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A consensus model for group decision making problems with linguistic interval fuzzy preference relations

J.M. Tapia García [a], M.J. del Moral [b], M.A. Martínez [c], E. Herrera-Viedma [d,*]

[a] Dept. of Quantitative Methods in Economy and Enterprise, University of Granada, 18071 Granada, Spain
[b] Dept. of Statistics and Operational Research, University of Granada, 18071 Granada, Spain
[c] SECABA Lab., University of Granada, 18071 Granada, Spain
[d] Dept. of Computer Science, A.I University of Granada, 18071 Granada, Spain

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ABSTRACT

Sometimes, we find decision situations in which it is difficult to express some preferences by means of concrete preference degrees. In this paper, we present a consensus model for group decision making problems in which the experts use linguistic interval fuzzy preference relations to represent their preferences. This model is based on two consensus criteria, a consensus measure and a proximity measure, and on the concept of coincidence among preferences. We compute both consensus criteria in the three representation levels of a preference relation and design an automatic feedback mechanism to guide experts in the consensus reaching process.

1. Introduction

A group decision making (GDM) problem may be defined as a decision problem with several alternatives and a panel of decision makers or experts that try to achieve a common solution taking into account their opinions or preferences.

Usually, some problems present quantitative aspects which can be assessed by means of precise numerical values with the help of fuzzy Theory (Alonso, Herrera-Viedma, Chiclana, & Herrera, 2009b, 2010; Chiclana, Herrera, & Herrera-Viedma, 1998, 2001, 2002; Kacprzyk, Nurmi, & Fedrizzi, 1997). However, some problems present also qualitative aspects that are complex to assess by means of these numerical values. In such situations, a fuzzy linguistic approach (Alonso, Cabrerizo, Chiclana, Herrera, & Herrera-Viedma, 2009a; Cabrerizo, López-Gijón, Ruíz-Rodríguez, & Herrera-Viedma, 2010b, 2010c; Herrera & Herrera-Viedma, 2000a, 2000b; Herrera-Viedma, Martínez, Mata, & Chiclana, 2005; Herrera, Herrera-Viedma, & Martinez, 2008; Mata, Martínez, & Herrera-Viedma, 2009; Xu, 2005, 2007; Zadeh, 1975a, 1975b, 1975c) can be used to obtain a better solution. Our interest is focused on GDM problems in which the experts express their preferences by means of linguistic terms instead of precise numerical values.

Many of this problems use linguistic variables assessed in linguistic term set (labels) and each expert provides his/her opinions on the set of alternatives as a linguistic fuzzy preference relation. However, an expert could have a vague knowledge about the linguistic preference degree or label of the alternative i over j and could not estimate his/her preference with only one label. In such cases, it is useful to use linguistic interval fuzzy preference relations.

The resolution method for a GDM problem is composed by two different processes (Herrera, Herrera-Viedma, & Verdegay, 1995, 1996a; Pérez, Cabrerizo, & Herrera-Viedma, 2011):

1. *Consensus process:* This process refers to how to obtain the maximum degree of consensus or agreement among the experts on the solution alternatives.
2. *Selection process:* This process consists in how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts.

We can find some proposals of selection process for GDM problems under interval fuzzy preference relations (Genç, Boran, Akay, & Xu, 2010; Jiang, 2007; Xu, 2004; Xu & Chen, 2008). Up to date, however no investigation has been devoted to model the consensus in GDM problems under linguistic interval fuzzy preference relations. This paper is focused on the definition of a new consensus model for GDM problems with linguistic interval fuzzy preference relations. In Tapia-Garcia et al. (2012) we present a consensus model under fuzzy interval preference relations and, in this paper, we extended that proposal to work in a linguistic context.

Normally, in GDM problems, a group of experts initially can have disagreeing preferences and it is necessary to develop a consensus reaching process. Usually, a consensus reaching process

* Corresponding author.
E-mail address: viedma@decsai.ugr.es (E. Herrera-Viedma).

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can be viewed as a dynamic process where a moderator via exchange of information and rational arguments, tries the experts to update their opinions. In each step, the degree of actual consensus and the distance from an ideal consensus is measured. This is repeated until the distance to the ideal consensus is considered sufficiently small. Traditionally, the ideal consensus meant as a full and unanimous agreement of all experts' preferences. This type of consensus is an utopian consensus and it is very difficult to achieve it. This has led to the use and definition of a new concept called "soft" consensus degree (Kacprzyk, 1987; Kacprzyk & Fedrizzi, 1986; Kacprzyk & Fedrizzi, 1988) which assesses the consensus degree in a more flexible way. The soft consensus measures that allow to measure the closeness among experts' opinions are based on the concept of coincidence (Cabrerizo, Moreno, Pérez, & Herrera-Viedma, 2010a; Herrera, Herrera-Viedma, & Verdegay, 1997b). We can identify three different approaches to apply coincidence criteria to compute soft consensus measures (Cabrerizo et al., 2010a):

1. *Consensus models based on strict coincidence among preferences.* In this case, similarity criteria among preferences provided by the experts are used to compute the coincidence concept. Only two possible results are assumed: the total coincidence (value 1) or null coincidence (value 0) (Kacprzyk, 1987; Herrera, Herrera-Viedma, & Verdegay, 1996b, 1997a).
2. *Consensus models based on soft coincidence among preferences.* As stated above. However, in this case, a major number of possible coincidence degrees are considered. It is assumed that the coincidence concepts is a gradual concept, which could be assessed with different degrees defined in the unit interval $[0,1]$. These are the most popular consensus models (Bordogna, Fedrizzi, & Pasi, 1997; Cabrerizo, Alonso, & Herrera-Viedma, 2009, 2010b; Fedrizzi, Kacprzyk, & Nurmi, 1993; Herrera et al., 1997b; Herrera-Viedma et al., 2005, Herrera-Viedma, Alonso, Chiclana, & Herrera, 2007a; Kacprzyk, 1987; Kacprzyk & Fedrizzi, 1986, 1988; Pérez, Cabrerizo, & Herrera-Viedma, 2010, 2011).
3. *Consensus models based on coincidence among solutions.* In this case, similarity criteria among the solutions obtained from the experts' preferences are used to compute the coincidence concept and different degrees assessed in $[0,1]$ are assumed (Ben-Arieh & Chen, 2006; Herrera-Viedma, Herrera, & Chiclana, 2002).

The aim of this paper is to present a new consensus model based on soft coincidence among preferences for GDM problems under linguistic interval fuzzy preference relations. As in Herrera et al. (1996b, 1997a, 1997b) this new consensus model is based on two consensus criteria to guide the consensus reaching process:

(1) *A consensus measure.* This measure evaluates the agreement of all the experts. It is used to guide the consensus process until the final solution is achieved.
(2) *A proximity measure.* This measure evaluates the agreement between the experts' individual opinions and the group opinion. It is used to guide the group discussion in the consensus process.

We compute both measures on the three levels of representation of linguistic interval fuzzy preference relations: level of pair, level of alternative and level of relation. Then, we design an automatic feedback mechanism to guide experts in the consensus reaching process and substitute the moderator's activity.

This paper is set out as follows. The GDM problem based on linguistic interval fuzzy preference relations is described in Section 2. Section 3 presents the new consensus model. A practical example is given in Section 4. Finally, in Section 5 we draw our conclusions.

2. The GDM problem with linguistic interval fuzzy preference relations

This section briefly describes the GDM problems with linguistic interval fuzzy preference relations and the resolution process used to obtain the solution set of alternatives.

2.1. A fuzzy linguistic approach

As aforementioned, an expert could have some difficulties to estimate his/her preference degrees with exact numerical values. So, another possibility is to use linguistic labels (Carrasco, Villar, Hornos, & Herrera-Viedma, 2011; Herrera-Viedma, Herrera, Martinez, Herrera, & Lopez-Herrera, 2004; Herrera-Viedma & Lopez-Herrera, 2007, 2010; Herrera-Viedma & Peis, 2003; Morales-del-Castillo, Peis, Ruiz-Rodriguez, & Herrera-Viedma, 2010; Moreno, Morales del Castillo, Porcel, & Herrera-Viedma, 2010; Porcel & Herrera-Viedma, 2010).

According to Herrera and Herrera-Viedma (1997, 2000b), as the linguistic assessments are merely approximate ones given by the experts, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of these linguistic assessments. This representation is achieved by the 4-tuple $(a_i, b_i, \alpha_i, \beta_i)$. The first two parameters indicate the interval in which the membership value is 1.0; the third and fourth parameters indicate the left and right widths of the distribution.

We shall consider a finite and totally ordered label set $S = \{s_0, \ldots, s_T\}$ in the usual sense and with odd cardinality, where each label s_i represents a possible value for a linguistic real variable.

Example 1. We could use the following nine linguistic label set with their respective associated semantics to express the preferences:

s_8 = C	Certain	(1.00, 1.00, 0.00, 0.00)
s_7 = EL	Extremely_likely	(0.98, 0.99, 0.05, 0.01)
s_6 = ML	Most_likely	(0.78, 0.92, 0.06, 0.05)
s_5= MC	Meaningful_chance	(0.63, 0.80, 0.05, 0.06)
s_4 = IM	It_may	(0.41, 0.58, 0.09, 0.07)
s_3 = SC	Small_chance	(0.22, 0.36, 0.05, 0.06)
s_2 = VLC	Very_low_chance	(0.10, 0.18, 0.06, 0.05)
s_1 = EU	Extremely unlikely	(0.01, 0.02, 0.01, 0.05)
s_0 = I	Impossible	(0.00, 0.00, 0.00, 0.00)

2.2. The GDM problem

Let $X = \{x_1, x_2, \ldots, x_n\} (n \geqslant 2)$ be a finite set of alternatives to be evaluated by a finite set of experts, $E = \{e_1, e_2, \ldots, e_m\} (m \geqslant 2)$. The GDM process consists to find the best alternative according to the experts' preferences $\{P^1, P^2, \ldots, P^m\}$.

In a usual GDM problem we assume that the experts provide their preferences on X by means of the fuzzy preference relations, $P^k \subset X \times X$, with membership function

$$\mu_{pk} : X \times X \to S,$$

where $\mu_{pk}(x_i, x_j) = p_{ij}^k$ denotes the preference degree of the alternative x_i over x_j. In a linguistic context we could assume the following:

- $p_{ij}^k = s_{T/2}$ indicates indifference between x_i and x_j,
- $p_{ij}^k = s_T$ indicates that x_i is unanimously preferred to x_j, and
- $p_{ij}^k > s_{T/2}$ indicates that x_i is preferred to x_j.

Furthermore, it is usual to assume that P^k is reciprocal (Chiclana et al., 1998, Chiclana, Herrera-Viedma, Alonso, & Herrera, 2009b; Orlovsky, 1978; Tanino, 1988), i.e., $p_{ij}^k = s_i$ then $p_{ji}^k = s_{T-i}$ and $p_{ii}^k = -$ (undefined).

In this paper we assume that the experts' preferences on X are described by means of the linguistic interval fuzzy preference relations, $P^k \subset X \times X$, with membership function

$$\mu'_{pk} : X \times X \to S \times S,$$

where $\mu'_{pk}(x_i, x_j) = \left[p_{ij}^{k-}, p_{ij}^{k+}\right]$ denotes the linguistic interval fuzzy preference degree of the alternative x_i over x_j with $s_0 \leqslant p_{ij}^{k-} \leqslant p_{ij}^{k+} \leqslant s_T$. Furthermore, according to the ordering induced by set S, we define a natural function:

$$s : S \to \mathbb{N}$$

as

$$s\left(p_{ij}^z\right) = a \quad \text{if} \quad p_{ij}^z = s_a.$$

- if $s\left(p_{ij}^{k-}\right) + s\left(p_{ij}^{k+}\right) = T$ indicates indifference between x_i and x_j,
- if $s\left(p_{ij}^{k-}\right) + s\left(p_{ij}^{k+}\right) > T$ indicates that x_i is preferred to x_j,
- if $s\left(p_{ij}^{k-}\right) + s\left(p_{ij}^{k+}\right) < T$ indicates that x_j is preferred to x_i.

In this case, we assume that if $p_{ij}^{k-} = s_i$ then $p_{ji}^{k+} = s_{T-i}$, and $p_{ii}^{k+} = p_{ii}^{k-} = -$ (undefined).

2.3. The LOWA operator

We use the linguistic ordered weighted averaging (LOWA) operator (Herrera, Herrera-Viedma, & Verdegay, 1996c) based on the OWA operator defined by Yager (1988).

Let $\{a_1, \ldots, a_m\}$ be a set of labels to be aggregated, then the LOWA operator, ϕ, is defined as

$$\begin{aligned}\phi(a_1, \ldots, a_m) &= W \cdot B^T = C^m\{w_k, b_k, k = 1, \ldots, m\} \\ &= w_1 \odot b_1 \oplus (1 - w_1) \odot C^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\},\end{aligned}$$

where $W = [w_1, \ldots, w_m]$, is a weighting vector, such that, $w_i \in [0,1]$ and $\sum_i w_i = 1$; $\beta_h = w_h / \sum_2^m w_k, h = 2, \ldots, m$, and B is the associated ordered label vector. Each element $b_i \in B$ is the ith largest label in the collection $a_1, \ldots, a_m$. C^m is the convex combination operator of m labels and if $m = 2$, then it is defined as

$$C^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k, s_j, s_i\ S(j \geqslant i)$$

such that $k = \min\{T, i + \text{round}(w_1 \cdot (j - i))\}$, where round is the usual round operation, and $b_1 = s_j, b_2 = s_i$. If $w_j = 1$ and $w_i = 0$ with $i \neq j$, then the convex combination is defined as:

$$C^m\{w_i, b_i, k = 1, \ldots, m\} = b_j.$$

Yager suggested a way to compute the weights of the OWA aggregation operator using linguistic quantifiers, which, in the case of a non-decreasing proportional quantifier Q, is given by this expression (Yager, 1988, 1993):

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n,$$

where the membership function of Q can be represented as

$$Q(r) = \begin{cases} 0, & \text{if } r < a, \\ \frac{r-a}{b-a}, & \text{if } a \leqslant r \leqslant b, \\ 1, & \text{if } r > b, \end{cases}$$

with $a, b, r \in [0,1]$. When a fuzzy linguistic quantifier Q is used to compute the weights of the LOWA operator ϕ, it is symbolized by ϕ_Q.

2.4. Resolution process of the GDM problem

In this context, the resolution process of the GDM problem consists in obtaining a set of solution alternatives from the preferences given by the experts. Usually, the resolution process of the GDM problem consists in obtaining a set of solution alternatives from the preferences given by the experts. As aforementioned, usually this resolution process is composed of two phases: consensus phase and selection phase. If we assume that the experts express their individual preferences by means of the interval fuzzy preference relations, then the resolution process would be as it is shown in Fig. 1.

The selection process to obtain is composed by two procedures in order to obtain the solution set of alternatives (Herrera & Herrera-Viedma, 1997; Herrera et al., 1995, Herrera, Herrera-Viedma, & Verdegay, 1996a, 1996c): (i) aggregation and (ii) exploitation.

2.4.1. Aggregation phase

A collective linguistic interval fuzzy preference relation is obtained by means of the aggregation of all individual linguistic interval fuzzy preference relations in this phase. This collective relation, called U, indicates the global preference between every ordered pair of alternatives according to the majority experts' opinions. For example, a possibility to obtain U in the case of the linguistic interval fuzzy preference relations it would be as follows:

$$\begin{aligned}&U = (U_{ij}) \quad \text{for } i, j = 1, \ldots, n \text{ with} \\ &U_{ij} = U\left[p_{ij}^-, p_{ij}^+\right] = \left[\phi_-\left(p_{ij}^{k-}\right), \phi_+\left(p_{ij}^{k+}\right)\right] = \left[\min_k\left(p_{ij}^{k-}\right), \max_k\left(p_{ij}^{k+}\right)\right] \quad \text{for } k = 1, \ldots, n \\ &\text{with } w_- = \{0, \ldots, 0, 1\} \quad \text{in } \phi_- \quad \text{and } w_+ = \{1, 0, \ldots, 0\} \quad \text{in } \phi_+\end{aligned}$$

Example 2. Suppose two experts that using label set from Example 1 provide us the following preferences:

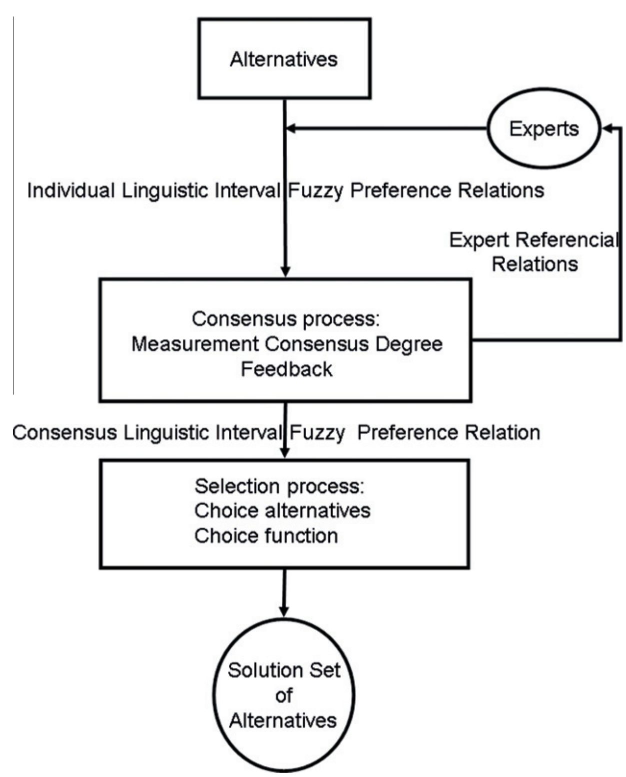

Fig. 1. Diagram of the GDM resolution process.

$$e^1 = \begin{pmatrix} - & [VLC, SC] & [IM, MC] \\ [MC, ML] & - & [ML, C] \\ [SC, IM] & [I, VLC] & - \end{pmatrix}$$
$$e^2 = \begin{pmatrix} - & [SC, IM] & [SC, MC] \\ [IM, ML] & - & [ML, EL] \\ [SC, MC] & [EU, VLC] & - \end{pmatrix}$$

Therefore, using the previous aggregation tool we would obtain the following collective preference relation U:

$$U = \begin{pmatrix} - & [VLC, IM] & [SC, MC] \\ [IM, ML] & - & [ML, C] \\ [SC, MC] & [I, VLC] & - \end{pmatrix}$$

2.4.2. Explotation phase

Global and collective information about the alternatives is transformed into a global ranking of them, and then we choose the set of solution alternatives. To do so, it is usual choice functions of alternatives which applied on the collective preference relation allow us to obtain the ranking of alternatives (Herrera & Herrera-Viedma, 2000a). For example, we could define choice functions using the dominance concept (Herrera & Herrera-Viedma, 2000a).

So, for each alternative x_i we could calculate its dominance degree px_i from the collective linguistic interval fuzzy preference relation as

$$px_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(s\left(p_{ij}^{-}\right) + s\left(p_{ij}^{+}\right) \right)$$

In such a way, we obtain a classification of the alternatives:

$if\ s(px_i) > s(px_j)$ then x_i is preferable to x_j.

Example 3. From the collective linguistic interval fuzzy preference relations obtained in Example 2 we could characterize each alternative with the following dominance degrees:

$$px_1 = 2 + 4 + 3 + 5 = 14$$
$$px_2 = 4 + 6 + 6 + 8 = 24$$
$$px_3 = 3 + 5 + 0 + 2 = 10$$

So these alternatives can be classified from highest to lowest preference as:

$x_2 > x_1 > x_3$

and therefore, the alternative x_2 is the recommended solution.

3. Consensus model under linguistic interval fuzzy preference relations

In Genç et al. (2010), Xu and Chen (2008) we can find different selection processes for GDM problems under interval fuzzy preference relations. As aforementioned, there not exist consensus models to deal with GDM problems under linguistic interval fuzzy preference relations. In the following section, we present a consensus process for GDM problems with linguistic interval fuzzy preference relations.

In this section we present a consensus model defined for GDM problems assuming that the experts express their preferences by means of the linguistic interval fuzzy preference relations. This model presents the following main characteristics:

1. It is based on two soft consensus criteria: a consensus measure and a proximity measure.
2. Both consensus criteria are defined using the coincidence among linguistic interval fuzzy preference relations provided by the experts (Cabrerizo et al., 2010a).
3. It incorporates a feedback mechanism that generates recommendations to the experts on how to change their linguistic interval fuzzy preference relations in the consensus reaching process.

Initially, we consider that in any non-trivial GDM problem the experts disagree in their opinions so that consensus has to be viewed as an iterate process, which means that the agreement is obtained only after many rounds of consultation. Then, in each round we calculate two consensus criteria, consensus measure and proximity measure (Herrera et al., 1996b, 1997a, 1997b). The former evaluates the level of agreement among all the experts and it guides the consensus process, and the latter evaluates the distance between the experts' individual preferences and the group or collective ones and it supports the discussion phase of the consensus process. To do so, we compute the coincidence among linguistic interval fuzzy preference relations.

As quantitative context, in this situation the main problem is how to find a way of making individual positions converge. To do this, a consensus level required for each decision situation is fixed in advance (A). When the consensus measure reaches this level then the decision making session is finished and the solution is obtained applying a selection process. If that is not the case, the experts' opinions must be modified. This is done in a group discussion session in which a feedback mechanism is used to supports the experts in changing their opinions. This feedback mechanism is defined using the proximity measures (Herrera-Viedma et al., 2002, 2005, 2007a; Mata et al., 2009). In order to avoid that the collective solution does not converge after several discussion rounds is possible fix a maximum number of rounds. The scheme of this consensus model for GDM is presented in Fig. 2. In the following subsections we present the components of this consensus model in detail, i.e., the consensus criteria and the feedback mechanism.

3.1. Consensus and proximity measures

We calculate both consensus indicators in the following steps:

1. Firstly, we calculate the consensus relations of each expert e^k, called $C\ k$, with respect to
$$C^k = \left(C_{ij}^k\right) \quad \text{with}$$
$$\left(\left| s\left(p_{ij}^{k-}\right) - s\left(p_{ij}^{-}\right) \right| + \left| s\left(p_{ij}^{k+}\right) - s\left(p_{ij}^{+}\right) \right| \right) / T \quad \text{for } i,j = 1, \ldots, n$$
In this consensus relation each value C_{ij}^k represents the agreement degree of the expert e^k with the group of experts on the preference p_{ij}.
2. Then, we define the *linguistic consensus degree on a preference* p_{ij} as
$$LCD_{ij} = 1 - \sum_{k=1}^{m} C_{ij}^k / m \quad \text{or} \quad LCD_{ij} = \left(1 - \sum_{k=1}^{m} C_{ij}^k / m\right) * 100\%.$$
We have a total consensus in the preference p_{ij} if $LCDij = 1$ or 100%.
3. We define the *linguistic consensus degree in the alternative* x_i as
$$LCDi = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{m} C_{ij}^k / ((n-1)m)$$
or
$$LCDi = \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{m} C_{ij}^k / ((n-1)m)\right) * 100\%.$$
We have a total consensus in the alternative x_i if $LCD_i = 1$ or 100%.

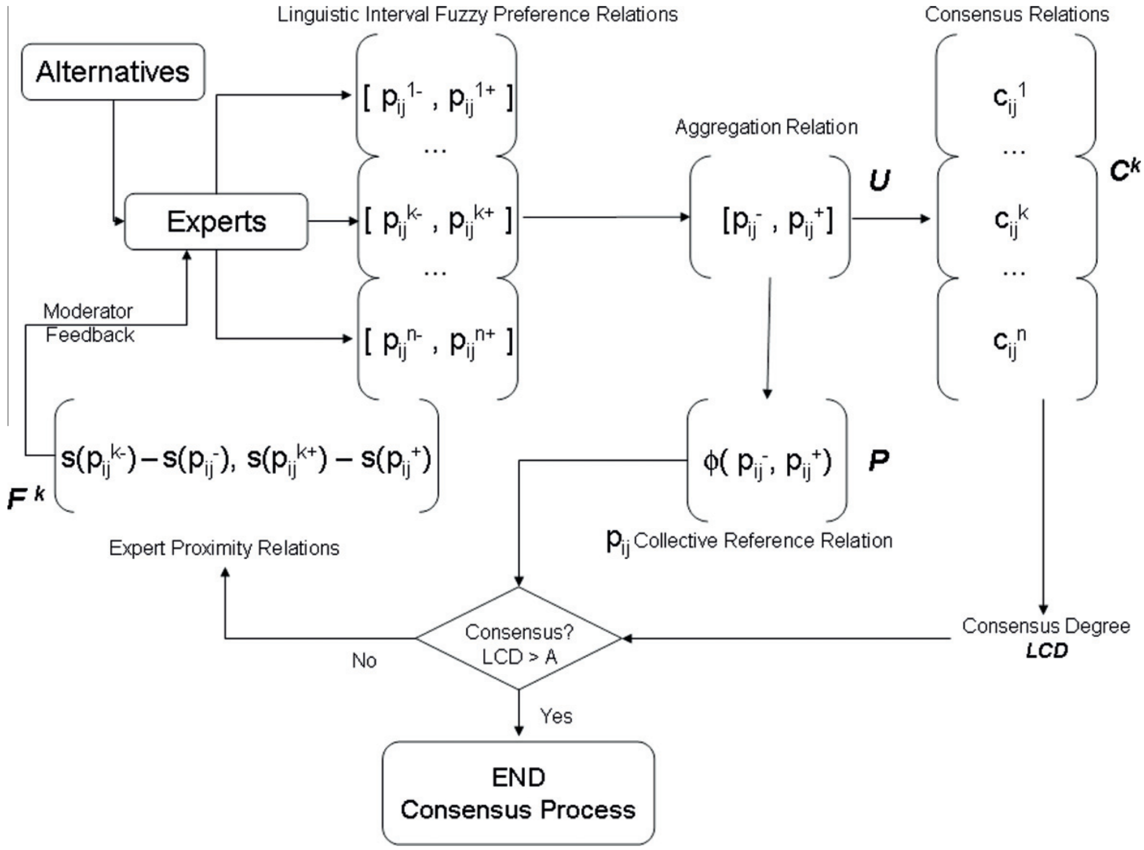

Fig. 2. Consensus Model for GDM with linguistic interval fuzzy preference relations.

Proposition 1.

$$\sum_{\substack{j=1\\ j\neq i}}^{n} LCD_{ij}/(n-1) = LCD_i$$

Proof. By definition $LCD_{ij} = 1 - \sum_{k=1}^{m} C_{ij}^{k}/m$ and then

$$\sum_{\substack{j=1\\ j\neq i}}^{n} LCD_{ij} = \sum_{\substack{j=1\\ j\neq i}}^{n}\left(1 - \sum_{k=1}^{m} C_{ij}^{k}/m\right) = \sum_{\substack{j=1\\ j\neq i}}^{n} 1 - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/m$$
$$= (n-1) - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/m$$

and therefore

$$\sum_{\substack{j=1\\ j\neq i}}^{n} LCD_{ij}/(n-1) = (n-1)/(n-1) - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/m/(n-1)$$
$$= 1 - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)m) = LCD_i.$$

4. We define the *linguistic global consensus degree, LCD,* as

$$LCD = 1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n^2-n)m)$$

or

$$LCD = \left(1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} C_{ij}^{k}\Big/((n^2-n)m)\right) * 100\%$$

In this case, $0 \leqslant LCD \leqslant 1$ or $0\% \leqslant LCD \leqslant 100\%$. We have a total consensus in the process if $LCD = 1$or $LCD = 100\%$. □

Proposition 2.

$$\sum_{i=1}^{n} LCD_i/n = LCD$$

Proof. Similarly, $LCD_i = 1 - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)m)$ by definition, and then

$$\sum_{i=1}^{n} LCD_i = \sum_{i=1}^{n}\left(1 - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)m)\right)$$
$$= \sum_{i=1}^{n} 1 - \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)m)$$
$$= n - \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)m)$$

and therefore

$$\sum_{i=1}^{n} LCD_i/n = n/n - \left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)m)\right)\Big/ n$$
$$= 1 - \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n-1)mn)$$
$$= 1 - \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n^2-n)m) = CD. \qquad \square$$

Example 4. From the collective preference relation obtained in Example 2, we obtain the following two consensus relations

$$C^1 = \begin{pmatrix} - & 0.125 & 0.125 \\ 0.125 & - & 0.000 \\ 0.125 & 0.000 & - \end{pmatrix} \text{ and } C^2 = \begin{pmatrix} - & 0.125 & 0.000 \\ 0.125 & - & 0.125 \\ 0.000 & 0.125 & - \end{pmatrix}$$

and therefore, the global consensus degree is $CD = 0.9167$ or CD = 91.67%, and for example, the consensus degree in the alternative x_1 is $CD_1 = 0.9063$ or $CD_1 = 90.63\%$, and the consensus degree on the preference p_{23} is CD_{23}= 0.9375 or CD_{23}= 93.75%.

5. Now, we continue the process in order to calculate the proximity measures. Firstly, we calculate the expert proximity relations, called F_k, with respect to the collective preference relation U as

$$F^k = \left(F^k_{ij}\right) \quad \text{with}$$

$$F^k_{ij} = \left(s\left(p^{k-}_{ij}\right) - s(p_{ij}), s\left(p^{k+}_{ij}\right) - s(p_{ij})\right) = \left(f^{k-}_{ij}, f^{k+}_{ij}\right)$$

for $i,j = 1,\ldots,n$ and $p_{ij} = \phi_Q\left(p^-_{ij}, p^+_{ij}\right)$ and $s(p_{ij}) = n$ if $p_{ij} = s_n$.

6. Then, we define the *proximity measure of the expert* e^k *on a preference* p_{ij} as

$$PM^k_{ij} = \left(|f^{k-}_{ij}| + |f^{k+}_{ij}|\right)/2T$$

7. Then, we define the *proximity measure of the expert* e^k *in an alternative* x_i as

$$PM^k_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} PM^k_{ij}/(n-1)$$

8. Then, we define the *global proximity measure of the expert* e^k as

$$PM^k = \sum_{i=1}^{n} PM^k_i/n$$

Example 5. Using the data given in Example 2 we obtain the following expert proximity relations for experts e^1 and e^2, respectively:

$$F^1 = \begin{pmatrix} - & (-1,0) & (0,+1) \\ (0,+1) & - & (-1,+1) \\ (-1,0) & (-1,+1) & - \end{pmatrix}$$

$$F^2 = \begin{pmatrix} - & (0,+1) & (-1,+1) \\ (-1,0) & - & (-1,0) \\ (-1,+1) & (0,+1) & - \end{pmatrix}$$

We obtain proximity measures for experts for each alternative,

$PM^1_1 = 0.063 \; PM^2_1 = 0.094$
$PM^1_2 = 0.094 \; PM^2_2 = 0.063$
$PM^1_3 = 0.094 \; PM^2_3 = 0.094$

and for the set of preferences

$PM^1 = 0.083 \quad PM^2 = 0.083$

3.2. Moderator/feedback process

We can apply a feedback mechanism to guide the change of the expert's opinions with use proximity matrices F^k (Herrera-Viedma et al., 2002, 2005, 2007a; Mata et al., 2009). This mechanism is able to help moderator in his/her tasks or even to substitute the moderator's actions in the consensus reaching process. In such a way, the feedback process helps experts to change their preferences in order to achieve an appropriate agreement degree. The main problem for the feedback mechanism is how to find a way of making individual positions converge and, therefore, how to support the experts in obtaining and agreeing with a particular solution (Herrera-Viedma et al., 2002).

Usually, the feedback process is carried out in two phases: identification phase, and recommendation phase.

1. *Identification phase*: It is necessary compare global consensus degree *LCD* and a consensus threshold *A*, previously fixed. Then, if *LCD* > *A* or *LCD* = *A* the consensus process will stop; on the other hand, if *LCD* < *A* a new consensus round it must be applied. If the agreement among all experts is low, then there exist a lot of experts' preferences in disagreement. In such a case, in order to bring the preferences closer to each other and so to improve the consensus situation, the number of changes in the experts' preferences should be high. However, if the agreement is high, the majority of preferences is close and only a low number of experts' preferences are is disagreement; it seems reasonable to change only these particular preferences. The procedure suggests modifying the preference values on all the pairs of alternatives where the agreement is not high enough. We finds out the set of preferences to be changed as follows:
   (a) Firstly, the pairs of alternatives with a consensus degree smaller than a threshold value *A* defined at level of pairs of alternatives, $CD_{ij} < A$, are identified.
   (b) Secondly, we identify the experts that will be required to modify the identified pairs of alternatives. To do that, we use the expert proximity measures PM^k and PM^k_i, and also we fix a value threshold *B*. The experts that are required to be modified are preferences whose $PM^k > B$.
2. *Recommendation phase*. In this phase we recommend expert changes of their preferences according to some rules to change the opinions. Once the preferences to be changed and experts to send recommendations have been identified, we develop a recommendation phase. In this phase we apply a recommendation rules that inform experts on the right direction of the changes in order to improve the agreement. We must find out the direction of change to be applied to the preference assessment p^{k+}_{ij} or p^{k-}_{ij} for each expert *k* on a preference. To do this, we define the following rules:
   (a) If $s\left(p^{k-}_{ij}\right) - s(p_{ij}) = f^{k-}_{ij} > 0$ then expert e_k should decrease the assessment associated to the pair of alternatives (x_i,x_j).
   (b) If $s\left(p^{k+}_{ij}\right) - s(p_{ij}) = f^{k+}_{ij} < 0$ then expert e_k should increase the assessment associated to the pair of alternatives (x_i,x_j).
   (c) If $f^{k-}_{ij} < 0 < f^{k+}_{ij}$ then expert e_k should increase p^{k-}_{ij} and decrease p^{k+}_{ij} in the assessments associated to the pair of alternatives (x_i,x_j).

4. Example

Suppose that we three experts looking for a solution among a set of four possible alternatives. The experts' preferences using linguistic interval fuzzy preference relations are:

$$e^1 = \begin{pmatrix} - & [I,EU] & [MC,MC] & [VLC,SC] \\ [EL,C] & - & [MC,C] & [IM,MC] \\ [SC,SC] & [I,SC] & - & [VLC,SC] \\ [MC,ML] & [SC,IM] & [MC,ML] & - \end{pmatrix}$$

$$e^2 = \begin{pmatrix} - & [SC,SC] & [SC,MC] & [EU,SC] \\ [MC,MC] & - & [MC,ML] & [MC,EL] \\ [SC,MC] & [VLC,SC] & - & [I,VLC] \\ [MC,EL] & [EU,SC] & [ML,C] & - \end{pmatrix}$$

$$e^3 = \begin{pmatrix} - & [SC,MC] & [VLC,IM] & [IM,IM] \\ [SC,MC] & - & [SC,IM] & [ML,C] \\ [IM,ML] & [IM,MC] & - & [MC,ML] \\ [IM,IM] & [I,VLC] & [VLC,SC] & - \end{pmatrix}$$

Then, we obtain the following collective linguistic interval fuzzy preference relation:

$$U = \begin{pmatrix} - & [I,MC] & [VLC,MC] & [EU,IM] \\ [SC,C] & - & [SC,C] & [IM,C] \\ [SC,ML] & [I,MC] & - & [I,ML] \\ [IM,EL] & [I,IM] & [VLC,C] & - \end{pmatrix}$$

Now, we calculate the consensus relations of each expert

$$C^1 = \begin{pmatrix} - & 0.5 & 0.375 & 0.25 \\ 0.5 & - & 0.25 & 0.375 \\ 0.375 & 0.25 & - & 0.625 \\ 0.25 & 0.375 & 0.625 & - \end{pmatrix}$$

$$C^2 = \begin{pmatrix} - & 0.625 & 0.125 & 0.125 \\ 0.625 & - & 0.5 & 0.25 \\ 0.125 & 0.5 & - & 0.5 \\ 0.125 & 0.25 & 0.5 & - \end{pmatrix}$$

$$C^3 = \begin{pmatrix} - & 0.375 & 0.125 & 0.375 \\ 0.375 & - & 0.5 & 0.25 \\ 0.125 & 0.5 & - & 0.625 \\ 0.375 & 0.25 & 0.625 & - \end{pmatrix}$$

Therefore, consensus degrees on the preferences $[p_{ij}]$ are

$$\begin{pmatrix} - & 0.5 & 0.792 & 0.75 \\ 0.5 & - & 0.583 & 0.708 \\ 0.792 & 0.583 & - & 0.417 \\ 0.75 & 0.708 & 0.417 & - \end{pmatrix}$$

and the global consensus degree is $CD = 0.625$ or $CD = 62.5\%$. If we fix a consensus threshold $A = 0.7$ then it seems unacceptable to finish the decision making process.

Then, we calculate F^k for each expert

$$F^1 = \begin{pmatrix} - & (-3,-2) & (+1,+1) & (-1,0) \\ (+1,+2) & - & (-1,+2) & (-2,-1) \\ (-2,-2) & (-3,0) & - & (-1,0) \\ (-1,0) & (+1,+2) & (0,+1) & - \end{pmatrix}$$

$$F^2 = \begin{pmatrix} - & (0,0) & (-1,+1) & (-2,0) \\ (-1,-1) & - & (-1,0) & (-1,+1) \\ (-2,0) & (-1,0) & - & (-3,-1) \\ (-1,+1) & (-1,+1) & (-1,+3) & - \end{pmatrix}$$

$$F^3 = \begin{pmatrix} - & (0,+2) & (-2,0) & (+1,+1) \\ (-3,-1) & - & (-3,-2) & (0,+2) \\ (-1,+1) & (+1,+2) & - & (+2,+3) \\ (-2,-2) & (-2,0) & (-3,-2) & - \end{pmatrix}$$

The proximity measures for experts are:

$$PM_1^1 = 0.167 \quad PM_1^2 = 0.083 \quad PM_1^3 = 0.125$$
$$PM_2^1 = 0.188 \quad PM_2^2 = 0.104 \quad PM_2^3 = 0.229$$
$$PM_3^1 = 0.167 \quad PM_3^2 = 0.146 \quad PM_3^3 = 0.209$$
$$PM_4^1 = 0.105 \quad PM_4^2 = 0.167 \quad PM_4^3 = 0.229$$

and

$$PM^1 = 0.157 \quad PM^2 = 0.125 \quad PM^3 = 0.198$$

Then applying the feedback mechanism we have:

- If we observe the preferences p_{12}, p_{23}, p_{34} and symmetrical ones do not present a reasonable consensus degree, i.e., they do not satisfy the threshold value 0.7.
- If we fix a threshold value 0.15 for identifying those experts that should change their assessments, expert 3 and expert 1 should change in alternatives p_{12}, p_{23}, p_{34} and symmetrical ones at least.
- For example, some recommendations could be the following ones: expert 3 in the preference p_{34} should decrease his/her preferences.

Suppose that after some rounds the experts' preferences are:

$$e^1 = \begin{pmatrix} - & [VLC,VLC] & [IM,IM] & [SC,SC] \\ [ML,ML] & - & [IM,MC] & [EL,C] \\ [IM,IM] & [SC,IM] & - & [VLC,SC] \\ [MC,MC] & [I,EU] & [MC,ML] & - \end{pmatrix}$$

$$e^2 = \begin{pmatrix} - & [VLC,VLC] & [IM,IM] & [SC,SC] \\ [EL,EL] & - & [MC,MC] & [ML,EL] \\ [IM,IM] & [SC,SC] & - & [VLC,VLC] \\ [MC,MC] & [VLC,VLC] & [ML,ML] & - \end{pmatrix}$$

$$e^3 = \begin{pmatrix} - & [SC,SC] & [IM,MC] & [SC,SC] \\ [ML,ML] & - & [MC,MC] & [MC,ML] \\ [SC,IM] & [SC,SC] & - & [SC,SC] \\ [MC,MC] & [VLC,SC] & [ML,ML] & - \end{pmatrix}$$

Then, we obtain the following collective linguistic interval fuzzy preference relation:

$$U = \begin{pmatrix} - & [VLC,SC] & [IM,MC] & [SC,SC] \\ [ML,EL] & - & [IM,MC] & [MC,C] \\ [SC,IM] & [SC,IM] & - & [VLC,SC] \\ [MC,MC] & [I,SC] & [MC,ML] & - \end{pmatrix}$$

Then we calculate the consensus relations of each expert

$$C^1 = \begin{pmatrix} - & 0.125 & 0.125 & 0 \\ 0.125 & - & 0 & 0.25 \\ 0.125 & 0 & - & 0 \\ 0 & 0.25 & 0 & - \end{pmatrix}$$

$$C^2 = \begin{pmatrix} - & 0.125 & 0.125 & 0 \\ 0.125 & - & 0.125 & 0.25 \\ 0.125 & 0.125 & - & 0.125 \\ 0 & 0.375 & 0.125 & - \end{pmatrix}$$

$$C^3 = \begin{pmatrix} - & 0.125 & 0 & 0 \\ 0.125 & - & 0.125 & 0.25 \\ 0 & 0.125 & - & 0.125 \\ 0 & 0.25 & 0.125 & - \end{pmatrix}$$

Therefore, consensus degrees on the preferences $[p_{ij}]$ are

$$\begin{pmatrix} - & 0.875 & 0.917 & 1 \\ 0.875 & - & 0.917 & 0.75 \\ 0.917 & 0.917 & - & 0.917 \\ 1 & 0.708 & 0.917 & - \end{pmatrix}$$

In this case, we obtain a global consensus degree $CD = 0.8925$ or $CD = 89.25\%$, which is acceptable.

So, from final collective interval fuzzy preference relations matrix U it is possible to obtain the following dominance degrees:

$$px_1 = 20 \quad px_2 = 35 \quad px_3 = 19 \quad px_4 = 24$$

So, these alternatives can be classified from highest to lowest preference as:

$$x_2 > x_4 > x_1 > x_3$$

5. Conclusions

We have presented a new consensus model to deal with GDM in a linguistic context with linguistic interval fuzzy preference relations. With this model we extent that presented in Tapia-Garcia et al. (2012) which was defined for interval fuzzy preference relations.

In the future we shall work on the management of incomplete information (Cabrerizo, Heradio, Pérez, & Herrera-Viedma, 2010d; Chiclana, Herrera-Viedma, & Alonso, 2009a; Herrera-Viedma et al., 2007a, Herrera-Viedma, Chiclana, Herrera, & Alonso, 2007b) in GDM with interval fuzzy preference relations.

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References

Alonso, S., Cabrerizo, F. J., Chiclana, F., Herrera, F., & Herrera-Viedma, E. (2009a). Group decision-making with incomplete fuzzy linguistic preference relations. *International Journal of Intelligent Systems, 24*(2), 201–222.

Alonso, S., Herrera-Viedma, E., Chiclana, F., & Herrera, F. (2009b). Individual and social strategies to deal with ignorance situations in multi-person decision making. *International Journal of Informations Technology Decisions Making, 8*(2), 313–333.

Alonso, S., Herrera-Viedma, E., Chiclana, F., & Herrera, F. (2010). A web based consensus support system for group decision making problems and incomplete preferences. *Informations Science, 180*(23), 4477–4495.

Ben-Arieh, D., & Chen, Z. (2006). Linguistic labels aggregation and consensus measure for autocratic decision-making using group recommendations. *IEEE Transactions Systems Man Cybernetics Part A System Human, 36*, 558–568.

Bordogna, G., Fedrizzi, M., & Pasi, G. (1997). A linguistic modelling of consensus for a fuzzy majority in group decision making. *IEEE Transactions System Man Cybernetics, 27*, 126–132.

Cabrerizo, F. J., Alonso, S., & Herrera-Viedma, E. (2009). A consensus model for group decision making problems with unbalanced fuzzy linguistic information. *International Journal Information Technology Decision Making, 8*, 109–131.

Cabrerizo, F. J., Heradio, R., Pérez, I. J., & Herrera-Viedma, E. (2010d). A selection process based on additive consistency to deal with incomplete fuzzy linguistic information. *Journal of Universal Computer Science, 16*(1), 62–81.

Cabrerizo, F. J., López-Gijón, J., Ruíz-Rodríguez, A. A., & Herrera-Viedma, E. (2010b). A model based on fuzzy linguistic information to evaluate the quality of digital libraries. *International Journal of Information Technology Decision Making, 9*(3), 455–472.

Cabrerizo, F. J., Moreno, J. M., Pérez, I. J., & Herrera-Viedma, E. (2010a). Analyzing consensus approaches in fuzzy group decision making: Advantages and drawbacks. *Soft Computing, 14*(5), 451–463.

Cabrerizo, F. J., Pérez, I. J., & Herrera-Viedma, E. (2010c). Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information. *Knowledge-Based Systems, 23*(2), 169–181.

Carrasco, R. A., Villar, P., Hornos, M. J., & Herrera-Viedma, E. (2011). A linguistic multi-criteria decision making model applied to the integration of education questionnaires. *International Journal of Computational Intelligence Systems, 4*(5), 946–959.

Chiclana, F., Herrera, F., & Herrera-Viedma, E. (1998). Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations. *fuzzy Sets and Systems, 97*(1), 33–48.

Chiclana, F., Herrera, F., & Herrera-Viedma, E. (2001). Multiperson decision making based on multiplicative preference relations. *European Journal of Operational Research, 129*, 372–385.

Chiclana, F., Herrera, F., & Herrera-Viedma, E. (2002). A note on the internal consistency of various preference representations. *fuzzy Sets and Systems, 131*(1), 75–78.

Chiclana, F., Herrera-Viedma, E., & Alonso, S. (2009a). A note on two methods for estimating missing pairwise preference values. *IEEE Transactions on Systems, Man and Cybernetics – Part B: Cybernetics, 39*(6), 1628–1633.

Chiclana, F., Herrera-Viedma, E., Alonso, S., & Herrera, F. (2009b). Cardinal consistency of reciprocal preference relations: a characterization of multiplicative transitivity. *IEEE Transactions on fuzzy Systems, 17*(1), 14–23.

Fedrizzi, M., Kacprzyk, J., & Nurmi, H. (1993). Consensus degrees under fuzzy majorities and fuzzy preferences using OWA (ordered weighed average) operators. *Control Cybernetics, 22*, 78–86.

Genç, S., Boran, F. E., Akay, D., & Xu, Z. (2010). Interval multiplicative transitivity for consistency, missing values and priority weights of interval fuzzy preference relations. *Information Science, 180*, 4877–4891.

Herrera, F., & Herrera-Viedma, E. (1997). Aggregation operators for linguistic weighted information. *IEEE Transactions on System Man Cybernetics, 27*, 646–656.

Herrera, F., & Herrera-Viedma, E. (2000a). Choice functions and mechanisms for linguistic preference relations. *European Journal of Operational Research, 120*, 144–161.

Herrera, F., & Herrera-Viedma, E. (2000b). Linguistic decision analysis: Steps for solving decisions problems under linguistic information. *fuzzy Sets and Systems, 115*, 67–82.

Herrera, F., Herrera-Viedma, E., & Martinez, L. (2008). A fuzzy linguistic methodology to deal with unbalanced linguistic term sets. *IEEE Transactions on fuzzy Systems, 16*(2), 354370.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1995). A sequential selection process in group decision making with linguistic assessment. *Information Science, 85*, 223–239.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996a). A linguistic decision process in group decision making. *Group Decisions and Negotiation, 5*, 165–176.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996b). A model of consensus in group decision making under linguistic assessments. *fuzzy Sets and Systems, 78*, 73–87.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996c). Direct approach processes in group decision making using linguistic OWA operators. *fuzzy Sets and Systems, 79*, 175–190.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1997a). A rational consensus model in group decision making under linguistic assessment. *fuzzy Sets and Systems, 88*, 31–49.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1997b). Linguistic measures based on fuzzy coincidence for reaching consensus in group decision making. *International Journal of Approximate Reasoning, 16*, 309–334.

Herrera-Viedma, E., Alonso, S., Chiclana, F., & Herrera, F. (2007a). A consensus model for group decision making with incomplete fuzzy preference relations. *IEEE Transactions on fuzzy Systems, 15*, 863–877.

Herrera-Viedma, E., Chiclana, F., Herrera, F., & Alonso, S. (2007b). Group decision-making model with incomplete fuzzy preference relations based on additive consistency. *IEEE Transactions on Systems, Man and Cybernetics, Part B, Cybernetics, 37*(1), 176–189.

Herrera-Viedma, E., Herrera, F., & Chiclana, F. (2002). A consensus model for multiperson decision making with different preference structures. *IEEE Transactions on Systems Man and Cybernetics Part A – Systems and Humans, 32*, 394–402.

Herrera-Viedma, E., Herrera, F., Martinez, L., Herrera, J. C., & Lopez-Herrera, A. G. (2004). Incorporating filtering techniques in a fuzzy linguistic multi-agent model for information gathering on the web. *fuzzy Sets and Systems, 148*(1), 61–83.

Herrera-Viedma, E., & Lopez-Herrera, A. G. (2007). A model of information retrieval system with unbalanced fuzzy linguistic information. *International Journal of Intelligent Systems, 22*(11), 1197–1214.

Herrera-Viedma, E., & Lopez-Herrera, A. G. (2010). A review on information accessing systems based on fuzzy linguistic modelling. *International Journal of Computational Intelligence Systems, 3*(4), 420–437.

Herrera-Viedma, E., Martínez, L., Mata, F., & Chiclana, F. (2005). A consensus support system model for group decision-making problems with multi-granular linguistic preference relations. *IEEE Transactions on fuzzy Systems, 13*, 644–645.

Herrera-Viedma, E., & Peis, E. (2003). Evaluating the informative quality of documents in SGML-format using fuzzy linguistic techniques based on computing with words. *Information Processing and Management, 39*(2), 233–249.

Jiang, Y. (2007). An approach to group decision making based on interval fuzzy preference relations. *Journal of Systems Science and Systems Engineering, 16*(1), 113–120.

Kacprzyk, J. (1987). On some fuzzy cores and "soft" consensus measures in group decision making. In J. Bezdek (Ed.), *The analysis of fuzzy information* (pp. 119–130). CRC Press.

Kacprzyk, J., & Fedrizzi, M. (1986). Soft consensus measure for monitoring real consensus reaching processes under fuzzy preferences. *Control Cybernetics, 15*, 309–323.

Kacprzyk, J., & Fedrizzi, M. (1988). A soft measure of consensus in the setting of partial (fuzzy) preferences. *European Journal of Operational Research, 34*, 316–323.

Kacprzyk, J., Nurmi, H., & Fedrizzi, M. (1997). *Consensus Under Fuzziness*. Boston: Kluwer Academics Publishers.

Mata, F., Martínez, L., & Herrera-Viedma, E. (2009). An adaptive consensus support model for group decision making problems in a multi-granular fuzzy linguistic context. *IEEE Transactions on fuzzy Systems, 17*(2), 279–290.

Morales-del-Castillo, J. M., Peis, E., Ruiz-Rodriguez, A. A., & Herrera-Viedma, E. (2010). Recommending biomedical resources: A fuzzy linguistic approach based on semantic web. *International Journal of Intelligent System, 25*, 11431157.

Moreno, J. M., Morales del Castillo, J. M., Porcel, C., & Herrera-Viedma, E. (2010). A quality evaluation methodology for health-related websites based on a 2-tuple fuzzy linguistic approach. *Soft Computing, 14*(8), 887–897.

Orlovsky, S. (1978). Decision-making with a fuzzy preference relation. *fuzzy Sets and Systems, 1*, 155–167.

Pérez, I. J., Cabrerizo, F. J., & Herrera-Viedma, E. (2010). A mobile decision support system for dynamic group decision making problems. *IEEE Transactions on Systems, Man and Cybernetics – Part A: Systems and Humans, 40*(6), 1244–1256.

Pérez, I. J., Cabrerizo, F. J., & Herrera-Viedma, E. (2011). Group decision making problems in a linguistic and dynamic context. *Expert Systems with Applications, 38*(3), 1675–1688.

Porcel, C., & Herrera-Viedma, E. (2010). Dealing with incomplete information in a fuzzy linguistic recommender system to disseminate information in university digital libraries. *Knowledge-Based Systems, 23*, 32–39.

Tanino, T. (1988). fuzzy preference relations in group decision making. In J. Kacprzyk & M. Roubens (Eds.), *Non-Conventional Preference Relations in Decision Making* (pp. 54–71). Springer-Verlag.

Tapia-Garcia J. M., Del Moral M. J., Martinez M. A., & Herrera-Viedma, E. (2012). A consensus model for group decision making problems with interval fuzzy preference relations. International Journal of Information Technology Decision Making, in press.

Xu, Z. S. (2004). On compatibility of interval fuzzy preference relations. *fuzzy Optimization and Decision Making, 3*, 217–225.

Xu, Z. S. (2005). An approach to group decision making based on incomplete linguistic preference relations. *International Journal of Information Technology Decision Making, 4*(1), 153–160.

Xu, Z. S. (2007). A method for multiple attribute decision making with incomplete weight information in linguistic setting. *Knowledge-Based Systems, 20*(8), 719–725.

Xu, Z. S., & Chen, J. (2008). Some models for deriving the priority weights from interval fuzzy preference relations. *European Journal of Operational Research, 184*, 266–280.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems Man Cybernetics, 18*, 183–190.

Yager, R. R. (1993). Families of OWA operators. *fuzzy Sets and Systems, 59*, 125–148.

Zadeh, L. A. (1975a). The concept of a linguistic variable and its applications to approximate reasoning Part I. *Information Science, 8*(3), 199–249.

Zadeh, L. A. (1975b). The concept of a linguistic variable and its applications to approximate reasoning Part II. *Information Science, 8*(4), 301–357.

Zadeh, L. A. (1975c). The concept of a linguistic variable and its applications to approximate reasoning Part III. *Information Science, 9*(1), 43–80.