Human Gait Modeling Using a Genetic Fuzzy Finite State Machine

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***Abstract*—Human gait modeling consists of studying the biomechanics of this human movement. Its importance lies in the fact that its analysis can help in the diagnosis of walking and movement disorders or rehabilitation programs, among other medical situations.**

Fuzzy finite state machines can be used to model the temporal evolution of this type of phenomenon. Nevertheless, the definition of details of the model in each particular case is a complex task for experts. In this paper, we present an automatic method for learning the model parameters based on the hybridization of fuzzy finite state machines and genetic algorithms leading to genetic fuzzy finite state machines. This new genetic fuzzy system automatically learns the fuzzy rules and membership functions of the fuzzy finite state machine while an expert defines the possible states and allowed transitions.

Our final goal is to obtain a specific model for each person's gait in such a way that it can generalize well with different gaits of the same person. The obtained model must become an accurate and human friendly linguistic description of this phenomenon, with the capability of identifying the relevant phases of the process.

A complete experimentation is developed to test the performance of the new proposal when dealing with datasets of 20 different people, comprising a detailed analysis of results which shows the advantages of our proposal in comparison with some other classical and computational intelligence techniques.

***Index Terms*—Human gait modeling, Fuzzy finite state machines, Fuzzy systems, Genetic algorithms, Genetic fuzzy systems.**

I. Introduction

Human gait modeling consists of studying the biomechanics of this human movement and can help in the detection of gait disorders, identification of balance factors, and assessment of clinical gait interventions and rehabilitation programs [1]. Typically, in human gait modeling there are a huge number of variables obtained by means of different measurement techniques such as height, limb length, walking speed, acceleration along axes, foot forces, etc., thus making the obtaining of an accurate model a very complex task.

Traditionally in system identification, engineers use differential equations to model the behavior of real-world systems (white-box models) [2], [3], [4], [5]. However, when the system grows in complexity, the number of variables and equations becomes intractable. In the last forty years, Fuzzy logic (FL)-based models [6], [7], [8], [9] have become a good alternative to deal with those systems where obtaining the appropriate differential equations is difficult or impossible.

FL is widely recognized for its ability for linguistic concept modeling and its use in system identification. On the one hand, semantic expressiveness, using linguistic variables [10] and rules [11], is quite close to natural language (NL) which reduces the effort of expert knowledge extraction. On the other hand, being universal approximators [12] fuzzy inference systems are able to perform nonlinear mappings between inputs and outputs. Thanks to these advantages, FL has been successfully applied in classification [13], [14], regression [15], [16], control [7], [17], [18], and system modeling [8], [9] achieving a good interpretability accuracy tradeoff.

Fuzzy finite state machines (FFSMs) are specially useful tools for modeling dynamical processes which change in time, becoming an extension of classical finite state machines (FSMs) [19], [20]. The main advantage of FFSMs is that their fuzziness allows them to handle imprecise and uncertain data, which is inherent to real-world phenomena, in the form of fuzzy states and transitions. The theoretical basics of FFSMs were established by [21] and later developed by [22], [23], [24]. In previous studies, we have learned that FFSMs are suitable tools for modeling signals that follow an approximately repetitive pattern. In [25], we explored the possibilities of using a FFSM to create the linguistic description of the temporal evolution of a signal by using a skin conductivity meter and accelerometers to model the activity of a person. Once we had checked the ability of FFSMs to deal with temporal data, we analyzed the chance to consider FFSMs for pattern recognition tasks such as human gait recognition [26], [27] and gesture recognition [28]. Finally, in [29] we used a FFSM for fusing information related to body posture and WiFi positioning [30], which consists of recording and processing signal strength information of WiFi networks to obtain the estimated position in indoor environments.

As any fuzzy system, FFSMs require the definition of a knowledge base (KB). It is well known that this is a complex task for experts as it was the case in the previous applications of FFSMs. In addition, the dynamic nature of FFSMs increases the complexity of the process. For this reason, in this contribution we consider the design of an automatic learning method for the fuzzy KB of FFSMs. In particular, we will take the use of genetic algorithms [31] as a base, which have proven largely their effectiveness and efficiency for this task during the last two decades in the so-called genetic fuzzy systems (GFSs) area [32], [33], [34], [35], [36].

In our approach, the fuzzy states and transitions will still be defined by the expert in order to keep the knowledge that

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she/he has over the whole system while the fuzzy rules and membership functions regulating the state changes will be derived automatically by the GFS, making a robust, accurate and human friendly model called genetic FFSM from now on. In addition, the use of this expert knowledge and the prefixed structure of the FFSM allows us to learn only the membership functions (MFs) and part of the rules to build its fuzzy KB, dealing with a reduced search space.

In the application presented, we show how the expert can combine her/his knowledge about human gait dynamics with the numerical data of the acceleration signals in order to produce an accurate linguistic description of the phenomenon. According to Licklider [37], our aim is to create a symbiotic relationship between the user and the computer, in such a way that human motivation and creativity is strengthened by the computer's greater memory storage and higher computational performance.

In the experimental phase, we have worked with gaits of 20 different people. Regarding to the human gait modeling problem, the goal is to obtain a specific model (FFSM) for each person in such a way that this FFSM can generalize well with different gaits of the same person. Each FFSM will be composed of a small set of linguistic fuzzy if-then rules in the transition function producing a linguistic description of the gait of this person while identifying the relevant states of the model. The design of the FFSM will be tackled in an automatic fashion by the proposed GFS. The performance of the obtained FFSMs will then be benchmarked against other system identification approaches.

To our mind, the current research constitutes an innovative application of fuzzy set theory as: i) up to our knowledge, it is the first time that genetic algorithms are used to design a FFSM, and thus it is also the first time that human gait modeling is tackled by means of an intelligent system of this kind, and ii) it outperforms other standard and non-fuzzy computational intelligence techniques, allowing us to produce a linguistic description of the human gait while identifying the relevant phases of the process with an accurate and human friendly model.

The remainder of this paper is organized as follows. Section II presents the human gait modeling problem. Section III describes how to use FFSMs for modeling the temporal evolution of a phenomenon. Section IV explains how to build FFSMs for modeling the human gait. The automatic method of learning the fuzzy KB of these FFSMs based on genetic algorithms is presented in Section V. Section VI describes the experimentation carried out, comparing the obtained results with other system identification tools. Finally, Section VII draws some conclusions and introduces some future research works.

II. Human gait modeling

Human gait modeling consists of studying the biomechanics of this human movement aimed at quantifying factors governing the functionality of the lower extremities. Gait is a complex integrated task which requires precise coordination of the neural and musculoskeletal system to ensure correct skeletal dynamics [38]. Therefore, its analysis can help in the diagnosis and treatment of walking and movement disorders, identification of balance factors, and assessment of clinical gait interventions and rehabilitation programs [1], [39].

The gait cycle is a periodical phenomenon which is defined as the interval between two successive events (usually heel contact) of the same foot [40]. It is characterized by a stance phase (60% of the total gait cycle), where at least one foot is in contact with the ground, and a swing phase (40% of the total gait cycle), during which one limb swings through the next heel contact (see Fig. 1). These phases can be quite different between individuals but when normalized to a percentage of the gait cycle they maintain close similarity, indicating the absence of disorders [41].

Typically, in human gait modeling there are a huge number of variables obtained by means of different measurement techniques. Most gait parameters can be categorized as anthropometric data which include height, weight, or limb length; spatiotemporal data comprising variables such as walking speed, step length, or phases times; kinematic data of measurements of joint angles, displacement, or acceleration along axes; kinetic data variables including foot force and torques; or electromyographic data which measures the muscle activation levels.

Our approach consists of identifying the relevant phases of the gait based on the accelerations produced during the process, i.e., we will develop human gait modeling by means of kinematic data. We have measured the accelerations using an accelerometer placed in the waist and centered in the back of the person, that provides us with the dorso-ventral acceleration (a_x), the medio-lateral acceleration (a_y), and the antero-posterior acceleration (a_z) at each instant of time. In this contribution, we only use a_x and a_y because a_z has to do with the walking speed and this speed can vary for the same person.

Fig. 1 shows three different synchronized pictures. The first one (at the top) illustrates the dorso-ventral acceleration (a_x) and the medio-lateral acceleration (a_y) obtained from the three-axial accelerometer. The middle picture plots a sketch of a person representing the different phases of the gait with the right limb boldfaced. Finally, the picture at the bottom represents the time period from one event (usually initial contact) of one foot to the subsequent occurrence of initial contact of the same foot.

III. Fuzzy finite state machines

In system identification, designers can choose among several paradigms to represent system models. One of the more expressive model structures is the state space representation [2]. In this approach, the designer must find out the necessary and sufficient subset of state variables $(x_1, x_2, \ldots x_n)$ to represent the entire state $X[t]$ of the system at the time instant t.

The designer uses her/his creativity and personal experience to choose the adequate set of state variables regarding the system goals. This set of variables emphasizes the relevant aspects of the system and hides the irrelevant ones.

When the system evolves in time, the current state $X[t]$ follows a trajectory in the state space. The general form of the model of a time-invariant discrete system in the state space is formulated by the following set of equations [2]:

$$\begin{cases} X[t+1] = f(X[t], U[t]) \\ Y[t] = g(X[t], U[t]) \end{cases} \quad (1)$$

where:

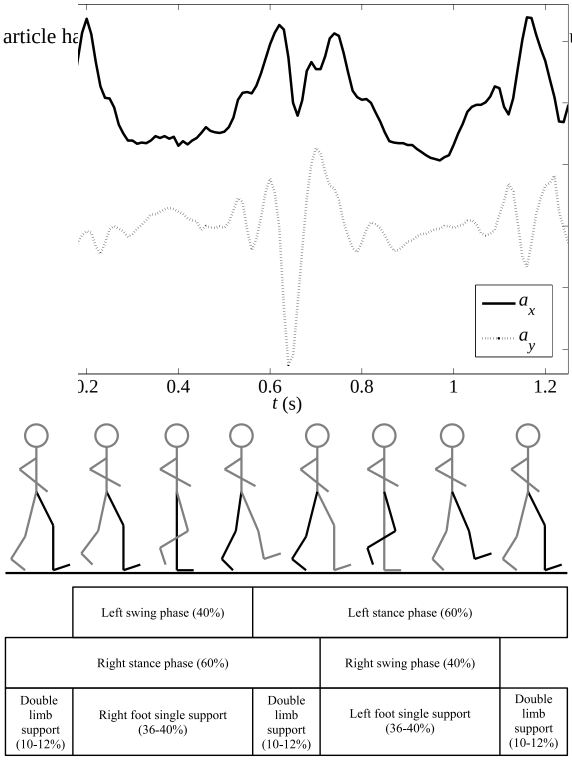

Fig. 1. One gait cycle illustrating the various phases and events and the dorso-ventral (a_x) and medio-lateral (a_y) accelerations.

- U is the input vector of the system: $(u_1, u_2, ..., u_{n_u})$, with n_u being the number of input variables.
- X is the state vector: $(x_1, x_2, ..., x_n)$, with n being the number of states. X_0 is the initial state of the system, i.e., $X_0 = X[t = 0]$.
- Y is the output vector: $(y_1, y_2, ..., y_{n_y})$, with n_y being the number of output variables.
- f is the function which calculates the state vector at time step $t + 1$.
- g is the function which calculates the output vector at time step t.

Unfortunately, for many systems in our environment, we are unable, or it is very costly, to obtain the differential equations corresponding to the functions f and g. This situation is described by the Zadeh's Principle of Incompatibility: "as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" [6].

This is to say that, when systems to be modeled grow in complexity, we have no other option but to work with imprecise models. There have been several attempts to deal with this type of problems by means of linguistic fuzzy models, which are models where at least one variable is fuzzy [42].

During the last years, FL-based models have grown in complexity as a consequence of the modeling requirements in terms of accuracy and interpretability. The number of variables and the number of needed rules to create a fuzzy model have grown up until making models difficult to understand, and consequently, difficult to apply. Currently, researchers in the field work to establish the formalism that will make the designed fuzzy models more human friendly [43], [44], [45], [46], [47].

In this work, we follow Zadeh's computing with words and perceptions paradigm [48]. The idea consists of extending FL to create system models based on the way that humans make descriptions using NL. The aim is the use of complex structures of NL to make robust imprecise models of complex systems.

As said, we will consider a FFSM to deal with the human gait modeling problem. The initial concept of FFSM was introduced by Santos [21] and developed by different authors (see, e.g., [22]). This family of FFSMs was characterized by having fuzzy states but crisp inputs. Later, this initial model was extended to have fuzzy inputs [23], [24]. Although the basic concept of FFSM used in this paper is much related to the latter one, the initial conception is quite different. The model of FFSM presented is inspired by the concepts of fuzzy state and fuzzy system developed by Zadeh [49], [50]. More specifically, it can be considered an implementation of the general idea of *input-output fuzzy models of dynamic systems* proposed by Yager [51], where the set of Equations 1 is implemented using sets of fuzzy rules. In addition, we focus our contribution on the practical challenge of developing a mechanism for learning automatically the set of rules and membership functions of the FFSM.

In this section, we introduce the main concepts and elements of our paradigm for system modeling allowing experts to build comprehensible linguistic fuzzy models in an easier way. In our framework, a FFSM is a tuple $\{Q, U, f, Y, g\}$, where:

- Q is the set of states of the system.
- U is the set of input vectors of the system.
- f is the transition function which calculates the set of states of the system.
- Y is the set of output vectors of the system.
- g is the output function which calculates the set of output vectors of the system.

Each of these components is described in detail in the following subsections. The interested reader can refer to [26], [27], [29] to find several previous applications of this FFSM model.

A. Fuzzy states (Q)

Q is the set of states of the system, which is defined as a linguistic variable [10] that takes its values in the set of linguistic labels $\{q_1, q_2, \ldots, q_n\}$, with n being the number of fuzzy states. Every fuzzy state represents the pattern of a repetitive situation. The concept of fuzzy state was introduced by Zadeh in [6]. Numerically, the state of the FFSM is represented by a state activation vector:

$S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$, where $s_i[t] \in [0, 1]$.

S_0 is defined as the initial value of the state activation vector, i.e., $S_0 = S[t = 0]$. The FFSM implementation verifies $\sum_{i=1}^{n} s_i[t] = 1$ in such way that we maintain compatibility with classical FSMs where only one state can be activated with degree 1 at each time instant. Hence, in order to maintain the latter characteristic in our FFSM model, the activation degree of the states must sum up to 1 for any system input. This restriction has been applied in previous fuzzy extensions of crisp phenomena such as fuzzy clustering [52], [53] where the sum of the membership value of a pattern to the different clusters must also sum up to 1. This decision of design is easily implemented using Equation 2 as we will show in Section III-C1.

B. Input vectors (U)

U is the set of input vectors: $(u_1, u_2, ..., u_{n_u})$, with n_u being the number of input variables. U is a set of linguistic variables obtained after fuzzification of numerical data. Typically, u_i can be directly obtained from sensor data or by applying some calculations to the raw measures, e.g., the derivative or integral of the signal, or the combination of several signals.

The expert summarizes the domain of numerical values representing them by a set of linguistic labels which define all the possible values that u_i can take:

$A_{u_i} = \{A^1_{u_i}, A^2_{u_i}, \ldots, A^{n_i}_{u_i}\}$, with n_i being the number of linguistic labels of the linguistic variable u_i.

C. Transition function (f)

f is the state transition function that calculates, at each time instant, the next value of the state activation vector: $S[t+1] = f(U[t], S[t])$.

This function is implemented by means of a fuzzy KB. Once the expert has identified the relevant states in the model, she/he must define the fuzzy rules that govern the temporal evolution of the system among these states.

We can distinguish between rules R_{ii} to remain in a state q_i, and rules R_{ij} to change from state q_i to state q_j. Fuzzy rules will only be associated to allowed transitions, i.e., if a transition is forbidden in the FFSM, it will have no fuzzy rules associated.

A generic expression of a rule R_{ij} is formulated as follows:

R_{ij}: **IF** ($S[t]$ is q_i) **AND** C_{ij} **THEN** $S[t+1]$ is q_j

where:

- The first term in the antecedent ($S[t]$ is q_i) involves the computation of the degree of activation of the state q_i in the time instant t, i.e., $s_i[t]$. With this mechanism, we only allow the FFSM to change from the state q_i to the state q_j (or to remain in state q_i, when $i = j$) if it was previously in that state.
- The second term in the antecedent C_{ij} describes the constraints imposed on the input variables that are required to change from the state q_i to the state q_j (or to remain in state q_i, when $i = j$). For example: $C_{ij} = (u_1[t]$ is $A^3_{u_1})$ **AND** $(u_2[t]$ is $A^4_{u_2}$ **OR** $A^5_{u_2})$[1].
- Finally, the consequent of the rule is the next value of the state activation vector $S[t+1]$. It consists of a vector with a zero value in all of its components but in $s_j[t]$, where it takes value one.

1) Fuzzy reasoning mechanism: The next value of the state activation vector is calculated as a weighted average of the individual rules. The weight of a rule k is calculated from its firing degree ω_k. To calculate the value of ω_k we use the minimum t-norm ($\top_{\min}(a, b) = \min\{a, b\}$) for the AND operator, and the bounded sum of Lukasiewicz t-conorm ($\perp_{\text{Luk}}(a, b) = \min\{a + b, 1\}$) for the OR operator [58], e.g., the constraint $C_k = (u_1$ is $A^3_{u_1})$ **AND** $(u_2$ is $A^4_{u_2}$ **OR** $A^5_{u_2})$ will produce a firing degree $\omega_k = \min\left\{A^3_{u_1}(u_1), \min\left\{1, A^4_{u_2}(u_2) + A^5_{u_2}(u_2)\right\}\right\}$.

As we have explained above, a certain output of a rule k predicting state q_i will be of the form $(0, \ldots, s_i[t] = 1, \ldots, 0)_k$. To calculate the total output of the rules and therefore, the state activation vector ($S[t+1]$), a weighted average of the individual outputs of each rule is computed as defined in Equation 2:

$$S[t+1] = \begin{cases} \dfrac{\sum_{k=1}^{\#Rules} \omega_k \cdot (s_1, \ldots, s_n)_k}{\sum_{k=1}^{\#Rules} \omega_k} & \text{if} \quad \sum_{k=1}^{\#Rules} \omega_k \neq 0 \\ S[t] & \text{if} \quad \sum_{k=1}^{\#Rules} \omega_k = 0 \end{cases} \tag{2}$$

This expression is a typical defuzzification mechanism applied to a set of Mamdani-type fuzzy rules where the linguistic labels of the consequent are singletons (see, e.g., [51]). With this fuzzy reasoning mechanism, the state activation vector always verifies the two constraints demanded in Section III-A: $s_i[t] \in [0, 1]$ and $\sum_{i=1}^{n} s_i[t] = 1$. Moreover, it keeps the system in its previous state if no rule is fired.

Notice that, the similarity between the FFSM's fuzzy rule structure and a fuzzy classification rule can easily be recognized. Among the three existing fuzzy classification rule structures, which mainly differ on the composition of the consequent, the simplest one is based on the use of a single class (the other two variants either include the class and a certainty degree or a certainty degree for each possible class) [59], [14]. Besides, a significant relation can be identified between the fuzzy reasoning mechanism used by the FFSM and that usually applied in fuzzy-rule based classification systems based on the latter kind of rules [59]. In fact, the computation of the next state for the FFSM can be considered as a classification problem where the set of possible fuzzy states are taken as the classes and the fuzzy system provides a membership degree to each of them by means of a single selection or an aggregation of the firing degree of the fuzzy rules matching the class and the input pattern. Nevertheless, the main difference between both fuzzy reasoning mechanisms is that, while the membership degree to all the possible fuzzy states must sum up to 1 in any case in a FFSM, there is no such restriction for the existing class labels in a fuzzy rule-based classification system.

D. Output vectors (Y)

Y is the set of output vectors: $(y_1, y_2, ..., y_{n_y})$, with n_y being the number of output variables. Y is a summary of the characteristics of the system evolution that are relevant for the application, e.g., each y_i can be obtained as the average of certain parameters of the system while the model remained in state q_i.

E. Output function (g)

g is the output function: $Y[t] = g(U[t], S[t])$. It calculates, at each time instant, the value of the output vector $Y(t)$.

[1]Notice that, this fuzzy rule structure corresponds to a disjunctive normal form (DNF), which has been largely used in fuzzy modeling and fuzzy classification [54], [55], [56], [57]

The most simple implementation of q is $Y[t] = S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$. In this contribution, the output is the current fuzzy state of the system represented by the state activation vector. An application example of a complex output function can be found in [27].

IV. Fuzzy finite state machine for human gait modeling

This section presents the design of the main elements needed to build a FFSM to model the human gait.

A. Fuzzy states

As stated in Section III-A, every state represents the pattern of a repetitive situation. According to the diagram at the bottom of Fig. 1 and using our own knowledge about the process, we can define four different fuzzy states which explain when double limb support, right limb single support, or left limb single support are produced. Therefore, we can easily define the possible set of fuzzy states as follows:

- $q_1 \rightarrow$ The right foot is in stance phase and the left foot is in stance phase (double limb support).
- $q_2 \rightarrow$ The right foot is in stance phase and the left foot is in swing phase (right limb single support).
- $q_3 \rightarrow$ The right foot is in stance phase and the left foot is in stance phase (double limb support but different of q_1 because the feet position).
- $q_4 \rightarrow$ The right foot is in swing phase and the left foot is in stance phase (left limb single support).

B. Input variables

As we have explained in Section II, we only use two of the three available accelerations, a_x and a_y. Therefore, the set of input variables is: $U = \{a_x, a_y\}$. We will build two different FFSMs, where each input variable will have three (FFSM 3) or five (FFSM 5) associated linguistic labels because, as we will show in the experimental results, they are enough to achieve a good accuracy keeping a high interpretability. The linguistic labels for each linguistic variable in the FFSM 3 are: $\{S_{a_x}, M_{a_x}, B_{a_x}\}$ and $\{S_{a_y}, M_{a_y}, B_{a_y}\}$, where S, M and B are linguistic terms representing small, medium, and big, respectively. While the linguistic labels for each linguistic variable in the FFSM 5 are: $\{VS_{a_x}, S_{a_x}, M_{a_x}, B_{a_x}, VB_{a_x}\}$ and $\{VS_{a_y}, S_{a_y}, M_{a_y}, B_{a_y}, VB_{a_y}\}$, where the additional terms VS, and VB are linguistic terms representing very small, and very big, respectively.

C. Transition function

As showed in Section III-C, the only thing required to determine the structure of the fuzzy rule base (RB) is the definition of which transitions are allowed and which are not. This is easily represented by means of a state diagram. Fig. 2 shows the proposed state diagram of the FFSM for the human gait cycle. This state diagram is very simple because the accelerations produced during the human gait are quasi-periodic, i.e., they are repeated with approximately similar values and periods. Moreover, all the states are correlative, i.e., they always follow the same activation order. Therefore, it is rather easy to define the allowed transitions and the forbidden ones.

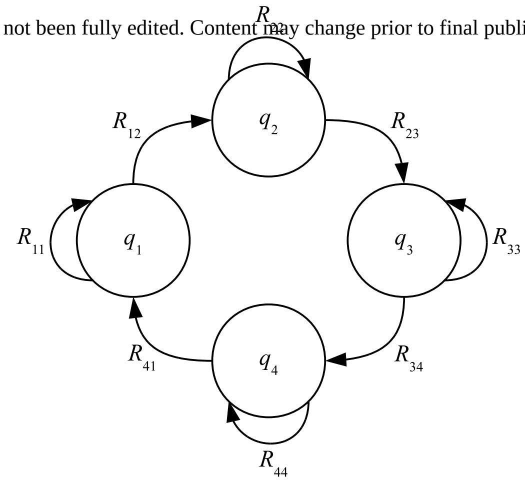

Fig. 2. State diagram of the FFSM for the human gait cycle.

From the state diagram represented in Fig. 2 it can be recognized that there are 8 fuzzy rules in total in the system, 4 rules to remain in each state and other 4 to change between states. Therefore, the RB will have the following structure:

R_{11}: **IF** ($S[t]$ is q_1) **AND** C_{11} **THEN** $S[t+1]$ is q_1
R_{22}: **IF** ($S[t]$ is q_2) **AND** C_{22} **THEN** $S[t+1]$ is q_2
R_{33}: **IF** ($S[t]$ is q_3) **AND** C_{33} **THEN** $S[t+1]$ is q_3
R_{44}: **IF** ($S[t]$ is q_4) **AND** C_{44} **THEN** $S[t+1]$ is q_4
R_{12}: **IF** ($S[t]$ is q_1) **AND** C_{12} **THEN** $S[t+1]$ is q_2
R_{23}: **IF** ($S[t]$ is q_2) **AND** C_{23} **THEN** $S[t+1]$ is q_3
R_{34}: **IF** ($S[t]$ is q_3) **AND** C_{34} **THEN** $S[t+1]$ is q_4
R_{41}: **IF** ($S[t]$ is q_4) **AND** C_{41} **THEN** $S[t+1]$ is q_1

D. Output vector and output function

In the current contribution, we simply consider $Y[t] = S[t]$, i.e., the output of the FFSM is the degree of activation of each state.

V. Genetic fuzzy system

Fuzzy systems have showed their ability to deal with a huge number of applications. In most of cases, the key for the success was the ability of fuzzy systems to incorporate human expert knowledge [45], [60], [61]. However, the lack of learning capabilities has generated a strong interest for the study of fuzzy systems with added learning capabilities. One of the most popular approaches is the hybridization between fuzzy logic and artificial neural networks [62] leading to the well known area of neuro-fuzzy systems [63], [64]. Another very extended hybrid computational intelligence system is based on the use of genetic algorithms (GAs) (and, in general, evolutionary algorithms) to learn the components of a fuzzy system leading to the field of GFSs [32], [33], [34], [35], [36]. This section introduces a new fusion framework of FFSMs, a fuzzy system type, and GAs, which will be called genetic fuzzy finite state machines (GFFSMs) from now on. Basically, a GFFSM is a FFSM augmented by a learning process based on a GA. In particular, the current section is devoted to present the GFS developed to learn the KB of the FFSM designed for human gait modeling.

When using a GA for learning a rule-based system, we can cover different levels of complexity according to the structural changes produced in the learning system by the search algorithm [65], i.e., we can do parameter optimization which is the simplest case or we can learn the complete rule set of a fuzzy rule-based system (FRBS). The KB is usually the object of study in the GFS framework. From the view point of optimization, the task of finding an appropriate KB for a particular problem is equivalent to parameterize the KB and to find those parameter values that are optimal with respect to the optimization criterion. The KB parameters constitute the search space, which is transformed into a suitable genetic representation on which the search process operates [33], [35].

As seen in Section III, the FFSM is a fuzzy system and, more specifically, a FRBS as the transition function is implemented by means of fuzzy if-then rules. Therefore, we can define a GFS to learn the main components of this fuzzy system.

In our approach, we allow the expert to introduce her/his own knowledge over the whole system by defining the states and transitions and specifying the general structure of the fuzzy rules defining the state transitions. The fuzzy rules themselves and the MFs of the input variables' linguistic labels will be automatically derived by the GFS, thus making a robust, accurate and human friendly model. Therefore, according to the different approaches presented in [32], [33], [35], we will develop a complete learning of the KB, i.e., our GFS will learn the MF shapes associated with the linguistic terms and the fuzzy rules simultaneously, although dealing with a reduced search space thanks to the incorporated expert knowledge.

The joint learning of the RB and the MFs associated with the input variables in the data base (DB) can be used as a cooperative way to obtain a FFSM that is not only accurate but also compact. We have opted by this genetic learning scheme since we consider that the joint learning of DB and RB deals with the interactions existing between both KB components in a better way than following a multi-stage learning based on first deriving the RB and later refining the preliminary DB definition [32], [33], [34], [35]. Moreover, in real complex problems, most of the effort developed in a RB learning problem is typically devoted to increase the performance of some wrong rules rather than to improve the performance of the overall system by performing a complex MF parameter learning process [66]. Nevertheless, learning the DB and the RB concurrently can make the search space so large that suboptimal models are generated [67]. Fortunately, in our case the combination of the use of expert knowledge and the prefixed structure of the FFSM allows us to deal with a more reduced search space size, thus allowing the derivation of good performing KBs.

The following subsections will describe in detail the structure of the different components of our GFS to learn the KB of FFSMs devoted to human gait modeling.

A. Representation scheme and initial population generation

Since we are developing a complete learning of the KB, we have divided the representation scheme into two parts: the RB part and the DB part. In the following, we explain each of these representations.

1) RB part: Once we have the complete rule set defined in Section IV-C, we codify the whole rule set in a chromosome following the Pittsburgh approach [68] because the evaluation of the FFSM requires a complete execution cycle. Moreover, the fixed size and structure of the rules (where the consequent and the first term of the antecedent are known) and the predefined structure of the constraints imposed on the input variables showed in Section III-C allow us to use the classical disjunctive normal form (DNF) representation based on a binary string coding [56], [57] to codify only the remaining part of the antecedent. For each of the two input variables a_x and a_y, the rule representation consists of a binary sub-string of the same length as the number of labels that refers to its linguistic term set. Each bit has a one (zero) which denotes the presence (absence) of each linguistic term in the rule. Moreover, the feature selection capability of this representation is used since an input variable is omitted in the rule if all of its bits in the representation become zeros or ones.

As an example of how this representation is developed in the GFFSM 3, let us define a rule R_k with the following constraint over the input variables:

C_k = ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is M_{a_y} **OR** B_{a_y}).

Therefore, the representation of this DNF fuzzy rule will be of the form: $\{010 : 011\}$, where in the first sub-string the second digit indicates the presence of the linguistic term M_{a_x} and the zeros indicate the absence of the terms S_{a_x} and B_{a_x}. The second sub-string has ones in the second and third positions indicating the presence of the linguistic terms M_{a_y} and B_{a_y}, and a zero in the first position, indicating the absence of the linguistic term S_{a_y}.

The RB part of the chromosome will thus be composed of 8 rules $\times$ 2 linguistic variables $\times$ l = 16 $\times$ l binary-coded genes, being l the number of linguistic terms per input variable.

2) DB part: Once we have decided the number of linguistic terms for each input variable (see Section IV-B), we can show how to represent the DB part of the KB, i.e., the representation of the MFs definition.

We have used strong fuzzy partitions (SFPs) [52] to define the fuzzy partitions. In a SFP, the membership degree forms a partition of unity. SFPs allow us to reduce the number of parameters to tune, in such way that the normalization constraint is easily satisfied by only coding the modal points of the MFs (one point for triangular MFs and two points for trapezoidal shapes). Moreover, when we calculate the OR between two linguistic labels using Łukasiewicz's bounded sum as explained in Section III-C1, the resulting linguistic label will be a convex fuzzy set without sawtooth shapes that would be produced if we use the maximum. From the interpretability point of view, SFPs satisfy semantic constraints and helps to get comprehensible fuzzy partitions [45].

We have used trapezoidal SFPs that are defined in the whole domain of discourse of the input variable. Since the fuzzy partition of each input variable is generically comprised by l linguistic labels, we have to code $2 \times [(l-2) \times 2 + 2]$ real parameters, $(l-2) \times 2 + 2$ per input variable where one parameter is enough to codify the first and last linguistic labels and two parameters are needed to codify each intermediate linguistic label. In particular, working with 3 or 5 linguistic labels, the DB part of the chromosome will be composed of 8 or 16 real-coded genes

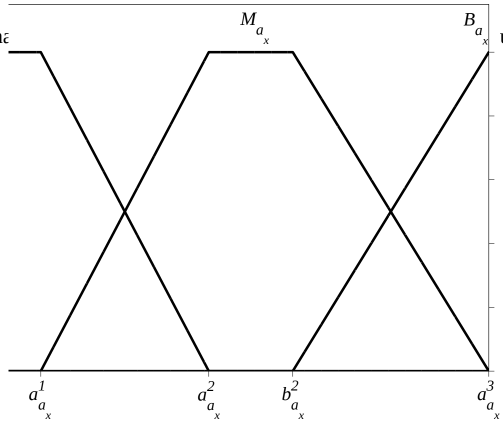

Fig. 3. Parameters that form all the linguistic labels of the linguistic variable a_x in the GFFSM 3, which are trapezoidal or triangular MFs.

respectively:

$$\text{GFFSM 3}\begin{cases} a_x \rightarrow \{a^1_{a_x}, a^2_{a_x}, b^2_{a_x}, a^3_{a_x}\} \\ a_y \rightarrow \{a^1_{a_y}, a^2_{a_y}, b^2_{a_y}, a^3_{a_y}\} \end{cases}$$

$$\text{GFFSM 5}\begin{cases} a_x \rightarrow \{a^1_{a_x}, a^2_{a_x}, b^2_{a_x}, a^3_{a_x}, b^3_{a_x}, a^4_{a_x}, b^4_{a_x}, a^5_{a_x}\} \\ a_y \rightarrow \{a^1_{a_y}, a^2_{a_y}, b^2_{a_y}, a^3_{a_y}, b^3_{a_y}, a^4_{a_y}, b^4_{a_y}, a^5_{a_y}\} \end{cases}$$

Fig. 3 shows the graphical representation of the fuzzy partition related with the linguistic input variable a_x in the GFFSM 3. For the first linguistic label S_{a_x} we only need one parameter $a^1_{a_x}$. The same stands for the last one B_{a_x} whose parameter is $a^3_{a_x}$. For the intermediate linguistic label we need two parameters $a^2_{a_x}$ and $b^2_{a_x}$. Note that we have chosen trapezoidal MFs because triangular MFs are a particular case of trapezoidal MFs, e.g. the linguistic label B_{a_x} will be triangular-shaped when the value $a^3_{a_x}$ reaches the limits of the domain of discourse of the input variable a_x.

We should remark that this learning problem demands a real-coded representation and, therefore, we have to implement real-coded crossover and mutation genetic operators. Moreover, to define the variation interval of each allele we have considered that each parameter can be only modified within the interval defined by its previous and next parameter, e.g., in Fig. 3 the definition/variation interval of parameter $a^2_{a_x}$ is $[a^1_{a_x}, b^2_{a_x}]$ while that of parameter $a^3_{a_x}$ is $[b^2_{a_x}, \max(a_x)]$ (with $\max(a_x)$ being the maximum value taken by the input variable a_x).

Hence, the final chromosome encoding a candidate problem solution will be comprised by $48 + 8 = 56$ genes in the GFFSM 3, and $80 + 16 = 96$ genes in the GFFSM 5. Fig. 4 shows the shape of the complete chromosome encoding the RB and DB part of the GFFSM 3.

We have initialized the first population by generating all the individuals at random. However, in order to include our previous knowledge about the problem, the DB part of the first individual of the population will encode uniform fuzzy partitions for both linguistic variables a_x and a_y. Then, the following individuals are created at random to introduce diversity.

B. Fitness function

The fitness function measures the quality of the candidate problem solution encoded in each chromosome. In the case of our GFFSM for human gait modeling, the dependence of the next state on the previous state makes it strictly necessary to test the FFSM over the whole data set and for each chromosome, which is very computationally expensive. This problem also appears when learning fuzzy logic controllers, where the fitness measure must be evaluated by simulating how the plant is controlled [69], [70], [71].

We have chosen the minimization of the mean absolute error (MAE) defined in Equation 3 as fitness function:

$$\text{MAE} = \frac{1}{n} \cdot \frac{1}{T} \cdot \sum_{i=1}^{n} \sum_{j=0}^{T} |s_i[j] - s_i^*[j]| \quad (3)$$

where:

- n is the number of states, i.e., $n = 4$ for the human gait modeling problem (see Section IV).
- T is the dataset size (i.e., the considered time interval duration).
- $s_i[j]$ is the degree of activation of state q_i at time $t = j$.
- $s_i^*[j]$ is the expected degree of activation of state q_i at time $t = j$.

The MAE is a very informative measure of the quality of the candidate solution because it directly measures the difference between the expected state activation vector ($S^*[t]$) and the obtained one ($S[t]$). However, we need to define an expected activation vector $S^*[t]$ for each input data set that we want to learn, i.e., a training data set in the context of a supervised learning problem to design our human gait FFSM-based model. This definition could be problematic and must be done carefully because sometimes, must be defined at each time instant more than one state, activated with certain degree in the interval $[0, 1]$. In the following subsection, this issue is explained in detail.

C. Defining the training data set

As described above in Section V-B, in order to learn a FFSM for different gaits of the same person, there is a need to define an expected activation vector $S^*[t]$ for each one of the gaits we want to learn. Hence, we have to create a training vector which consists of $a_x[t]$, $a_y[t]$ and $S^*[t]$, i.e., $(a_x[t], a_y[t], s_1^*[t], s_2^*[t], s_3^*[t], s_4^*[t])$.

To define the training vector, we have developed a user-friendly graphical interface that allows the expert to select the relevant points where each state starts and ends using her/his knowledge about the human gait process. For instance, "the double limb support that comes after the right single support starts just after the heel contact" can be translated as "state q_1 must start when a_x increases drastically and a_y tends to decrease" [72]. The fuzzy definition of the states is based on the imprecision of the expert when defining the start and the end of each state which she/he must identify and label within the time series associated to the measured signals. We have defined the training vectors for data sets which consist of five complete cycles of the human gait. For each state q_i, we will have ten different points, five comprising the beginning (b_i^m) and another five comprising the end (e_i^m) of each state, with $m = 1, \ldots, 5$. In the current FFSM involving four fuzzy states, the expert will have to tag each sample of five cycles with 40 points: $\{b_1^1, b_1^2, \ldots, e_4^4, e_4^5\}$.

As an example, let us consider the definition of the degree of activation of state q_2 specified by Equation 4. Between the end time of q_1 (e_1^m) and the start time of q_2 (b_2^m), the

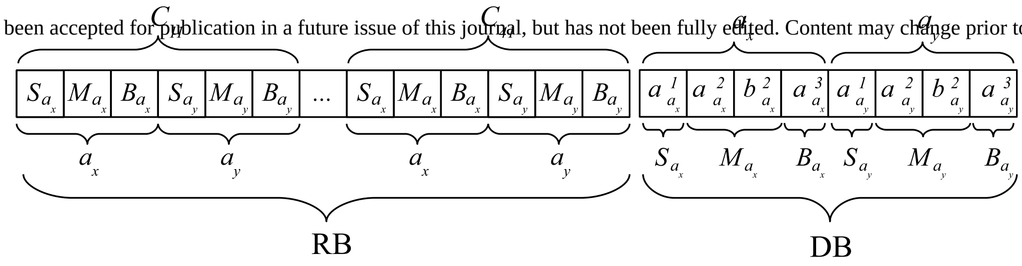

Fig. 4. Chromosome which codifies the RB and DB part of the GFFSM 3.

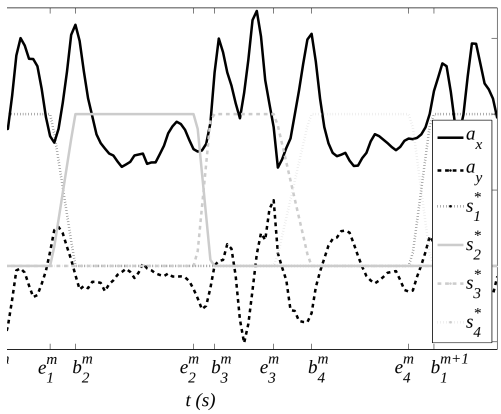

Fig. 5. Construction of the vectors of training data based on the start and end points given by the user.

activation of the state q_2 is rising from 0 to 1. Between the start (b_2^m) and the end time (e_2^m) of q_2 defined by the user, the activation has the maximum of 1, and afterwards the activation drops till zero at the start of q_3 (b_3^m). Otherwise, the activation is zero.

$$s_2^*[t] = \begin{cases} \frac{t-e_1^m}{b_2^m-e_1^m} & \text{if} \quad e_1^m < t < b_2^m \\ 1 & \text{if} \quad b_2^m \leq t \leq e_2^m \\ \frac{b_3^m-t}{b_3^m-e_2^m} & \text{if} \quad e_2^m < t < b_3^m \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Fig. 5 shows an example of how a part of the training vector is labeled based on the beginning and the end points given by the expert.

D. Genetic operators

The definition of the genetic operators considered in our GFFSM for human gait modeling are showed as follows:

1) Selection mechanism: To select the parents that will undergo crossover and mutation, a binary tournament selection is considered. This operator is very useful since it does not require any global knowledge of the population [31]. The idea is to select at random two parents and choose the best one with respect to the fitness function, repeating this process until a complete parents set is built.

2) Crossover: The classical two-point crossover has been used for the RB (binary-coded) part of the chromosome and BLX-α crossover [73] for the DB (real-coded) part. The BLX-α crossover is applied twice over a pair of parents in order to obtain a new pair of chromosomes. When a pair of chromosomes is chosen for crossover based on a single crossover probability, we separately crossover the binary part and the real part. Notice that, the proposed genetic operators can be independently applied in both chromosome parts ensuring the obtaining of an offspring encoding a coherent FFSM KB definition. That does not always happen when working with GFSs learning the whole KB using a representation scheme based on two information levels (the DB and RB parts) since those two parts can be so related that the action of a genetic operator in one of them can cause the appearance of meaningless chromosomes because the information encoded in the other part is no longer valid (see, for example, [74], [75]). Nevertheless, that is not the case in the current coding scheme.

3) Mutation: For the binary-coded RB part, the classical bitwise mutation has been selected. For the real-coded DB part, the corresponding mutation operator called uniform mutation [31] has been chosen. It consists of changing the allele value of each gene randomly within its definition interval. As for the crossover, the same mutation probability defined at gene level is considered for both chromosome parts.

4) Replacement mechanism: In our approach, we directly replace the current population by the offspring one (generational replacement) keeping elitism.

5) Termination condition: In this contribution, we have implemented three different termination conditions. First, the search is stopped when the algorithm has obtained a fitness value equal to zero, which is the best value that the fitness function can take. However, this condition is almost impossible to be reached. Therefore, we have decided to set a maximum number of evaluations and also to stop the search when, for a certain number of evaluations, the fitness value of the best individual is not improved.

VI. Experimentation

This section presents the experimentation carried out to validate our proposal. First, the experimental setup, which comprises the data acquisition and the GFFSM parameter values, is explained. The second subsection contains a brief description of two alternative modeling approaches used for human gait modeling. Finally, the third and fourth subsections report the obtained results and their analysis, respectively.

A. Experimental Setup

1) Data acquisition: To evaluate the proposed approach, we have collected the acceleration signals of 20 different people in order to create a specific FFSM to model the gait of each person. The group of people consisted of healthy adults, 5 women and 15 men, with ages ranging between 23 and 52 years (with an average age of 30 years) and weights between 45 and 97 Kg (with an average of 76 Kg).

We attached to a belt, centered in the back of the person, a three-axial accelerometer with Bluetooth capabilities that provided measurements of the three orthogonal accelerations with a frequency of 100 Hz. We programmed a personal digital agenda (PDA) to receive the data via a Bluetooth connection and to record it with a timestamp. Therefore, every record contained the information: (t, a_x, a_y, a_z) where t is each instant of time, a_x is the dorso-ventral acceleration, a_y is the medio-lateral acceleration, and a_z is the antero-posterior acceleration. As explained in Section II, in this work we only use a_x and a_y.

We asked each person to walk a certain distance at a self-selected walking speed which comprises around ten complete gait cycles in such a way that we were able to extract five complete gait cycles discarding the first and last steps which are not very stable. This process was repeated ten times for each person producing a total of ten datasets of five complete cycles for each person. These datasets were then processed as explained in Section V-C in order to define all the fuzzy states. Therefore, once we captured and tagged all the signals, we had ten different datasets for each person with the following components:

$$(a_x[t], a_y[t], s_1^*[t], s_2^*[t], s_3^*[t], s_4^*[t])$$

Where:

- $a_x[t]$ is the dorso-ventral acceleration at time instant t.
- $a_y[t]$ is the medio-lateral acceleration at time instant t.
- $s_1^*[t]$ is the expected degree of activation of state q_1 at time instant t.
- $s_2^*[t]$ is the expected degree of activation of state q_2 at time instant t.
- $s_3^*[t]$ is the expected degree of activation of state q_3 at time instant t.
- $s_4^*[t]$ is the expected degree of activation of state q_4 at time instant t.

2) Parameter values for the GFFSM: Two different granularity levels have been considered for the fuzzy partitions, 3 and 5 (noted as GFFSM 3 and GFFSM 5 respectively). The parameter values used by both GFFSMs are as follows. Quite standard values are considered and a preliminary experimentation was developed to check their good performance.

- Population size $\rightarrow 100$ individuals.
- Crossover probability $\rightarrow p_c = 0.8$.
- Value of alpha (BLX-α parameter) $\rightarrow \alpha = 0.3$.
- Mutation probability per bit $\rightarrow p_m = 0.02$.
- Termination conditions:
  - Fitness value reached $\rightarrow$ MAE $= 0$.
  - Maximum number of evaluations $\rightarrow$ 40000 for the GFFSM 3 and 60000 for the GFFSM 5.
  - Evaluations without improvement of the fitness function $\rightarrow$ 10000.

B. Alternative modeling approaches

In order to compare the two GFFSM results with other system identification approaches, we have considered two different techniques commonly used in system modeling of time-dependent systems: autoregressive linear models (ARX) [3] and neural networks (NN) [62].

MODEL	n_A	n_B	COMPLEXITY
ARX 1	1	1	0
ARX 2	2	2	0.010
ARX 3	5	5	0.040
ARX 4	10	10	0.091
ARX 5	20	20	0.192
ARX 6	25	25	0.242
ARX 7	50	50	0.495
ARX 8	75	75	0.747
ARX 9	80	80	0.798
ARX 10	100	100	1

TABLE I
PARAMETER VALUES CONSIDERED FOR THE DIFFERENT ARX MODELS AND THEIR COMPLEXITY.

1) Autoregressive linear models (ARX): We have defined a multiple-input multiple-output (MIMO) ARX model with the structure defined by Equation 5.

$$\begin{aligned} Y[t] &= A_1 \cdot Y[t-1] + \ldots + A_{n_A} \cdot Y[t-n_A] \\ &+ B_0 \cdot U[t] + \ldots + B_{n_B} \cdot U[t-n_B] \end{aligned} \quad (5)$$

where:

- $Y[t] = (s_1[t], s_2[t], s_3[t], s_4[t])$ is the current output vector.
- $Y[t-1], \ldots, Y[t-n_A]$ are the previous output vectors on which the current output vector depends.
- $U[t] = (a_x[t], a_y[t]), \ldots, U[t-n_B]$ are the current and delayed input vectors on which the current output vector depends.
- n_A is the number of previous output vectors on which the current output vector depends.
- n_B is the number of previous input vectors on which the current output vector depends.
- $A_1, \ldots, A_{n_A}, B_0, \ldots, B_{n_B}$ are the matrices that define the models. They are estimated using the least squares method.

We have tested the performance of this model for ten different values of the parameters n_A and n_B in order to obtain several models with a different accuracy-complexity tradeoff.

For the first parameter values, we have selected a simple model similar to the delay of our GFFSM, i.e., $n_A = n_B = 1$ resulting in the ARX model number 1 defined by Equation 6:

$$Y[t] = A_1 \cdot Y[t-1] + B_0 \cdot U[t] \quad (6)$$

Then, another nine different values (with a maximum delay of 100) were used to progressively increase the complexity of the model. A linear complexity index is defined in such a way that the complexity of the basic model with $n_A = n_B = 1$ is zero and the complexity for the most complex model with $n_A = n_B = 100$ is one. The different parameter values for each model together with the model complexity are showed in Table I.

2) Neural networks: As for the ARX models, we have built ten different feed-forward NN architectures representing different levels of complexity. The first and simplest one (NN 1) consists of 2 neurons in the input layer which represent the two input variables $a_x[t]$ and $a_y[t]$, one hidden layer, and four output neurons in the output layer corresponding to the four components of the state activation vector $(s_1[t], s_2[t], s_3[t], s_4[t])$.

The other NN models, representing different levels of complexity, are determined by the number of delayed input variables. Moreover, in order to avoid NNs with a huge number of input neurons (which leads to overfitting and big training times), we have considered delayed input variables separated by a fixed interval of ten samples. E.g., the second NN architecture (NN 2) has $a_x[t], a_x[t-10], a_y[t]$, and $a_y[t-10]$ as inputs; while the most complex one (NN 10) has twenty inputs that cover a delay of 90 samples: $a_x[t], a_x[t-10], \ldots, a_x[t-90]$, and $a_y[t], a_y[t-10], \ldots, a_y[t-90]$.

The NN weights have been estimated using the Levenberg-Marquardt method during 500 epochs. The number of neurons in the hidden layer was chosen to minimize the test error of each specific architecture. The architectures of the two extreme NNs are represented in Fig. 6.

Similarly to the ARX models, a complexity index is defined in such a way that the complexity of the NN with 2 inputs (NN 1) is zero and the complexity of the NN with 20 inputs (NN 10) is one. The different models, their input variables, and their complexity are showed in Table II.

C. Results

To test the performance of the two GFFSMs and the alternative modeling approaches, we have done a leave-one-out cross validation [76] for each of the 10 datasets of each person. As an example, Table III shows the MAE obtained for each fold of the leave-one-out corresponding to the first person's experiments. It also depicts the average value of the MAE and its standard deviation for the ten folds.

As a global summary of the results obtained, Table IV reports, for each of the leave-one-out applications for the 10 datasets of each person, the average (MEAN) and standard deviation (STD) of the MAE for eight different models: those two corresponding to our proposal (GFFSM 3 and GFFSM 5), three ARX models comprising a good tradeoff model (ARX 4) and the two extreme ones (ARX 1 and ARX 10), and three NN models comprising a good tradeoff model (NN 4) and the two extreme ones (NN 1 and NN 10).

To select the best accuracy-complexity tradeoff model for both NN and ARX models we compute 1000 random weights $\omega_i \in [0, 1]$. We calculate the average MAE for each model for the 20 people ($\overline{\text{MAE}}$) and normalize the resulting set of MAEs in the interval $[0, 1]$. We take the average value of the aggregation function of both the normalized MAE ($\widetilde{\text{MAE}}$) and the complexity index value (COMPLEXITY) of each model as showed in Equation 7. Finally, the model with the lowest aggregated value is selected as that with the best tradeoff:

$$Q_{\text{MODEL}} = \sum_{i=1}^{1000} \omega_i \cdot \widetilde{\text{MAE}}_{\text{MODEL}} + (1-\omega_i) \cdot \text{COMPLEXITY}_{\text{MODEL}} \quad (7)$$

Moreover, since our final goal is to obtain a specific model (FFSM) for each person's gait, Table V shows the average of the MAE for each one of the person's models (FFSMs) generated during the leave-one-out procedure when the input data is the whole set of gaits of each person. The aim of these results is to check if the generated models are significantly fitted to the specific person's gait than to the other persons' gaits, as expected and desired.

As can be seen, that is clearly the case. For example, notice that, the models GFFSM 3 and GFFSM 5 for the first person (P1) corresponding to the first two rows get an average MAE (boldfaced) of 0.088 and 0.076 respectively with its own person's gait (first row, first column) while they get large average MAE values for the gaits of the rest of the people (the rest of the columns in the first row). This fact can also be checked for the models of the rest of the people. In addition, the last column of the table (MEAN^{-}) shows boldfaced the average value of all the MAEs obtained by each person's FFSM model with the gaits of all the people except its own input gaits. It can be easily seen that these values are much greater than the ones obtained with the gaits of each person's model (boldfaced in the diagonal cells of the table).

D. Discussion

This subsection aims to present a discussion about four different issues of our proposed model: its accuracy, its interpretability, its computational cost, and the importance of the use of expert knowledge.

1) Accuracy analysis: The results given in Tables III and IV show that the GFFSM models exhibit better accuracies when compared to the simplest competing models, namely ARX 1 and NN 1. Besides, it can be seen how the best tradeoff model ARX 7 is able to outperform our proposal, although it needs a big delay of 50 samples to do so. On the opposite, the best tradeoff model NN 4 shows a similar accuracy to our models. Its results are slightly better than those of GFSSM 3 and slightly worse than those of GFFSM 5. In fact, the GFFSM 5 is better than the GFFSM 3 for the majority of the people due to its higher granularity in the number of linguistic labels, which provides it with additional freedom degrees for the modeling task.

In order to assess whether significant differences exist among the results of all models, we use the Wilcoxon signed-rank test [77] for pairwise comparison between our models (GFFSM 3 and GFSSM 5) and the rest of competing models. We choose this test because it does not assume normal distributions and because it has been commonly used to compare performance of methods in computational intelligence [78], [79]. To perform the test we use the standard confidence level of $\alpha = 0.05$.

We have run the Wilcoxon signed rank test for three different hypotheses, if the average MAEs of our proposed approaches ($\mu_{\text{GFFSM}\,3}$ and $\mu_{\text{GFFSM}\,5}$) are *equal*, *less*, or *greater* than those obtained by the other modeling techniques (μ_{MOD}). We conclude that our proposal is better (denoted by [+]) if the test rejects both null hypotheses $H_0 : \mu_{\text{GFFSM}} > \mu_{\text{MOD}}$ and $H_0 : \mu_{\text{GFFSM}} = \mu_{\text{MOD}}$. We conclude that our proposal is worse (denoted by [-]) if the test rejects both null hypotheses $H_0 : \mu_{\text{GFFSM}} < \mu_{\text{MOD}}$ and $H_0 : \mu_{\text{GFFSM}} = \mu_{\text{MOD}}$. In all other cases we do not draw any conclusion (denoted by [=]).

Table VI shows the obtained p-values and the drawn conclusions. The results particularly indicate that GFFSM 3

MODEL	INPUT VARIABLES	COMPLEXITY
NN 1	$a_x[t], a_y[t]$	0
NN 2	$a_x[t], a_x[t-10], a_y[t], a_y[t-10]$	0.111
NN 3	$a_x[t], a_x[t-10], a_x[t-20], a_y[t], a_y[t-10], a_y[t-20]$	0.222
NN 4	$a_x[t], a_x[t-10], \ldots, a_x[t-30], a_y[t], a_y[t-10], \ldots, a_y[t-30]$	0.333
NN 5	$a_x[t], a_x[t-10], \ldots, a_x[t-40], a_y[t], a_y[t-10], \ldots, a_y[t-40]$	0.444
NN 6	$a_x[t], a_x[t-10], \ldots, a_x[t-50], a_y[t], a_y[t-10], \ldots, a_y[t-50]$	0.556
NN 7	$a_x[t], a_x[t-10], \ldots, a_x[t-60], a_y[t], a_y[t-10], \ldots, a_y[t-60]$	0.667
NN 8	$a_x[t], a_x[t-10], \ldots, a_x[t-70], a_y[t], a_y[t-10], \ldots, a_y[t-70]$	0.778
NN 9	$a_x[t], a_x[t-10], \ldots, a_x[t-80], a_y[t], a_y[t-10], \ldots, a_y[t-80]$	0.889
NN 10	$a_x[t], a_x[t-10], \ldots, a_x[t-90], a_y[t], a_y[t-10], \ldots, a_y[t-90]$	1

TABLE II
THE TEN DIFFERENT NN ARCHITECTURES WITH THEIR INPUT VARIABLES AND COMPLEXITIES.

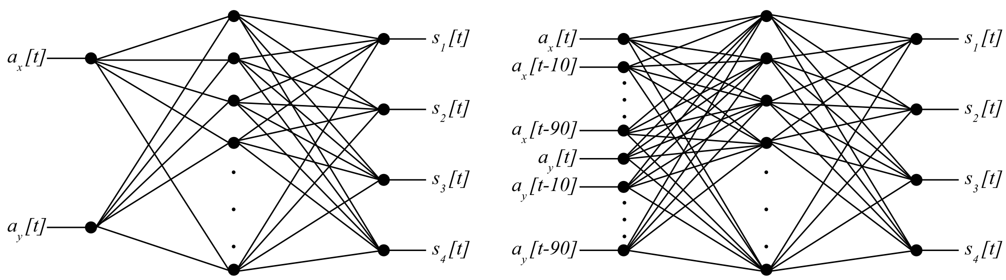

Fig. 6. Architectures of the simplest neural network (NN 1) and the most complex one (NN 10) designed for human gait modeling. All of them have a single hidden layer and the difference arises in the number of (delayed) inputs.

FOLD	GFFSM 3	ARX 1	ARX 2	ARX 3	ARX 4	ARX 5	ARX 6	ARX 7	ARX 8	ARX 9	ARX 10
1	0.089	0.337	0.316	0.293	0.262	0.201	0.172	0.068	0.065	0.066	0.068
2	0.066	0.340	0.316	0.293	0.261	0.192	0.154	0.058	0.051	0.051	0.058
3	0.135	0.341	0.320	0.296	0.267	0.224	0.162	0.060	0.044	0.044	0.060
4	0.108	0.343	0.318	0.291	0.256	0.212	0.166	0.054	0.049	0.048	0.054
5	0.133	0.337	0.320	0.298	0.259	0.217	0.215	0.054	0.039	0.040	0.053
6	0.078	0.338	0.315	0.300	0.258	0.184	0.169	0.059	0.056	0.057	0.058
7	0.101	0.345	0.316	0.299	0.256	0.219	0.182	0.058	0.051	0.048	0.058
8	0.149	0.345	0.317	0.298	0.274	0.234	0.164	0.107	0.105	0.105	0.107
9	0.086	0.340	0.311	0.288	0.253	0.241	0.221	0.117	0.117	0.115	0.116
10	0.081	0.335	0.313	0.291	0.259	0.195	0.196	0.070	0.079	0.079	0.070
MEAN	0.103	0.340	0.316	0.295	0.261	0.212	0.180	0.070	0.065	0.065	0.070
STD	0.028	0.003	0.003	0.004	0.006	0.019	0.023	0.023	0.027	0.026	0.023

FOLD	GFFSM 5	NN 1	NN 2	NN 3	NN 4	NN 5	NN 6	NN 7	NN 8	NN 9	NN 10
1	0.107	0.218	0.138	0.098	0.084	0.077	0.076	0.065	0.061	0.060	0.059
2	0.081	0.209	0.114	0.087	0.068	0.067	0.057	0.056	0.048	0.049	0.046
3	0.077	0.221	0.125	0.089	0.072	0.065	0.056	0.051	0.053	0.043	0.041
4	0.085	0.219	0.123	0.096	0.084	0.070	0.066	0.053	0.054	0.058	0.050
5	0.085	0.221	0.132	0.092	0.074	0.073	0.060	0.061	0.055	0.050	0.051
6	0.063	0.219	0.146	0.110	0.087	0.084	0.069	0.060	0.062	0.055	0.058
7	0.083	0.209	0.123	0.088	0.078	0.070	0.063	0.061	0.061	0.054	0.051
8	0.142	0.236	0.144	0.113	0.085	0.089	0.079	0.073	0.075	0.068	0.074
9	0.076	0.212	0.131	0.107	0.085	0.077	0.074	0.074	0.066	0.057	0.065
10	0.080	0.207	0.119	0.080	0.067	0.062	0.057	0.056	0.050	0.045	0.046
MEAN	0.088	0.217	0.129	0.096	0.078	0.073	0.066	0.061	0.059	0.054	0.054
STD	0.022	0.009	0.011	0.011	0.008	0.009	0.008	0.008	0.008	0.008	0.010

TABLE III
MAE OF THE LEAVE-ONE-OUT FOR THE DATASETS OF THE FIRST PERSON, WITH THE AVERAGE (MEAN) AND STANDARD DEVIATION (STD) FOR EACH OF THE EVALUATED MODELS.

model is significantly better than the first five ARX models and the first two NNs (while for ARX 6, NN 3, and NN 4 the testings do not provide clear conclusions). The GFFSM 5 model is significantly better than the GFFSM 3, the first six ARX models, and the first three NNs (while for NN 4, NN 5, NN 6, NN 7, NN 8, and NN 10 the testings do not provide clear conclusions). In view of these results, the accuracy of the GFFSMs, in particular that of GFFSM 5, is competitive with almost every NN (all but NN 9) and the first six ARX models.

The good accuracy of our model is also illustrated in Fig. 7. The vertical axis depicts each component of the state activation vector for a gait of the first person obtained using our proposal ($S[t]^{GFFSM\ 3}$ and $S[t]^{GFFSM\ 5}$), the THRIFT-FFSM model ($S[t]^{THRIFT}$, see later Section VI-D4), the best tradeoff ARX model (ARX 7) ($S[t]^{ARX\ 7}$), and the best tradeoff NN architecture (NN 4) ($S[t]^{NN\ 4}$). The actual values ($S^*[t]$) are reported in the bottom line for comparison. The activation value is represented by means of a gray intensity scale (black means one and white means zero). Notice that, both ARX 7 and NN 4 calculate the activation vector after the first 0.5 and 0.3 seconds respectively due to the fact that they need the first 50 and 30 samples to operate (0.5 and 0.3 seconds with a sampling frequency of 100 Hz). It can be seen how the GFFSM 3 and the GFFSM 5 are able to follow the appropriate sequence of states with the correct

PERSON	GFFSM 3		GFFSM 5		ARX 1		ARX 7		ARX 10		NN 1		NN 4		NN 10	
	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD
1	0.103	0.028	0.088	0.022	0.340	0.003	0.070	0.023	0.070	0.023	0.217	0.009	0.078	0.008	0.054	0.010
2	0.063	0.025	0.055	0.020	0.379	0.011	0.064	0.027	0.064	0.027	0.194	0.007	0.061	0.009	0.053	0.008
3	0.091	0.018	0.077	0.022	0.331	0.007	0.103	0.041	0.102	0.041	0.238	0.010	0.080	0.010	0.053	0.008
4	0.150	0.100	0.143	0.040	0.328	0.006	0.110	0.050	0.109	0.050	0.257	0.007	0.118	0.012	0.078	0.011
5	0.071	0.047	0.055	0.031	0.254	0.006	0.071	0.016	0.071	0.016	0.248	0.005	0.093	0.008	0.063	0.010
6	0.106	0.030	0.117	0.022	0.323	0.014	0.107	0.040	0.106	0.040	0.258	0.006	0.115	0.014	0.077	0.011
7	0.170	0.045	0.159	0.034	0.355	0.008	0.074	0.021	0.074	0.021	0.255	0.009	0.094	0.012	0.052	0.007
8	0.065	0.019	0.067	0.034	0.352	0.004	0.065	0.020	0.065	0.020	0.211	0.003	0.072	0.005	0.048	0.006
9	0.098	0.041	0.121	0.075	0.319	0.009	0.115	0.076	0.115	0.076	0.243	0.010	0.120	0.020	0.088	0.016
10	0.121	0.056	0.098	0.050	0.382	0.012	0.088	0.030	0.088	0.030	0.237	0.005	0.108	0.008	0.068	0.008
11	0.101	0.039	0.110	0.032	0.376	0.004	0.094	0.018	0.090	0.025	0.221	0.003	0.109	0.012	0.088	0.010
12	0.303	0.143	0.229	0.131	0.339	0.002	0.078	0.018	0.076	0.028	0.301	0.021	0.261	0.048	0.257	0.065
13	0.281	0.104	0.263	0.126	0.348	0.022	0.083	0.046	0.086	0.059	0.282	0.029	0.268	0.085	0.296	0.094
14	0.059	0.020	0.066	0.044	0.370	0.005	0.070	0.029	0.066	0.026	0.204	0.005	0.062	0.011	0.046	0.014
15	0.279	0.125	0.209	0.127	0.339	0.020	0.089	0.028	0.080	0.030	0.290	0.026	0.249	0.075	0.253	0.082
16	0.059	0.038	0.066	0.064	0.233	0.009	0.085	0.069	0.070	0.020	0.238	0.010	0.093	0.012	0.055	0.007
17	0.215	0.135	0.153	0.120	0.333	0.004	0.114	0.038	0.124	0.051	0.297	0.034	0.233	0.110	0.233	0.124
18	0.088	0.046	0.070	0.015	0.343	0.006	0.069	0.026	0.073	0.020	0.210	0.009	0.077	0.009	0.052	0.008
19	0.105	0.029	0.106	0.041	0.384	0.007	0.101	0.044	0.104	0.044	0.243	0.006	0.110	0.012	0.080	0.012
20	0.142	0.053	0.120	0.036	0.366	0.018	0.117	0.056	0.101	0.039	0.270	0.005	0.099	0.012	0.062	0.007
MEAN	0.133	0.057	0.119	0.054	0.340	0.009	0.088	0.036	0.087	0.034	0.246	0.011	0.125	0.025	0.103	0.026

TABLE IV

AVERAGE (MEAN) AND STANDARD DEVIATION (STD) OF THE MAE FOR EACH ONE OF THE LEAVE-ONE-OUT FOR THE 10 DATASETS OF EACH PERSON.

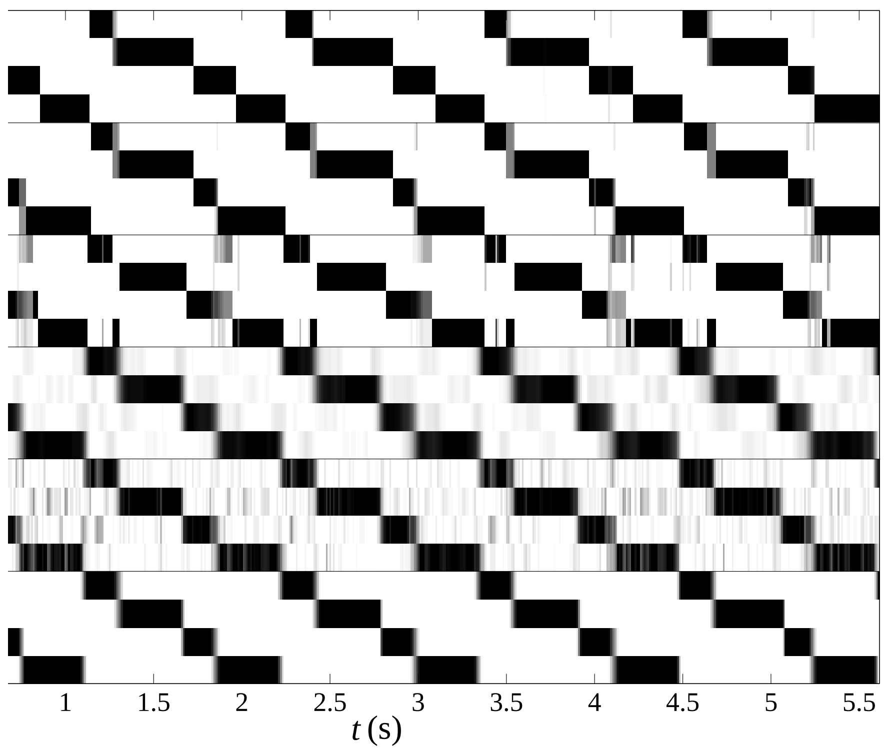

Fig. 7. Comparison of the state activation vector $(s_1[t], s_2[t], s_3[t], s_4[t])$ values obtained by means of the two GFFSMs, the THRIFT-FFSM, the ARX 7 model, the NN 4 model, with the actual values $(s_1^*[t], s_2^*[t], s_3^*[t], s_4^*[t])$ with respect to time.

activation degree.

2) Interpretability analysis: From the interpretability point of view, both NNs and the ARX models are black-box models difficult to be understood by human experts, even more if they have a big number of delayed input variables or a high number of inputs. Nevertheless, our GFFSMs are able to describe and model the human gait phenomenon by means of only eight linguistic fuzzy if-then rules (whose input variables have only three or five associated linguistic labels) achieving a good interpretability-accuracy tradeoff. As an example of how our proposal is describing linguistically the temporal evolution of the accelerations produced during the human gait, a complete RB learned for the GFFSM 3 in one of the executions of the GA is showed as follows:

R_{11}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x})
THEN $S[t+1]$ is q_1
R_{22}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is M_{a_y} **OR** B_{a_y})
THEN $S[t+1]$ is q_2
R_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is M_{a_y})
THEN $S[t+1]$ is q_3
R_{44}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is S_{a_x} **OR** B_{a_x}) **AND** ($a_y[t]$ is S_{a_y} **OR** M_{a_y})
THEN $S[t+1]$ is q_4
R_{12}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is S_{a_y} **OR** M_{a_y})
THEN $S[t+1]$ is q_2
R_{23}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is S_{a_y})
THEN $S[t+1]$ is q_3
R_{34}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is M_{a_x} **OR** B_{a_x}) **AND** ($a_y[t]$ is M_{a_y} **OR** B_{a_y})
THEN $S[t+1]$ is q_4

MODEL	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	MEAN
	INPUT																				
$P1_{GFFSM\,3}$	**0.088**	0.276	0.347	0.336	0.373	0.357	0.319	0.167	0.335	0.331	0.288	0.335	0.335	0.417	0.325	0.343	0.349	0.357	0.361	0.346	**0.331**
$P1_{GFFSM\,5}$	**0.076**	0.341	0.370	0.333	0.357	0.380	0.348	0.260	0.353	0.357	0.301	0.366	0.360	0.361	0.366	0.312	0.360	0.366	0.358	0.364	**0.348**
$P2_{GFFSM\,3}$	0.400	**0.060**	0.331	0.415	0.405	0.419	0.393	0.402	0.394	0.344	0.400	0.395	0.362	0.404	0.392	0.435	0.405	0.399	0.417	0.407	**0.396**
$P2_{GFFSM\,5}$	0.405	**0.047**	0.305	0.394	0.403	0.418	0.388	0.400	0.390	0.362	0.395	0.383	0.341	0.351	0.386	0.410	0.386	0.404	0.404	0.406	**0.386**
$P3_{GFFSM\,3}$	0.407	0.319	**0.082**	0.400	0.375	0.407	0.397	0.410	0.405	0.383	0.224	0.381	0.369	0.425	0.374	0.400	0.386	0.398	0.447	0.399	**0.384**
$P3_{GFFSM\,5}$	0.380	0.321	**0.072**	0.377	0.363	0.384	0.386	0.312	0.399	0.373	0.254	0.380	0.369	0.409	0.382	0.403	0.382	0.386	0.429	0.367	**0.371**
$P4_{GFFSM\,3}$	0.284	0.309	0.409	**0.103**	0.279	0.287	0.333	0.300	0.358	0.374	0.323	0.347	0.313	0.325	0.342	0.224	0.383	0.365	0.243	0.372	**0.325**
$P4_{GFFSM\,5}$	0.252	0.315	0.360	**0.109**	0.345	0.285	0.290	0.310	0.400	0.389	0.316	0.340	0.308	0.342	0.355	0.249	0.365	0.369	0.251	0.376	**0.327**
$P5_{GFFSM\,3}$	0.410	0.399	0.467	0.345	**0.046**	0.360	0.369	0.372	0.380	0.352	0.395	0.362	0.395	0.229	0.398	0.360	0.376	0.386	0.347	0.382	**0.373**
$P5_{GFFSM\,5}$	0.378	0.368	0.409	0.373	**0.039**	0.319	0.356	0.411	0.352	0.345	0.317	0.365	0.387	0.355	0.381	0.364	0.372	0.363	0.366	0.347	**0.365**
$P6_{GFFSM\,3}$	0.380	0.362	0.388	0.327	0.242	**0.091**	0.339	0.334	0.400	0.423	0.429	0.348	0.305	0.382	0.316	0.219	0.364	0.327	0.252	0.329	**0.340**
$P6_{GFFSM\,5}$	0.363	0.298	0.360	0.342	0.256	**0.085**	0.345	0.285	0.419	0.418	0.410	0.349	0.331	0.406	0.363	0.259	0.354	0.347	0.261	0.330	**0.342**
$P7_{GFFSM\,3}$	0.278	0.215	0.229	0.287	0.381	0.267	**0.135**	0.213	0.379	0.396	0.240	0.335	0.317	0.411	0.350	0.302	0.333	0.310	0.328	0.378	**0.313**
$P7_{GFFSM\,5}$	0.306	0.230	0.284	0.302	0.389	0.237	**0.124**	0.248	0.354	0.374	0.276	0.338	0.317	0.376	0.355	0.317	0.339	0.295	0.310	0.371	**0.317**
$P8_{GFFSM\,3}$	0.264	0.239	0.327	0.332	0.327	0.333	0.311	**0.061**	0.345	0.369	0.322	0.339	0.336	0.397	0.332	0.349	0.337	0.350	0.371	0.328	**0.332**
$P8_{GFFSM\,5}$	0.216	0.275	0.315	0.337	0.380	0.335	0.317	**0.056**	0.337	0.395	0.291	0.358	0.358	0.395	0.340	0.374	0.331	0.378	0.370	0.361	**0.340**
$P9_{GFFSM\,3}$	0.402	0.409	0.438	0.379	0.379	0.411	0.374	0.379	**0.073**	0.356	0.387	0.332	0.373	0.251	0.365	0.377	0.340	0.357	0.359	0.378	**0.371**
$P9_{GFFSM\,5}$	0.384	0.426	0.447	0.384	0.381	0.393	0.362	0.408	**0.075**	0.306	0.408	0.322	0.373	0.208	0.367	0.374	0.348	0.398	0.368	0.385	**0.371**
$P10_{GFFSM\,3}$	0.395	0.375	0.385	0.385	0.354	0.432	0.341	0.392	0.252	**0.093**	0.390	0.345	0.382	0.330	0.311	0.415	0.352	0.401	0.432	0.332	**0.369**
$P10_{GFFSM\,5}$	0.342	0.362	0.407	0.368	0.323	0.362	0.341	0.367	0.236	**0.073**	0.334	0.321	0.347	0.353	0.311	0.360	0.367	0.383	0.358	0.318	**0.345**
$P11_{GFFSM\,3}$	0.279	0.254	0.243	0.312	0.334	0.305	0.357	0.251	0.404	0.389	**0.075**	0.341	0.322	0.393	0.366	0.353	0.355	0.418	0.354	0.363	**0.337**
$P11_{GFFSM\,5}$	0.258	0.265	0.266	0.320	0.323	0.300	0.311	0.210	0.354	0.337	**0.072**	0.332	0.309	0.337	0.348	0.318	0.331	0.373	0.324	0.341	**0.314**
$P12_{GFFSM\,3}$	0.325	0.335	0.315	0.344	0.335	0.317	0.335	0.311	0.298	0.331	0.338	**0.174**	0.317	0.351	0.319	0.292	0.329	0.342	0.316	0.340	**0.326**
$P12_{GFFSM\,5}$	0.348	0.362	0.317	0.348	0.336	0.298	0.348	0.281	0.365	0.345	0.351	**0.133**	0.307	0.402	0.313	0.287	0.319	0.316	0.347	0.325	**0.332**
$P13_{GFFSM\,3}$	0.333	0.380	0.342	0.308	0.369	0.350	0.344	0.328	0.360	0.340	0.337	0.333	**0.188**	0.393	0.320	0.318	0.311	0.325	0.369	0.353	**0.343**
$P13_{GFFSM\,5}$	0.330	0.338	0.286	0.322	0.341	0.335	0.320	0.284	0.359	0.356	0.325	0.336	**0.179**	0.384	0.319	0.358	0.322	0.310	0.318	0.344	**0.331**
$P14_{GFFSM\,3}$	0.383	0.416	0.451	0.406	0.358	0.415	0.388	0.385	0.318	0.342	0.382	0.383	0.388	**0.053**	0.401	0.399	0.394	0.384	0.372	0.399	**0.388**
$P14_{GFFSM\,5}$	0.414	0.412	0.444	0.410	0.385	0.411	0.401	0.426	0.296	0.347	0.419	0.387	0.398	**0.041**	0.402	0.386	0.380	0.392	0.366	0.398	**0.393**
$P15_{GFFSM\,3}$	0.368	0.401	0.372	0.329	0.360	0.328	0.353	0.375	0.303	0.378	0.351	0.312	0.344	0.398	**0.180**	0.352	0.333	0.313	0.327	0.360	**0.350**
$P15_{GFFSM\,5}$	0.363	0.375	0.369	0.306	0.358	0.319	0.335	0.319	0.327	0.422	0.306	0.309	0.327	0.411	**0.137**	0.331	0.332	0.319	0.301	0.334	**0.340**
$P16_{GFFSM\,3}$	0.327	0.360	0.363	0.293	0.322	0.342	0.338	0.318	0.359	0.369	0.336	0.330	0.329	0.359	0.353	**0.044**	0.349	0.339	0.233	0.338	**0.334**
$P16_{GFFSM\,5}$	0.292	0.363	0.342	0.319	0.330	0.339	0.340	0.301	0.354	0.348	0.315	0.329	0.334	0.356	0.354	**0.034**	0.347	0.342	0.235	0.344	**0.331**
$P17_{GFFSM\,3}$	0.299	0.358	0.261	0.340	0.423	0.358	0.361	0.267	0.380	0.386	0.260	0.339	0.312	0.349	0.330	0.346	**0.134**	0.294	0.310	0.393	**0.335**
$P17_{GFFSM\,5}$	0.319	0.327	0.254	0.353	0.416	0.355	0.362	0.190	0.368	0.390	0.260	0.359	0.306	0.433	0.353	0.342	**0.121**	0.320	0.323	0.401	**0.338**
$P18_{GFFSM\,3}$	0.323	0.336	0.337	0.366	0.372	0.377	0.374	0.344	0.385	0.409	0.383	0.350	0.351	0.399	0.349	0.326	0.368	**0.070**	0.341	0.336	**0.359**
$P18_{GFFSM\,5}$	0.394	0.343	0.362	0.352	0.416	0.399	0.361	0.366	0.393	0.412	0.389	0.378	0.352	0.388	0.363	0.326	0.405	**0.061**	0.284	0.391	**0.372**
$P19_{GFFSM\,3}$	0.303	0.350	0.387	0.291	0.350	0.343	0.353	0.310	0.372	0.367	0.365	0.371	0.328	0.391	0.385	0.312	0.364	0.287	**0.082**	0.352	**0.346**
$P19_{GFFSM\,5}$	0.298	0.322	0.375	0.300	0.343	0.332	0.346	0.316	0.383	0.342	0.359	0.377	0.352	0.403	0.384	0.310	0.373	0.303	**0.079**	0.343	**0.345**
$P20_{GFFSM\,3}$	0.369	0.408	0.385	0.386	0.314	0.404	0.419	0.366	0.315	0.276	0.385	0.333	0.367	0.341	0.331	0.373	0.329	0.343	0.388	**0.107**	**0.360**
$P20_{GFFSM\,5}$	0.383	0.428	0.395	0.396	0.329	0.380	0.429	0.379	0.318	0.294	0.386	0.313	0.361	0.314	0.310	0.376	0.365	0.365	0.392	**0.086**	**0.364**

TABLE V
AVERAGE OF THE MAE FOR EACH ONE OF THE PERSON'S FFSM MODELS WHEN THE INPUT DATA IS THE WHOLE SET OF GAITS OF EACH PERSON.

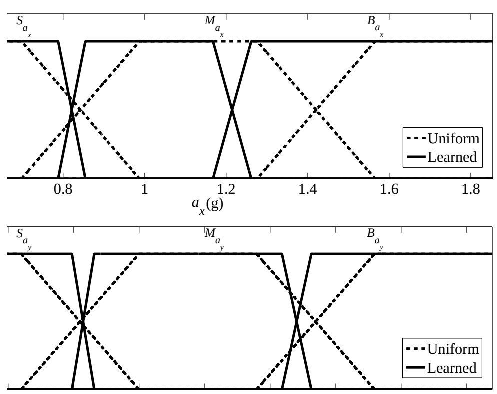

Fig. 8. MFs which comprise the learned DB compared with the original uniformly distributed MFs.

R_{41}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is S_{a_x} **OR** $M_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_1

Fig. 8 shows the graphical representation of the learned DB associated with this RB. The initial DB is also plotted, which consists of uniformly distributed MFs. In both cases, the use of SFPs produce comprehensible fuzzy partitions which allow us to get an interpretable fuzzy system.

As mentioned above, the main advantage of our model of human gait is its interpretability. The chance to properly understand the obtained model can report a large number of benefits for the designer. For example, in [27], we took advantage of this interpretability to create a model aimed to compare the characteristics of different human gaits, a different but related problem. With this aim, we elaborated upon two relevant measures of human gait based on the degree of activation and duration of successive states. We called these measures symmetry and homogeneity. Symmetry is a measure of the similarity among accelerations produced by steps given by the right leg (states q_1 and q_2) and accelerations produced by steps of the left leg (states q_3 and q_4). Homogeneity is a measure of how the same pattern of accelerations is repeated on time, i.e., it is a measure of similarity between each two steps and the following ones. Empirically, we have observed that these measures are characteristic of the style of walking of each person. In the paper mentioned above, we showed how to use these parameters to authenticate one person among 11 individuals (the interested reader is referred to a research work with the same goal but with a different approach in [80]). As in [1], [39], we think that the model presented in that paper could be used to detect and analyze pathological disorders in the gait in the

GFFSM 3				
ALG	$H_0: \mu_{\mathrm{GFFSM\ 3}} = \mu_{\mathrm{MOD}}$	$H_0: \mu_{\mathrm{GFFSM\ 3}} < \mu_{\mathrm{MOD}}$	$H_0: \mu_{\mathrm{GFFSM\ 3}} > \mu_{\mathrm{MOD}}$	CONCLUSION
GFFSM 5	0.021	0.010	0.990	[-]
ARX 1	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 2	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 3	$3.81 \cdot 10^{-6}$	1	$1.91 \cdot 10^{-6}$	[+]
ARX 4	$2.67 \cdot 10^{-5}$	1	$1.34 \cdot 10^{-5}$	[+]
ARX 5	0.004	0.998	0.002	[+]
ARX 6	0.231	0.892	0.115	[=]
ARX 7	0.018	0.009	0.991	[-]
ARX 8	0.005	0.002	0.998	[-]
ARX 9	0.004	0.002	0.998	[-]
ARX 10	0.010	0.005	0.995	[-]
NN 1	$1.20 \cdot 10^{-4}$	1	$6.01 \cdot 10^{-5}$	[+]
NN 2	$8.20 \cdot 10^{-5}$	1	$4.10 \cdot 10^{-5}$	[+]
NN 3	0.121	0.939	0.061	[=]
NN 4	0.232	0.116	0.884	[=]
NN 5	0.022	0.011	0.989	[-]
NN 6	0.003	0.002	0.999	[-]
NN 7	0.002	$8.01 \cdot 10^{-4}$	0.999	[-]
NN 8	$5.93 \cdot 10^{-4}$	$2.96 \cdot 10^{-4}$	1	[-]
NN 9	$1.68 \cdot 10^{-4}$	$8.39 \cdot 10^{-5}$	1	[-]
NN 10	$3.22 \cdot 10^{-4}$	$1.61 \cdot 10^{-4}$	1	[-]

GFFSM 5				
ALG	$H_0: \mu_{\mathrm{GFFSM\ 5}} = \mu_{\mathrm{MOD}}$	$H_0: \mu_{\mathrm{GFFSM\ 5}} < \mu_{\mathrm{MOD}}$	$H_0: \mu_{\mathrm{GFFSM\ 5}} > \mu_{\mathrm{MOD}}$	CONCLUSION
GFFSM 3	0.021	0.990	0.010	[+]
ARX 1	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 2	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 3	$8.84 \cdot 10^{-5}$	1	$4.42 \cdot 10^{-5}$	[+]
ARX 4	$8.84 \cdot 10^{-5}$	1	$4.42 \cdot 10^{-5}$	[+]
ARX 5	$2.10 \cdot 10^{-4}$	1	$1.05 \cdot 10^{-4}$	[+]
ARX 6	0.040	0.980	0.020	[+]
ARX 7	0.033	0.017	0.983	[-]
ARX 8	0.005	0.003	0.997	[-]
ARX 9	0.005	0.002	0.998	[-]
ARX 10	0.016	0.008	0.992	[-]
NN 1	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
NN 2	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
NN 3	$2.61 \cdot 10^{-4}$	1	$1.31 \cdot 10^{-4}$	[+]
NN 4	0.179	0.911	0.089	[=]
NN 5	0.601	0.300	0.700	[=]
NN 6	0.255	0.127	0.873	[=]
NN 7	0.151	0.075	0.925	[=]
NN 8	0.073	0.037	0.963	[=]
NN 9	0.042	0.021	0.979	[-]
NN 10	0.090	0.045	0.959	[=]

TABLE VI
P-VALUES OBTAINED FOR THE THREE DIFFERENT WILCOXON SIGNED-RANK TESTS AND THEIR NULL HYPOTHESES.

same way. It seems evident that symmetry and homogeneity will be affected by the presence of gait disorders, e.g., we can check this point measuring the symmetry of gait when a person is carrying a heavy bag in one hand and when she/he is free of that heavy unbalancing load.

Focusing on the current contribution, the expert analysis of the RB and the DB obtained of the human gait GFFSM model constitutes another approach to detect gait disorders. The antecedents of the learned rules in conjunction with the MFs of each variable can provide relevant information about the quality of the gait of a person, e.g., by showing abnormal membership values of the dorso-ventral acceleration (a_x) or inconsistent rules not compatible with the expert's knowledge. Moreover, regarding to the topic of gait modeling, it is worth noting that, the interpretability of the model allows us to calculate relevant temporal features of the gait, i.e., the duration of the states and their temporal sequence. With this information, we can easily count the number of steps and the duration of each of them and therefore the instantaneous walking speed. This is a significant issue in gait disorder analysis due to the fact that, e.g., patients tend to alter speed in order to accommodate loads applied on the knee.

3) Computational cost analysis: We have already compared the different algorithms in terms of the complexity and accuracy. Nevertheless, it is also interesting to evaluate their computational cost. The average times needed for building a GFFSM 3 and a GFFSM 5 model were 4076 and 6022 seconds respectively, while the ARX 1, ARX 7 and ARX 10 models took 0.25, 72, and 630 seconds respectively. The NN 1, NN 4 and NN 10 took 40, 118, and 314 seconds respectively. All the methods were run in a single computer, with 4 GB RAM and an Intel Core 2 Quad Q8400 with 2.66 GHz.

As expected, the GFFSMs spent a larger run time as they do not only involve parameter estimation but also structure identification. As said in Section V-B, the dependence of the next state on the previous state in our GFFSM makes it strictly necessary to test the FFSM over the whole data set for each chromosome evaluation, which is very computationally expensive. Nevertheless, the additional interpretability advantage makes this computational cost increase worthy. In addition, while NNs and ARX models are implemented in well established and optimized libraries, the GFFSMs were programmed in not optimized Matlab code (more refined implementations could be done in the future).

4) Importance of the use of expert knowledge analysis: Our GFFSMs are designed to take advantage from the available expert knowledge, exploiting the power of fuzzy systems which are capable of integrating this knowledge with machine learning techniques. The possibility of merging expert information with the information derived from data using GAs allows us to obtain a rough linguistic description of the gait, i.e., the final set of fuzzy rules obtained provide a linguistic description of the phenomenon. In the current GFFSMs, the designer has chosen a model of human gait with four basic fuzzy states easily recognized when we observe a walking person (see Fig. 1). Applying this constraint in the model, the designer makes the model easily understandable. Then, the GA explores possibilities into this restricted framework to define the final model structure and to estimate its parameters.

Even so, we have also decided to check whether the proposed GFFSM method is powerful enough to handle the overall learning problem, i.e., to extract the whole model (fuzzy rule set) structure from scratch along with the relevant labels and MFs in the case of 3 linguistic labels per input variable. We have assumed full ignorance of the RB and tried to build a FFSM using the classical genetic learning method proposed by Thrift [81] to derive the RB of the FFSM keeping the previous derivation of the DB based on a GA with real-coded chromosomes (from now on, this method will be called THRIFT-FFSM).

Thrift's RB derivation method is based on encoding all the cells of the complete decision table in the chromosomes. In our case, we have three antecedents: the current state (with four different possibilities corresponding to the number of possible states, the only information provided by the expert in the current experiment, together with the granularity of the fuzzy partitions), the input variable a_x (with three different linguistic labels corresponding to S_{a_x}, M_{a_x}, and B_{a_x}), and the input variable a_y (with another three different linguistic labels corresponding to S_{a_y}, M_{a_y}, and B_{a_y}). Therefore, the decision table will be a three-dimensional structure of size $4 \times 3 \times 3$ consisting of a total of 36 possible rules. Each cell of the decision table will represent the output of each fuzzy rule by means of an integer coding scheme represented by the set $\{0, 1, 2, 3, 4\}$, where 0 indicates the absence of the rule and $1, 2, 3$ or 4 indicates that the next state will be q_1, q_2, q_3, or q_4 respectively. Hence, we substitute the first part of the chromosome in Fig. 4 (RB part), composed of 48 binary genes (see Section V-A), by an integer-coded array of size 36, encoding the consequents for each possible rule. The resulting coding scheme has thus 44 genes (36 for the RB part plus 8 for the DB part).

We have used the same genetic operators for the GA as explained in Section V and the same parameter values showed in Section VI-A2, except the bitwise mutation (designed for binary-coded chromosomes) which was replaced by the original Thrift's mutation operator that randomly adds or subtracts 1 (with equal probability) to the current value of the allele within the set $\{0, 1, 2, 3, 4\}$.

Table VII shows the MAE obtained for each fold of the leave-one-out corresponding to the first and second person using the Thrift's RB derivation keeping the DB derivation (THRIFT-FFSM) compared to our expert information-based proposal (GFFSM 3). It also depicts the average value of the MAE and its standard deviation for the ten folds. As it can be seen, the lack of expert knowledge pays the cost of larger test errors. Moreover, the average training time for each THRIFT-FFSM model is 10855 seconds while the original GFFSM takes an average of 4076.

FOLD	PERSON 1		PERSON 2	
	THRIFT-FFSM	GFFSM 3	THRIFT-FFSM	GFFSM 3
1	0.115	0.089	0.054	0.049
2	0.085	0.066	0.091	0.072
3	0.115	0.135	0.072	0.065
4	0.102	0.108	0.107	0.047
5	0.205	0.133	0.092	0.072
6	0.172	0.078	0.107	0.037
7	0.083	0.101	0.169	0.048
8	0.111	0.149	0.051	0.066
9	0.152	0.086	0.079	0.124
10	0.083	0.081	0.121	0.046
MEAN	0.122	0.103	0.094	0.063
STD	0.041	0.028	0.035	0.025

TABLE VII

MAE OF THE LEAVE-ONE-OUT FOR THE DATASETS OF THE FIRST AND SECOND PERSON, WITH THE AVERAGE (MEAN) AND STANDARD DEVIATION (STD) FOR THE TWO EVALUATED MODELS.

We can also examine whether the RB extracted by the THRIFT-FFSM model resembles to the expert knowledge based one. As an example, the RB obtained for the seventh fold of the first person (with a MAE of 0.083) is showed as follows:

R^1_{11}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is S_{a_y}) **THEN** $S[t+1]$ is q_1

R^2_{11}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_1

R^3_{11}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_1

R^4_{11}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_1

R^5_{22}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_2

R^6_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_3

R^7_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is S_{a_y}) **THEN** $S[t+1]$ is q_3

R^8_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_3

R^9_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_3

R^{10}_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_3

R^{11}_{44}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_4

R^{12}_{12}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is S_{a_y}) **THEN** $S[t+1]$ is q_2

R^{13}_{14}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_4

R^{14}_{23}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is S_{a_y}) **THEN** $S[t+1]$ is q_3

R^{15}_{24}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_4

R^{16}_{34}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_4

R^{17}_{34}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_4

R^{18}_{41}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_1

R^{19}_{41}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_1

R^{20}_{42}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is S_{a_y}) **THEN** $S[t+1]$ is q_2

R^{21}_{42}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is M_{a_y}) **THEN** $S[t+1]$ is q_2

R^{22}_{42}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is B_{a_y}) **THEN** $S[t+1]$ is q_2

It consists of 22 rules (14 rules were automatically discarded) which, as can be seen in the state diagram showed

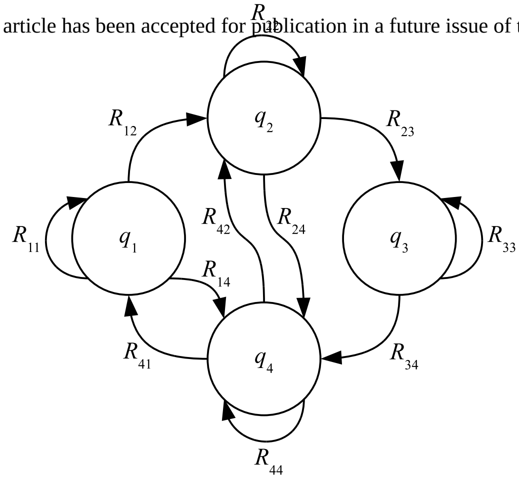

Fig. 9. State diagram obtained with the Thrift's RB derivation.

in Fig. 9, are not able to capture the expert knowledge represented by the state diagram of the human gait showed in Fig. 2. It presents some weird transitions as that represented in rule number 13 from state q_1 to state q_4 or the transitions between states q_2 and q_4 in rules 15, 20, 21, and 22. The effects of these transitions are reflected in Fig. 7, where the state activation vector corresponding with the THIRFT-FFSM model ($S[t]^{THRIFT}$) activates q_4 when going from state q_1 to the state q_2.

In summary, it is clear that the full consideration of the expert knowledge is the best way to design a FFSM for the human gait modeling problem by means of GAs.

VII. Concluding Remarks

We have presented a practical application where we described how to build FFSMs to model the human gait of a set of people by using GAs and expert knowledge. We defined the principal elements of the human gait cycle and developed a genetic learning procedure for FFSMs to model the gait cycle for each person. It has been showed how this GFS can obtain automatically the fuzzy rules and the fuzzy MFs associated to the linguistic terms of the FFSM while the states and transitions are defined by the expert, thus maintaining the knowledge that she/he has about the problem. To incorporate this expert knowledge, we have designed a user-friendly graphical interface for defining the fuzzy states of the human gait. The results obtained showed the goodness of our proposal.

We have increased the capabilities of FFSMs with a novel GA-based procedure for the automatic definition of its KB. Therefore, a great number of opportunities arise. We can set out new applications of system modeling by means of GFFSMs. The ability of our proposal to combine the available expert knowledge with the accuracy achieved by the learning process can be used to study several signals where the human interaction is demanded. Examples of application could range from biomedical engineering (e.g., electroencephalogram or electrocardiogram signals) to other time series analysis (e.g., econometrics or natural processes).

Our next research work in this direction consists of developing a model of the human gait where gait symmetry and homogeneity can be analyzed in detail. This work will include the automatic generation of linguistic reports about the human gait quality.

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