Consistency of Reciprocal Fuzzy Preference Relations

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Abstract

Consistency is related with rationality, which is associated with the *transitivity property*. For fuzzy preference relations many properties have been suggested to model transitivity and, consequently, consistency may be measured according to which of these different properties is required to be satisfied. In this contribution the consistency of reciprocal preference relations is studied and we will show that many of the suggested properties are not appropriate for reciprocal preference relations. We put forward a functional equation to model consistency of reciprocal preference relations, and show that self-dual uninorms operators are the solutions to it. In particular, Tanino's multiplicative transitivity property being an example of such type of uninorms seems to be an appropriate consistency property amongst the many proposed for fuzzy reciprocal preferences.

1 Introduction

To reach a satisfactory solution in a decision problem, a set of experts $E = \{e_1, \ldots, e_m\}$ are usually required to provide a set of evaluations over the set of alternatives $X = \{x_1, \ldots, x_n\}$. Those evaluations can be expressed using several different models. However, preference relations, where each alternative is compared with the rest of alternatives, are a very widely used preference representation format [2, 3, 7, 9, 10, 11, 13, 18, 19, 21].

Given a pair of alternatives there are three basic possibilities for an expert: one may be preferred to another, they may be equally preferred (the expert is indifferent to them) or they may be incomparable. Two possible mathematical models have been developed to represent the above cases. In the first one a preference relation is defined for each one of the above three preference states. The second one, integrates the three preference states in a single preference relation. Further to this, in each case two different representation could be adopted: the use of binary or *crisp preference relations* or the use of *fuzzy preference relations.*

The main advantage of pairwise comparison is that of focusing exclusively on two alternatives at a time and on how they are related. However, this approach generates more information that is needed and therefore inconsistent information may be generated. In a crisp context the concept of consistency has traditionally been defined in terms of acyclicity [20]. Clearly, this condition is closely related to the transitivity of the corresponding binary preference relation, R , in the sense that if alternative x_i is preferred to alternative x_j and this one to x_k then alternative x_i should be preferred to x_k. In a fuzzy context, the traditional requirement to characterize consistency has followed the way of extending the classical requirements of binary preference relations. Thus, consistency is also based on the notion of transitivity. However, the main difference in this case with respect to the classical one is that transitivity of a fuzzy preference relation has been modelled in many different ways due to the role that the intensity of preference has [7, 18, 21, 23, 10, 13, 14, 15], and consequently, consistency may be measured according to which of these different properties is required to be satisfied. One of these properties is the additive transitivity property, which is equivalent to Saaty's consistency property for multiplicative preference relations [10].

In this contribution we will show that this consistency property is in conflict with the corresponding scale used for providing the preference values. In order to overcome this conflict, a set of conditions will be put forward for a fuzzy preference relation to be considered 'fully consistent'. Under this set of conditions we show that consistency of preferences should be modelled using uninorm operators. In particular, Tanino's multiplicative transitivity property [21], being an example of such type of uninorms, seems to be an appropriate consistency property for fuzzy reciprocal preferences.

The rest of the paper is set as follows. Preliminaries on the consistency of preferences are provided in Section 2. In Section 3, a set of conditions for a fuzzy preference relation to be considered 'fully consistent' will be established. Self-dual uninorms operators are shown to be the solutions to this set of conditions in Section 4. Finally, some conclusions and future works are drawn in Section 5.

2 Consistency of Fuzzy Preference Relations

Preference relations are usually assumed to model experts' preferences in decision making problems [6]. To implement the degree of preference between alternatives, which may be essential in many situations, we use fuzzy preference relations [2, 5, 7, 21]:

Definition: A fuzzy preference relation R on a set of alternatives X is a fuzzy set on the product set $X \times X$, that is characterized by a membership function $\mu_R : X \times X \longrightarrow [0, 1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $R = (r_{ij})$ being $r_{ij} = \mu_R(x_i, x_j)\ \ \forall i, j \in \{1, \ldots, n\}$.

In this approach, given two alternatives an expert provides

i. a value in the range $(0.5, 1]$ to quantify the "degree or strength of preference" of an alternative when preferred to another;

ii. the value 0.5 when the two alternatives are indifferent to him;

iii. no value when he is uncertain as to his preference between the alternatives or he is unable to compare them [9].

There are three fundamental and hierarchical levels of rationality assumptions when dealing with preference relations [12]:

- The first level requires indifference between any alternative x_i and itself.
- The second one requires that if an expert prefers x_i to x_j, that expert should not simultaneously prefer x_j to x_i. This asymmetry condition is viewed as an "obvious" condition/criterion of consistency for preferences [6]. This rationality condition is modelled by the reciprocity property in the pairwise comparison between any two alternatives, which is seen by Saaty as basic in making paired comparisons [18].
- Finally, the third one is associated with the transitivity in the pairwise comparison among any three alternatives.

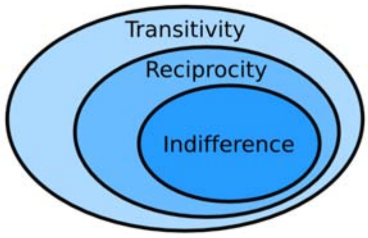


Figure 1: Levels of Rationality for Pref. Relations

This hierarchical structure (depicted in figure 1) also requires for a particular level of rationality to be compatible with the upper ones: the third level of rationality should imply or be compatible with the second level, and this with the first one. This necessary compatibility between the rationality assumptions could be used as a criterion for considering a particular condition modelling any of the rationality levels as adequate or inadequate.

A preference relation verifying the third level of rationality is usually called a *consistent preference relation* and any property that guarantees the transitivity of the preferences is called a consistency property. The lack of consistency in decision making can lead to inconsistent conclusions; that is why it is important, in fact crucial, to study conditions under which consistency is satisfied [19].

In a crisp context, where an expert provides his/her opinion on the set of alternatives X by means of a binary preference relation, R, the concept of consistency has traditionally been defined in terms of acyclicity [20], i.e. the absence of sequences such as $x_1, x_2, \ldots, x_k (x_{k+1} = x_1)$ with $x_j R x_{j+1} \forall j = 1, \ldots, k$. This condition is closely related to the transitivity of the binary relation and its corresponding binary indifference relation.

In a fuzzy context, the traditional requirement to characterize consistency has followed the way of extending the classical requirements of binary preference relations. Thus, consistency is also based on the notion of transitivity, in the sense that if alternative x_i is preferred to alternative x_j and this one to x_k then alternative x_i should be preferred to x_k, which is normally referred in this context as *weak transitivity*. However, the main difference in this case with respect to the classical one is that consistency has been modelled in many different ways due to the role that the intensity of preference has [7, 18, 21, 23, 10, 13, 14, 15]. Indeed, many properties or conditions have been suggested as rational ones for a fuzzy preference relation to be considered a consistent one. Among these properties we can cite:

- Max-min trans.[4, 23]:$r_{ik} \geq \min\{r_{ij}, r_{jk}\}$
- Restricted max-min transitivity [21]: $\min\{r_{ij}, r_{jk}\} \geq 0.5 \Rightarrow r_{ik} \geq \min\{r_{ij}, r_{jk}\}$
- Max-max trans[4, 23]:$r_{ik} \geq \max\{r_{ij}, r_{jk}\}$
- Restricted max-max transitivity [21]: $\min\{r_{ij}, r_{jk}\} \geq 0.5 \Rightarrow r_{ik} \geq \max\{r_{ij}, r_{jk}\}$
- Multiplicative trans. [21]: $\frac{r_{ji}}{r_{ij}} \cdot \frac{r_{kj}}{r_{jk}} = \frac{r_{ki}}{r_{ik}}$
- Additive transitivity [21]: $(r_{ij} - 0.5) + (r_{jk} - 0.5) = r_{ik} - 0.5$

We note that these conditions are stronger than weak transitivity, and therefore a fuzzy preference relation might be transitive but not consistent (see [17, 21, 10]).

As aforementioned, the value 0.5 is usually used to model the first level of rationality in the case fuzzy preference relations, and therefore we have $r_{ii} = 0.5 \; \forall i$.

The second level of rationality of fuzzy preferences is modelled using the following reciprocity property $r_{ij} + r_{ji} = 1 \; \forall i, j$.

Clearly, reciprocity property implies indifference, and therefore both properties are compatible.

Max-max transitivity cannot be verified under reciprocity. Indeed, if $R = (r_{ij})$ is reciprocal and verifies max-max transitivity, then:

$$\begin{aligned} \forall i,j,k: \quad r_{ki} &= 1 - r_{ik} \\ &\leq 1 - \max\{r_{ij}, r_{jk}\} \\ &= \min\{1 - r_{ij}, 1 - r_{jk}\} \\ &= \min\{r_{ji}, r_{kj}\} \end{aligned}$$

From max-max transitivity we have that:

$$\forall i,j,k : r_{ki} \geq \max\{r_{kj}, r_{ji}\}$$

and therefore we have that max-max transitivity and reciprocity can be verified only when

$$\forall i,j,k : r_{ik} = r_{ij} = r_{jk} = 0.5.$$

Max-min transitivity and reciprocity imply:

$$\begin{aligned} \forall i,j,k: \quad r_{ik} &= 1 - r_{ki} \\ &\leq 1 - \min\{r_{kj}, r_{ji}\} \\ &= \max\{1 - r_{kj}, 1 - r_{ji}\} \\ &= \max\{r_{jk}, r_{ij}\} \end{aligned}$$

Therefore, max-min transitivity under reciprocity can be rewritten as

$$\forall i,j,k : \min\{r_{ij}, r_{jk}\} \leq r_{ik} \leq \max\{r_{ij}, r_{jk}\}.$$

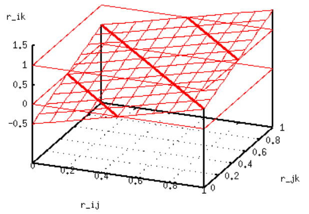


Figure 2: Additive Transitivity

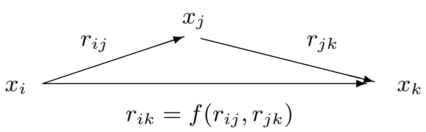
Even though the restricted versions of max-max and max-min are not incompatible with the reciprocity property, they do not directly imply it. In fact, a fuzzy preference relation can be reciprocal and still verify both restricted transitivity properties. The same applies to max-min transitivity and multiplicative transitivity.

Additive transitivity implies both reciprocity and indifference and thus we might conclude that it is the most adequate property among the above list to model consistency of fuzzy preferences. However, additive transitivity is in conflict with the scale used for providing the preference values. Indeed, for a set of three alternatives $\{x_i, x_j, x_k\}$, if $(r_{ij} + r_{jk} > 1.5) \vee (r_{ij} + r_{jk} < 0.5) \Rightarrow r_{ik} \notin [0,1]$ (figure 2). Therefore, additive transitivity might not be considered the most suitable condition to model consistency of fuzzy preference relations.

3 Consistency Function of Reciprocal Fuzzy Preference Relations

The assumption of experts being able to quantify their preferences in the domain [0,1] instead of $\{0,1\}$ or a set with finite cardinality, underlies unlimited computational abilities and resources from the experts. Taking these unlimited computational abilities and resources into account we may formulate that an expert's preferences are consistent when for any three alternatives x_i, x_j, x_k their preference values are related in the 'exact' form

$$r_{ik} = f(r_{ij}, r_{jk}) \quad \forall i,j,k \tag{1}$$



being f a function $f\colon [0,1] \times [0,1] \to [0,1]$.

In practical cases expression (1) might obviously not be verified even when the preference values of a preference relation are transitive, i.e., they comply with the weak transitivity property. However, the assumption of modelling consistency using expression (1) can be used to introduce levels of consistency, which in group decision making situations could be exploited by assigning a relative importance weight to each one of the experts in arriving to a collective preference opinion.

In what follows we will set out a set of conditions or properties to be verified by such a function f.

The first condition to impose to function f is that it must be monotonic (increasing), i.e. if any of the preference values r_{ij}, r_{jk} increases while the other remains fixed then the preference value r_{ik} will not decrease.

Property 1 (Monotonicity)
$f(x,y) \geq f(x',y')$ *if* $x \geq x'$ *and* $y \geq y'$

Equation (1) implies that $f(r_{ij}, r_{jk}) = f(r_{il}, r_{lk})$ $\forall i,j,k,l$. On the other hand we have $r_{ij} = f(r_{il}, r_{lj})$ and $r_{lk} = f(r_{lj}, r_{jk})$. Putting these expressions together we have

$$f(f(r_{il}, r_{lj}), r_{jk}) = f(r_{il}, f(r_{lj}, r_{jk})) \quad \forall i,j,k,l.$$

Thus function f must be associative.

Property 2 (Associativity)
$f(f(x,y),z) = f(x, f(y,z)) \quad \forall x,y,z \in [0,1]$

The application of equation (1) and the assumed reciprocity property of preferences give

$$\forall i,j,k: \quad \begin{array}{l} r_{ki} = f(r_{kj}, r_{ji}) = f(1-r_{jk}, 1-r_{ij}) \\ r_{ki} = 1 - r_{ik} = 1 - f(r_{ij}, r_{jk}) \end{array}$$

and hence

$$f(1-r_{jk}, 1-r_{ij}) = 1 - f(r_{ij}, r_{jk}) \quad \forall i,j,k.$$

Property 3 (Reciprocity)
$f(x,y) + f(1-y, 1-x) = 1 \quad \forall x,y \in [0,1]$

Making $y = 1-x$ and $x = y = 0.5$ in property 3 we have respectively:

Property 4 (Indifference)
$f(x, 1-x) = 0.5 \quad \forall x \in [0,1]$

Property 5 (Transitivity of Indifference)
$f(0.5, 0.5) = 0.5$

From properties 2 and 4 we obtain:

$$\begin{aligned} \forall i,k: \quad f(0.5, r_{ik}) = & \; f(f(r_{ik}, 1-r_{ik}), r_{ij}) \\ = & \; f(r_{ik}, f(1-r_{ik}, r_{ik})) \\ = & \; f(r_{ik}, 0.5) \end{aligned}$$

From equation (1) and property 2 we have that

$$\begin{aligned} \forall i,k: \quad r_{ik} = & \; f(r_{ij}, r_{jk}) \\ = & \; f(r_{ij}, f(r_{ji}, r_{ik})) \\ = & \; f(f(r_{ij}, r_{ji}), r_{ik}). \end{aligned}$$

By property 4, $f(r_{ij}, r_{ji}) = 0.5$ which reduces the previous expression to

$$r_{ik} = f(0.5, r_{ik}) \; \forall i,k.$$

Therefore, we have that 0.5 must be the identity element of function f.

Property 6 (Identity element)
$f(0.5, x) = f(x, 0.5) = x \quad \forall x \in [0,1]$

The following result can be easily proved from properties 1 and 6:

Proposition 1

$$\min\{r_{ij},\; r_{jk}\} \geq 0.5 \Rightarrow f(r_{ij}, r_{jk}) \geq \max\{r_{ij},\; r_{jk}\}$$

$$\max\{r_{ij},\; r_{jk}\} \leq 0.5 \Rightarrow f(r_{ij}, r_{jk}) \leq \min\{r_{ij},\; r_{jk}\}$$

$$r_{ij} \leq 0.5 \leq r_{jk} \Rightarrow r_{ij} \leq f(r_{ij}, r_{jk}) \leq r_{jk}$$

This result means that a reciprocal preference relation that verifies expression (1) also verifies restricted max-min and restricted max-max transitivity properties. Clearly, this result rules out max-min transitivity property as a candidate for modelling the consistency of reciprocal preference relations.

From proposition 1 we derive:

Corollary 1

$$\begin{aligned} x \geq 0.5 \Rightarrow & f(x,1) = f(1,x) = 1 \\ x \leq 0.5 \Rightarrow & f(x,0) = f(0,x) = 0 \end{aligned}$$

Corollary 2 $f(0,0) = 0 \;\wedge\; f(1,1) = 1$

A problem arises when $(x,y) \in \{(0,1),(1,0)\}$. Indeed, on the one hand, by property 4 we would have that $f(0,1) = f(1,0) = 0.5$. On the other hand, properties 2, 6 and corollary 1 imply

$$\begin{aligned} x \geq 0.5 \Rightarrow x =& f(0.5, x) = f(f(0,1), x) \\ =& f(0, f(1,x)) = f(0,1) = 0.5 \\ x \leq 0.5 \Rightarrow x =& f(x, 0.5) = f(x, f(0,1)) \\ =& f(f(x,0), 1) = f(0,1) = 0.5 \end{aligned}$$

Thus, the value $f(0,1) = f(1,0) = 0.5$ implies that $0.5 = x \; \forall x \in [0,1]$. Therefore, properties 3 and 4 must be true for $(x,y) \in [0,1]^2 \backslash \{(0,1),(1,0)\}$.

If $f(0,1)$ ($f(1,0)$) exists then

$$f(0,1) = f(0, f(1,x)) = f(f(0,1), x) \; \forall x \geq 0.5$$

$$f(0,1) = f(f(x,0), 1) = f(x, f(0,1)) \; \forall x \geq 0.5$$

There are two alternative cases to the value 0.5. If $f(0,1) > 0.5$ then $f(0,1) = f(f(0,1),1) = 1$, while if $f(0,1) < 0.5$ then $f(0,1) = f(0, f(0,1)) = 0$. Therefore, we have that in all cases:

Proposition 2 $f(0,1), f(1,0) \in \{0,1\}$

Another desirable property to be verified by function f should be that of continuity as it is expected that a slight change of the values in (r_{ij}, r_{jk}) should produce a slight change in the value r_{ik}. Continuity is not possible to be achieved in $(0,1)$ nor in $(1,0)$. Indeed, the following is true

$$\lim_{x \to 0} f(x, 1-x) \neq f(0,1) \wedge \lim_{x \to 0} f(1-x, x) \neq f(1,0).$$

To conclude this section of properties of function f, we note that if there exist alternatives x_j, x_k and x_l such that

$$f(r_{ij}, r_{jk}) = f(r_{ij}, r_{jl}) \quad \forall i$$

then applying properties 6, 4 and 2 we have

$$\begin{aligned} r_{jk} =& f(0.5, r_{jk}) = f(f(r_{ji}, r_{ij}), r_{jk}) \\ =& f(r_{ji}, f(r_{ij}, r_{jk})) = f(r_{ji}, f(r_{ij}, r_{jl})) \\ =& f(f(r_{ji}, r_{ij}), r_{jl}) = f(0.5, r_{jl}) = r_{jl} \end{aligned}$$

Obviously, when $f(r_{kj}, r_{ji}) = f(r_{lj}, r_{ji}) \quad \forall i$ then we also obtain $r_{kj} = r_{lj}$.

This property is usually known with the name of "cancellative". Due to the problems with the definition of function f when $(x, y) \in \{(0,1),(1,0)\}$, we have that:

Property 7 (Cancellative)

$$f(x, y) = f(x, z) \ \forall x \in]0,1[\Rightarrow y = z$$

$$f(y, x) = f(z, x) \ \forall x \in]0,1[\Rightarrow y = z$$

Summarising, a solution to the functional equation (1) for reciprocal preference values is any function $f\colon [0,1]\times[0,1] \longrightarrow [0,1]$ with the following properties:

- f is continuous, monotonic increasing, associative, cancellative and reciprocal in $[0,1]^2 \backslash \{(0,1),(1,0)\}$.
- $f(0,1), f(1,0) \in \{0,1\}$.
- f has 0.5 as its identity element.

4 Uninorms and Consistency of Reciprocal Fuzzy Preference Relations

Uninorms were introduced by Yager and Rybalov in 1996 [22] as a generalisation of the t-norm and t-conorm. Uninorms share the properties commutativity, associativity and monotonicity with t-norms and t-conorms. It is the boundary condition or identity element the one that is used to generalise t-norms and t-conorms. The identity element of t-norms is the number 1, while for t-conorms the identity element is 0. Uninorms can have an identity element lying anywhere in the unit interval $[0,1]$.

Clearly, function f in the previous section share all properties of a uninorm except perhaps commutativity, which cannot be directly derived from the above set of properties. However, commutativity of f can be derived indirectly from associativity, cancellativity and continuity of f. Indeed, the following result was proved by Aczél in [1]:

Theorem 1 *Let I be a (closed, open, half-open, finite or infinite) proper interval of real numbers. Then $F\colon I^2 \longrightarrow I$ is a continuous operation on I^2 which satisfies the associativity equation*

$$F(F(x, y), z) = F(x, F(y, z)) \forall x, y, z \in I$$

and is cancellative, that is, $F(x_1, y) = F(x_2, y)$ or $F(y, x_1) = F(y, x_2)$ implies $x_1 = x_2$ for any $y \in I$ if, and only if, there exists a continuous and strictly monotonic function $\phi\colon J \longrightarrow I$ such that

$$F(x, y) = \phi\left[\phi^{-1}(x) + \phi^{-1}(y)\right] \ \ \forall x, y \in I \quad (2)$$

Here J is one of the real intervals

$$]-\infty, \gamma], \ ]-\infty, \gamma[, \ [\delta, \infty[, \ ]\delta, \infty[, \ \textit{or} \ ]-\infty, \infty[\quad (3)$$

for some $\gamma \leq 0 \leq \delta$. Accordingly I has to be open at least from one side.

The function in (2) is unique up to a linear transformation of the variable ($\phi(x)$ may be replaced by $\phi(Cx)$, $C \neq 0$ but by no other function).

We note that although function F in theorem 1 was not assumed to be commutative, the result (2) shows that it is. Also, function F is strictly monotonic as a result of Aczél theorem. Therefore, the assumption of modelling consistency of reciprocal preferences in $[0,1]$ using the functional expression (1) has as solution f a uninorm with identity element 0.5 which is strictly increasing.

Fodor, Yager and Rybalov in [8] provide a representation theorem for almost continuous uninorms U, i.e. uninorms with identity element in $]0,1[$ continuous on $[0,1]^2 \backslash \{(0,1),(1,0)\}$. This representation theorem coincides with (2), with generator function $\phi^{-1}\colon [0,1] \longrightarrow [-\infty, \infty]$ such that $h(0) = -\infty$, $h(1) = \infty$. Furthermore, such a uninorm must be self-dual with respect a strong negation N with fixed point e, i.e.

$$U(N(x), N(y)) = N(U(x, y))$$
$$N(e) = e$$

Indifference and reciprocity of preferences in $[0,1]$ is based on the use of the strong negation $N(x) = 1 - x$. Thus, the solutions to the

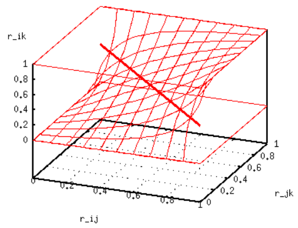


Figure 3: Multiplicative Transitivity

functional equation (1) for reciprocal preference values are self-dual uninorms with respect to $N(x) = 1 - x$.

Interestingly, multiplicative transitivity property introduced by Tanino

$$\frac{r_{ji}}{r_{ij}} \cdot \frac{r_{kj}}{r_{jk}} = \frac{r_{ki}}{r_{ik}} \quad \forall i, j, k.$$

when $r_{ij} > 0 \quad \forall i, j$, can be expressed, under the assumption of reciprocity, as

$$\begin{aligned}
\forall i,j,k : & r_{ij} \cdot r_{jk} \cdot (1 - r_{ik}) = r_{ik} \cdot r_{kj} \cdot r_{ji} && \Leftrightarrow \\
& r_{ij} \cdot r_{jk} - r_{ij} \cdot r_{jk} \cdot r_{ik} = r_{ik} \cdot r_{kj} \cdot r_{ji} && \Leftrightarrow \\
& r_{ik} \cdot r_{kj} \cdot r_{ji} + r_{ij} \cdot r_{jk} \cdot r_{ik} = r_{ij} \cdot r_{jk} && \Leftrightarrow \\
& r_{ik} \cdot (r_{kj} \cdot r_{ji} + r_{ij} \cdot r_{jk}) = r_{ij} \cdot r_{jk} && \Leftrightarrow \\
& r_{ik} = \frac{r_{ij} \cdot r_{jk}}{r_{ij} \cdot r_{jk} + r_{ji} \cdot r_{jk}} && \Leftrightarrow \\
& r_{ik} = \frac{r_{ij} \cdot r_{jk}}{r_{ij} \cdot r_{jk} + (1 - r_{ij}) \cdot (1 - r_{jk})}
\end{aligned} \tag{4}$$

This multiplicative transitivity of a reciprocal fuzzy preference relation (figure 3) has been studied by De Baets et al. in [3] under the name of 'isostochastic transitivity'. Clearly, multiplicative transitivity is the restriction to $[0,1]^2 \backslash \{(0,1),(1,0)\}$ of the well known andlike uninorm

$$U(x,y) = \begin{cases} 0, & (x,y) \in \{(0,1),(1,0)\} \\ \dfrac{xy}{xy + (1-x)(1-y)}, & \text{otherwise} \end{cases} \tag{5}$$

This 'multiplicative' uninorm is self-dual with respect to the negator operator $N(x) = 1 - x$ and has the generator function $\phi^{-1}(x) = \ln \frac{x}{1-x}$ [16]. The behaviour of uninorms on the squares $[0, 0.5] \times [0, 0.5]$ and $[0.5, 1] \times [0.5, 1]$ is closely related to t-norms and as t-conorms [8].

On the evidence obtained so far, we conclude that from the many properties or conditions suggested as rational ones for a fuzzy preference relation to be considered a consistent one, Tanino's multiplicative transitivity property is the most appropriate for the case of reciprocal fuzzy preference relations.

5 Conclusions and Future Works

Rationality is related with consistency, which is associated with the *transitivity property*. For fuzzy preference relations many properties have been suggested to model transitivity. However, it has been shown that also many of them are not appropriate as they are in conflict with the corresponding scale used for providing the preferences or because they are incompatible with the reciprocity and indifference properties, which are seen as basics in making paired comparisons. In this paper we have proved that under a set of conditions consistency of preferences is to be modelled using uninorm operators. In particular, for reciprocal fuzzy preference relations we have that consistency should be modelled by Tanino's multiplicative transitivity property.

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