable of estimating some of the missing values that the previous procedure was not capable to estimate.

3.1 Additive Consistency Measure

As it has been said in the previous sections additive consistency is a property that can be satisfied by fuzzy preference relations. However, it is usually difficult for an expert to express its preferences in a completely consistent way. In this section we will develop a measure of the additive consistency property for the fuzzy preference relations, that is, a measure which gives a degree on which they are more or less additive consistent.

Expression 2 presented above can be used to calculate the value of a preference degree p_{ik} using other preference degrees in a fuzzy preference relation. Indeed,

$$cp_{ik}^{j} = p_{ij} + p_{jk} - 0.5 \tag{3}$$

where cp_{ik}^{j} means the calculated value of p_{ik} via j, that is, using p_{ij} and p_{jk}. Obviously, if the information provided in a fuzzy preference relation is completely consistent then cp_{ik}^{j}, $\forall j \in \{1, \ldots, n\}$ and p_{ik} coincide. However, the information given by an expert usually does not fulfil *Equation 2*, because experts are not always fully consistent. In these cases, the value

$$\varepsilon p_{ik} = \frac{\sum_{j=1; j \neq i,k}^{n} \left| cp_{ik}^{j} - p_{ik} \right|}{n-2} \tag{4}$$

can be used to measure the error expressed in a preference degree between two options. This error can be interpreted as the consistency level between the preference degree p_{ik} and the rest of the preference values of the fuzzy preference relation. Clearly, when $\varepsilon p_{ik} = 0$ then there is no inconsistency at all, and the higher the value of εp_{ik} the more inconsistent is p_{ik} with respect to the rest of information.

The *Consistency Level* for the whole fuzzy preference relation P is defined as follows:

$$CL_P = \frac{\sum_{i,k=1; i \neq k}^{n} \varepsilon p_{ik}}{n^2 - n} \tag{5}$$

When $CL_P = 0$, then the preference relation P is fully consistent, otherwise, the higher CL_P the more inconsistent is P.

We also introduce the following sets:

$$A = \{(i, j) \mid i, j \in \{1, \ldots, n\} \wedge i \neq j\} \tag{6}$$

$$MV = \{(i, j) \mid p_{ij} = x, (i, j) \in A\} \tag{7}$$

$$EV = A \setminus MV \tag{8}$$

MV is the set of pairs of alternatives for which the preference degree of the first alternative over the second one is unknown or missing; EV is the set of pairs of alternatives for which the expert provide preference values. Notice that we do not take into account the preference value of one alternative over itself as this is always assumed to be equal to 0.5.

When working with an incomplete fuzzy preference relation, we note that *Expression 4* cannot be to estimate the error on a particular pair of alternatives. An obvious consequence of this is the need to extend the above definition of CL_P to include the cases when the fuzzy preference relation is incomplete. We do this as follows:

$$H_{ik} = \{j \mid (i, j), (j, k) \in EV\} \; \forall i \neq k \tag{9}$$

$$\varepsilon p_{ik} = \frac{\sum_{j \in H_{ik}} \left| cp_{ik}^{j} - p_{ik} \right|}{\# H_{ik}} \tag{10}$$

$$CE_P = \{(i, k) \in EV \mid \exists j : (i, j), (j, k) \in EV\} \tag{11}$$

$$CL_P = \frac{\sum_{(i,k) \in CE_P} \varepsilon p_{ik}}{\# CE_P} \tag{12}$$

We call CE_P the *Computable Error* set because it contains all the elements for which we can compute every εp_{ik}. Clearly, this redefinition of CL_P is an extension of *Expression 5*. Indeed, when a fuzzy preference relation is complete, both CE_P and A coincide and thus $\# CE_P = n^2 - n$.

3.2 An Iterative Procedure to Estimate Missing Values

We have developed an iterative procedure capable of estimating missing values on an incomplete fuzzy preference relation, only using the information expressed in the proper preference relation and the additive consistency property. This procedure, opposed to classical approaches where missing information is acquired using information from other experts, does not make use of information from other sources, and tries to maintain the consistency level expressed by the expert.

To develop the iterative procedure to estimate missing values two different tasks have to be carried out:

A) To establish the elements that can be estimated in each step of the procedure, and

B) To produce the particular expression that will be used to estimate a particular missing value.

A) Elements to be estimated in step h

The subset of the missing values MV that can be estimated in step h of our procedure is denoted by EMV_h (Estimable Missing Values) and defined as follows:

$$EMV_h = \{(i,k) \in MV \setminus \bigcup_{l=0}^{h-1} EMV_l \mid \exists j : (i,j),(j,k) \in EV \cup \left(\bigcup_{l=0}^{h-1} EMV_l\right)\}$$

with $EMV_0 = \emptyset$.

When $EMV_{maxIter} = \emptyset$ with $maxIter > 0$ the procedure will stop as there will not be any more missing values to be estimated. Furthermore, if $\bigcup_{l=0}^{maxIter} EMV_l = MV$ then all missing values are estimated and consequently the procedure was successful in the completion of the fuzzy preference relation.

B) Expression to estimate a particular p_{ik} value

In order to estimate a particular value p_{ik} with $(i,k) \in EMV_h$, in iteration h, we propose the application of the the following three steps function:

function estimate_p(i,k)

1. $I_{ik} = \left\{ j \mid (i,j),(j,k) \in EV \cup \left(\bigcup_{l=0}^{h-1} EMV_l\right)\right\}$
2. Calculate $cp'_{ik} = \frac{\sum_{j \in I_{ik}} cp^j_{ik}}{\# I_{ik}}$
3. Make $p_{ik} = cp'_{ik} + z$ with $z \in [-CL_P, CL_P]$ randomly selected, subject to $0 \leq p_{ik} + z \leq 1$

end function

With this procedure, a missing value p_{ik} is estimated using *Expression 3* when there is at least one chained pair of known preference values p_{ij}, p_{jk} that allow this. If there is more than one pair of preference values that allow to estimate p_{ik} using that equation, then we use the average value of all them as an estimate of the missing value, cp'_{ik}. Finally, we add a random value $z \in [-CL_P, CL_P]$ to this estimation in order to maintain the consistency level of the expert, but obviously forcing the estimated value to be in the range of fuzzy preference values $[0,1]$.

The *iterative estimating procedure pseudo-code* is as follows:

0. $EMV_0 = \emptyset$
1. $h = 1$
2. while $EMV_h \neq \emptyset$ {
3. for every $(i,k) \in EMV_h$ {
4. learn_p(i,k)
5. }
6. h++
7. }

When this procedure ends successfully all missing values that could be estimated would have been calculated. However, as we have previously mentioned, there are cases when not every missing value of an incomplete fuzzy preference relation can be estimated using this iterative procedure. In the following, we provide an example illustrating this situation.

3.3 Some Missing Values Cannot Be Estimated By The Iterative Procedure

In this section we provide sufficient conditions to assure the estimation of all missing values in the incomplete fuzzy preference relation and an example where not all missing values can be estimated.

A) Sufficient conditions to be able to estimate all missing values

As we will see later, there are cases where all missing information can not be learnt using our procedure. However, to obtain conditions that guarantee that all missing information in an incomplete fuzzy preference relation can be estimated is of great importance. In the following, we provide sufficient conditions on the success of the above estimating procedure.

It is clear that if there exist a value j such that for all $i \in \{1,2,\ldots,n\}$ both (i,j) and (j,k) do not belong to MV, then all missing information can be estimated in the first iteration of our procedure ($EMV_1 = MV$) because for every $p_{ik} \in MV$ we can use at least the pair of preference values p_{ij} and p_{jk} to estimate it.

In [5], a different sufficient condition that guarantees the estimation of all missing values was given. This condition states that any incomplete fuzzy preference relation can be converted into a complete one when the set of $n-1$ values $\{p_{12}, p_{23}, \ldots, p_{n-1n}\}$ is known.

Missing value (i, k)	**Pairs to estimate** p_{ik}
$(1, 4)$	$(1, 2), (2, 4)$
$(2, 3)$	$(2, 1), (1, 3)$
$(2, 5)$	$(2, 4), (4, 5)$
$(4, 2)$	$(4, 1), (1, 2)$
$(4, 3)$	$(4, 1), (1, 3); (4, 5), (5, 3)$
$(5, 1)$	$(5, 4), (4, 1)$

Tabla 1: Pairs of values which allow to estimate missing values in iteration 1 of the procedure.

An equivalent condition to this one is obtained when at least one value different to those from the leading diagonal is known from each different row (column) of the incomplete fuzzy preference relation. However, in these two last cases the additive reciprocity property is also assumed.

B) Impossibility to estimate all missing values

The following is an illustrative example of an incomplete fuzzy preference relation where our procedure is unable to estimate all missing values.

Suppose an expert that provides the following incomplete fuzzy preference relation

$$P = \begin{pmatrix} - & e & e & x & x \\ e & - & x & e & x \\ x & x & - & x & x \\ e & x & x & - & e \\ x & x & e & e & - \end{pmatrix}$$

over a set of five different alternatives, $X = \{x_1, x_2, x_3, x_4, x_5\}$, where x means "a missing value" and e means "a value is known".

Remark. We note that the actual values of the known preference values are not relevant for the purpose of this example.

At the beginning of our iterative procedure we get:

$$EMV_1 = \{(1, 4), (2, 3), (2, 5), (4, 2), (4, 3), (5, 1)\}$$

as we can find pairs of preference values that allow us to calculate the missing preference values in those positions. Indeed, the following table shows all the pairs of alternatives that are available to estimate each one of the above missing values:

The other missing values cannot be estimated in this first iteration of the procedure. If we substitute all the $x's$ values calculated in this iteration by a number 1 (indicating the step in which they have been estimated) we get:

Missing value (i, k)	**Pairs to estimate** p_{ik}
$(1, 5)$	$(1, 2), (2, 5); (1, 4), (4, 5)$
$(5, 2)$	$(5, 1), (1, 2); (5, 4), (4, 2)$

Tabla 2: Pairs of values which allow to estimate missing values in iteartion 2 of the procedure.

$$P = \begin{pmatrix} - & e & e & 1 & x \\ e & - & 1 & e & 1 \\ x & x & - & x & x \\ e & 1 & 1 & - & e \\ 1 & x & e & e & - \end{pmatrix}$$

In the next iteration, to construct the set EMV_2 we can use the values expressed directly by the expert as well as the values previously estimated in iteration 1. In our case we have $EMV_2 = \{(1, 5), (5, 2)\}$:

and the incomplete fuzzy preference relation at this point is:

$$P = \begin{pmatrix} - & e & e & 1 & 2 \\ e & - & 1 & e & 1 \\ x & x & - & x & x \\ e & 1 & 1 & - & e \\ 1 & 2 & e & e & - \end{pmatrix}$$

In the next iteration $EMV_3 = \emptyset$. The procedure ends and it does not succeed in the completion of the fuzzy preference relation. The reason of this failure is that the expert did not provide any preference degree of the alternative x_3 over the rest of alternatives. Fortunately, this kind of situation is not very common in real problems, and therefore the procedure will usually be successful in estimating all missing values. Clearly, if additive reciprocity is also assumed (this is a direct consequence of additive transitivity property) then the chances of succeeding in the estimation of every missing value would increase, as we will show in what follows.

3.4 Additive reciprocity property

In the literature, preference relations are usually assumed reciprocal. In particular, *additive reciprocity* is used in many decision models as one of the properties that fuzzy preference relations have to verify [1, 6]. Additive reciprocity is defined as:

$$p_{ij} + p_{ji} = 1 \;\; \forall i, j \in \{1, 2, \ldots, n\} \qquad (13)$$

Our iterative procedure does not imply any kind of reciprocity. In fact, it allows to estimate missing values in fuzzy preference relations when this condition is not satisfied. Furthermore, the procedure itself does not assure that the estimated values will fulfil the reciprocity property.

However, if we assume that the fuzzy preference relation has to be reciprocal, this would allow to calculate some of the missing values that were not possible to estimate without it. In the previous example all p_{3k} values that could not be estimated could have been easily calculated assuming additive reciprocity over the p_{k3} values.

In what follows, we describe how our procedure has to change to implement the use of the additive reciprocity, and to assure that the estimated values fulfil this property.

Firstly, we need to guarantee that the incomplete fuzzy preference relation given by the expert fulfils the reciprocity property, i.e., we have to check that $p_{ik} + p_{ki} = 1 \;\; \forall (i,k),(k,i) \in EV$.

Once this first check if carried out we have to complete the EV matrix by means of computing those missing values with a known reciprocal one as the first step of our procedure, i.e.

$$p_{ki} \leftarrow 1 - p_{ik} \;\; \forall (k,i) \in MV \wedge (i,k) \in EV. \qquad (14)$$

The following steps of our procedure will be as described in *Section 3.2* but restricted to the estimation of missing values above the leading diagonal of the incomplete fuzzy preference relation, i.e. to learn every p_{ik} with $i < k$. We define the $EMV_h^{\uparrow}$ sets as the Learnable Missing Values in every iteration h restricted to the values above the leading diagonal:

$$EMV_h^{\uparrow} = \{(i,k) \in EMV_h \mid (i < k)\} \qquad (15)$$

Once every p_{ik} value has been estimated we use again the reciprocity property to calculate its reciprocal value p_{ki} (and thus, the values under the leading diagonal are also estimated, and obviously the reconstructed fuzzy preference relation will obey the reciprocity property).

The *iterative estimation procedure that makes use of the additive reciprocity property pseudo-code* is as follows:

0. Check that $p_{ik} + p_{ki} = 1 \;\; \forall (i,k),(k,i) \in EV$ /* reciprocity property is satisfied in EV */
1. for every $(k,i) \in MV \wedge (i,k) \in EV$ {
2. $p_{ki} \leftarrow 1 - p_{ik}$ /* Computing missing values in EV directly using reciprocity property */
3. }
4. $EMV_0 = \emptyset$
5. $h = 1$
6. while $EMV_h^{\uparrow} \neq \emptyset$ {
7. for every $(i,k) \in EMV_h^{\uparrow}$ {
8. learn_p(i,k)
9. $p_{ki} \leftarrow 1 - p_{ik}$ /* Use reciprocity to learn values under the leading diagonal */
10. }
11. h++
12. }

As it has been shown in this section, in many cases we can estimate the missing values on incomplete fuzzy preference relations using our iterative procedure. Moreover, if we impose new conditions as additive reciprocity, we can make our procedure more efficient, allowing the estimation of some missing values that were not possible to obtain with the original procedure. Posteriorly this reconstructed fuzzy preference relation can be aggregated with the rest of the information provided by other experts using, for example, the AC-IOWA operator.

4 Integrating the Consistency Measure and the Estimation Procedure on a Decision Model

Once the fuzzy preference relations are completed we can apply a resolution process to obtain the solution of our GDM problem. To do so we need an aggregation operator to obtain a global fuzzy preference relation that summarizes all information from the different experts. In this section we present the AC-IOWA operator, capable of integrating fuzzy preference relations biasing the aggregation with the consistency level for every fuzzy preference relation, that is, our operator gives more importance to the most consistent preference relations, because consistent information is usually more valuable than information with contradictions. Laterly we will present the whole resolution process that must be followed to obtain the solution for a GDM problem with incomplete fuzzy preference relations.

4.1 Additive Consistency based IOWA Operator

In this work we assume that experts that are more consistent when expressing their preferences should have more influence on the final solution (consistent information is much more valuable that inconsistent one), that is, we deal with an heterogeneous GDM problem, and thus, the aggregation process should be biased by the consistency property.

In particular, we can measure the experts' consistency by means of the Consistency Level of the fuzzy preference relation (CL_{P^h}) given by each expert e_h as it was defined in *Section 3.1.* The general procedure for the inclusion of these Consistency Level values in the aggregation process involves the transformation of the preference values, p_{ik}^h, under the Consistency Level CL_{P^h} to generate a new value, $\overline{p}_{ik}^h$. This activity is carried out by means of a transformation function g:

$$\overline{p}_{ik}^h = g\left(p_{ik}^h, CL_{P^h}\right) \tag{16}$$

Examples of functions g used in these cases include the minimum operator [4] , the exponential function $g(x,y) = x^y$ [11], or generally any t-norm operator.

In our case we can implement this consistency level by an alternative method, which consists of using it as the order inducing variable of an IOWA operator.

The IOWA operator was defined by Yager and Filev [13] as an extension to the OWA operator to allow different reorderings of the values to be aggregated:

Definition. An IOWA operator of dimension n is a function

$$\Phi_W : (\Re \times \Re)^n \to \Re, \tag{17}$$

to which a set of weights or weighting vector is associated, $W = (w_1, ..., w_n)$, such that $w_i \in [0,1]$ and $\Sigma_i w_i = 1$, and it is defined to aggregate the set of second arguments of a list of n 2-tuples $\{\langle u_1, p_1\rangle, ..., \langle u_n, p_n\rangle\}$ according to the following expression,

$$\Phi_W(\langle u_1, p_1\rangle, ..., \langle u_n, p_n\rangle) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)} \tag{18}$$

being σ a permutation of $\{1, ..., n\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}, \forall i = 1, ..., n-1$, i.e., $\langle u_{\sigma(i)}, p_{\sigma(i)}\rangle$ is the 2-tuple with $u_{\sigma(i)}$ the highest value in the set $\{u_1, ..., u_n\}$.

In the above definition, the reordering of the set of values to be aggregated, $\{p_1, ..., p_n\}$, is induced by the reordering of the set of values $\{u_1, ..., u_n\}$ associated to them, which is based upon their magnitude. Due to this use of the set of values $\{u_1, ..., u_n\}$, Yager and Filev called them the values of an order inducing variable and $\{p_1, ..., p_n\}$ the values of the argument variable [15, 13, 14].

A natural question in the definition of the OWA and IOWA operators is how to obtain the associated weighting vector. In [12] Yager proposed two ways to obtain it for OWA operators, but it can as well be used for IOWA operators. The first approach is to use some kind of learning mechanism using some sample data; and the second approach is to try to give some semantics or meaning to the weights. The latter possibility has allowed multiple applications on areas of fuzzy and multi-valued logics, evidence theory, design of fuzzy controllers and the quantifier-guided aggregations.

We are interested in the area of quantifier-guided aggregations. Our idea is to calculate weights for the aggregation operations using linguistic quantifiers that represent the concept of *fuzzy majority.* In [12], Yager suggested an interesting way to compute the weights of the aggregation operator using fuzzy quantifiers, which, in the case of a non-decreasing relative quantifier Q, is given by the expression

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n. \tag{19}$$

When a fuzzy quantifier Q is used to compute the weights of the OWA operator ϕ, it is symbolized by ϕ_Q.

Our new IOWA operator, based on Additive Consistency property (AC-IOWA) uses the consistency level presented in *Section 3.1* as the order inducing variable, and thus, biases the aggregation process according to the consistency expressed by the experts in their fuzzy preference relations. More specifically it uses the opposite of the CL_{P^h} values, because the σ permutation in the IOWA operator needs the order inducing variable to be higher when the order of the value is low:

Definition. If a set of experts, $E = \{e_1, ..., e_m\}$, provide preferences about a set of alternatives, $X = \{x_1, ..., x_n\}$, by means of the fuzzy preference relations, $\{P^1, ..., P^m\}$, then, the AC-IOWA operator of dimension m, Φ_W^{AC}, is an IOWA operator whose set of order inducing values is the set $\{-CL_{P^1}, ..., -CL_{P^m}\}$.

This new operator is an example of a Consistency IOW operator (C-IOW) as it is defined in [2], where the set of consistency index values is obtained from the experts' own information, applying the additive consistency property over their fuzzy preference relations.

4.2 Resolution Process of a GDM with Incomplete Fuzzy Preference Relations

In this context, to obtain a set of solution alternatives $X_{sol} \subset X$, the first step of a resolution process of GDM problems with incomplete fuzzy preference relations might be the application of the iterative procedure to estimate the missing values. Therefore, the resolution process presents the scheme given in *Fig. 4.2*.

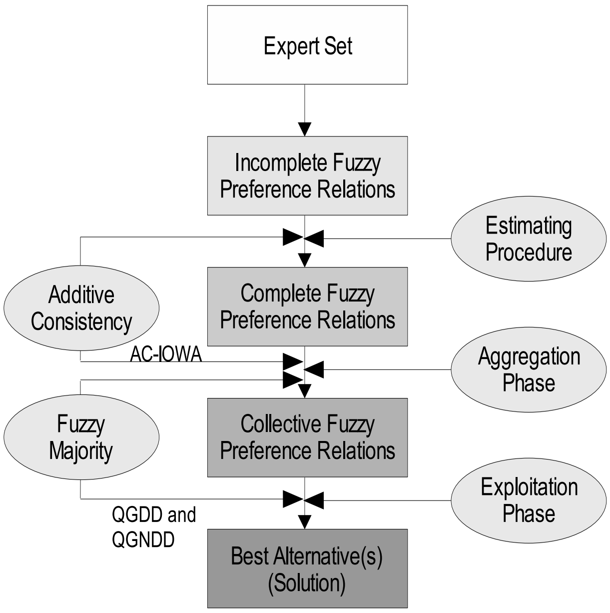

Figura 1: Resolution Process of a GDM with Incomplete FPR.

Once the experts provide their (incomplete) preference relations, two main steps are applied: (1) *Estimation of missing information*, and (2) *Application of a selection process*.

1. *Estimation of missing information*. In this step, incomplete fuzzy preference relations are completed by using the iterative procedure presented in *Section 3*.

2. *Application of a selection process*, which is carried out in two sequential phases:

   (a) *Aggregation phase*. A collective fuzzy preference relation is obtained by aggregating all the individual fuzzy preference relations. This aggregation is carried out by applying the AC-IOWA operator guided by a linguistic quantifier representing the concept of *fuzzy majority* (of experts) desired to implement in the resolution process.

   (b) *Exploitation phase*. Using again the concept of fuzzy majority (of alternatives), two chocie degrees of alternatives are used: the *quantifier-guided dominance degree(QGDD)* and the *quantifier-guided non-dominance degree (QGNDD)* [1]. These choice degrees will act over the collective preference relation resulting in a global ranking of the alternatives, from which the set of solution alternatives will be obtained.

5 Conclusions and Current Works

In this work we have presented some tools that are used to resolve GDM problems with incomplete information. We have centered our attention on incomplete fuzzy preference relations, and have developed an iterative procedure capable of estimating the missing information on them. We have also presented the AC-IOWA operator, capable of aggregating the information provided by different experts into a global one which summarizes all the opinions of the experts. Both the iterative procedure and the AC-IOWA operator are based on a new measure of the additive consistency property of the fuzzy preference relations. Finally we have briefly presented how can both tools be integrated into a global decision model to solve GDM problems.

We are also working on how to extend the iterative procedure to different kinds of preference relations: *multiplicative preference relations*, *interval-valued preference relations* and *linguistic preference relations* and in the design of a new decision model capable of integrating those different kinds of preference relations, that is, allowing the experts to express their preferences in their preferred kind of preference relation.

Another point where we are focusing our attention is on the developement of a consensus model for GDM problems with different kinds of incomplete preference relations. This consensus model will allow to follow a consensus process without a moderator by means of a feedback mechanism capable of substituting the actions of the moderator.

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