FINGRAMS: Visual Representations of Fuzzy Rule-Based Inference for Expert Analysis of Comprehensibility

David P. Pancho, Jose M. Alonso, *Member, IEEE,* Oscar Cordón, *Senior Member, IEEE,* Arnaud Quirin and Luis Magdalena *Senior Member, IEEE,*

Abstract—Since Zadeh's proposal and Mamdani's seminal ideas, interpretability is acknowledged as one of the most appreciated and valuable characteristics of fuzzy system identification methodologies. It represents the ability of fuzzy systems to formalize the behavior of a real system in a human understandable way, by means of a set of linguistic variables and rules with a high semantic expressivity close to natural language. Interpretability analysis involves two main points of view: readability of the knowledge base description (regarding complexity of fuzzy partitions and rules) and comprehensibility of the fuzzy system (regarding implicit and explicit semantics embedded in fuzzy partitions and rules, as well as the fuzzy reasoning method). Readability has been thoroughly treated by many authors who have proposed several criteria and metrics. Unfortunately, comprehensibility has usually been neglected because it involves some cognitive aspects related to the human reasoning which are very hard to formalize and to deal with. This paper proposes the creation of a new paradigm for fuzzy system comprehensibility analysis based on fuzzy systems' inference maps, so-called fuzzy inference-grams (fingrams) by analogy with scientograms used for visualizing the structure of science. Fingrams show graphically the interaction between rules at the inference level in terms of co-fired rules, i.e., rules fired at the same time by a given input. The analysis of fingrams offers many possibilities: measuring the comprehensibility of fuzzy systems, detecting redundancies and/or inconsistencies among fuzzy rules, identifying the most significant rules, etc. Some of these capabilities are explored in this work for the case of fuzzy models and classifiers.

Index Terms—Fuzzy Modeling, Interpretability-accuracy Trade-off, Comprehensibility Analysis, Expert Analysis, Information Visualization, Social Network Analysis.

I. INTRODUCTION

Interpretability of a fuzzy system involves the skill or talent of the specific end-user, i.e., the person who interprets its linguistic description with the aim of inferring (conceiving) the significance of the system behavior. In consequence, characterizing and assessing interpretability is a very subjective task which strongly depends on the background (experience,

D. P. Pancho, J. M. Alonso, O. Cordón and L. Magdalena are with European Centre for Soft Computing, Edificio de Investigación, C/ Gonzalo Gutiérrez Quirós s/n, 33600 Mieres, Asturias, Spain, e-mail1: {david.perez,jose.alonso}@softcomputing.es, e-mail2: {oscar.cordon,luis.magdalena}@softcomputing.es

O. Cordón is with DECSAI and CITIC-UGR, University of Granada, C/ Periodista Daniel Saucedo Aranda s/n, 18071 Granada, Spain, e-mail: ocordon@decsai.ugr.es

A. Quirin is with Galician Research and Development Center in Advanced Telecommunications (GRADIANT), Communications Area, Edif. CITEXVI, local 14, University of Vigo, 36310 Vigo, Spain, e-mail: aquirin@gradiant.org

preferences, knowledge, etc.) of the person who makes the evaluation [1].

Interpretability is a distinguishing capability of fuzzy systems that is really appreciated in most applications. Even more, it becomes an essential requirement for those applications that involve extensive interaction with human beings. Thus, we will focus on the so-called *humanistic systems*, defined by Zadeh [2] as those systems whose behavior is strongly influenced by human judgment, perception or emotions. For instance, decision support systems in medicine [3] must be easily understandable, for both physicians and patients, with the intention of being reliable, i.e., widely accepted and successfully applicable.

Unfortunately, fuzzy systems are not interpretable *per se*, they have to be designed carefully to fulfill that characteristic. Of course, the use of linguistic variables [2] and rules [4], [5] favors interpretability due to their high semantic expressivity close to natural language. Nevertheless, there are many different issues which must be taken into account in order to design interpretable fuzzy systems. Firstly, several interpretability constraints [6], [7] have to be imposed along the whole design process with the aim of producing fuzzy systems with the required interpretability level, i.e., systems capable of being understood, described or accounted for by a human being. As a result of these constraints, interpretability is usually achieved at the cost of penalizing accuracy. For this reason, most fuzzy systems are built jeopardizing interpretability, only paying attention to accuracy. Even in those cases, authors usually claim their fuzzy systems are much more interpretable than those systems based on black-box techniques, like neural networks, because they are based on fuzzy logic. Those claims are quite questionable and should be rejected because they are deceptive. Obtaining interpretable fuzzy systems is a matter of design which must be carefully considered. Unless this is done neatly, produced fuzzy systems will be hardly interpretable, becoming black-boxes in that interpretability sense.

The assessment of interpretability has to face two main issues [1]: (1) readability (transparency) of the system description, related to the view of the model structure as a gray-box, and (2) comprehensibility of the system explanation, which is closer to cognitive aspects because it is always related to human beings. Of course, the analysis has to take into account all elements included in a fuzzy system, from the lowest (fuzzy partitions) to the highest (fuzzy rules) abstraction levels [8]. Namely, the analysis must range from the design of each

individual linguistic term (and its related fuzzy set) to the analysis of the cooperation among several rules, what depends on the fuzzy inference mechanism.

Most previous works [9], [10] only analyze the readability of the designed fuzzy system. Moreover, the analysis of readability is usually reduced to a basic analysis of complexity, i.e., it consists of counting the number of elements included in the fuzzy knowledge base (number of rules, premises, linguistic terms, etc.). Other contributions also analyze structural properties of fuzzy partitions [6] such as distinguishability, coverage, and so on. Recently, a few authors have shown the importance of extending the analysis of readability to evaluate the implicit and explicit semantics embedded in a fuzzy knowledge base [11], [12]. Of course, keeping a small number of linguistic terms is appreciated due to the limits of human processing capabilities [13]. Nevertheless, not only the quantity but also the quality is very important. Thus, the selection of the right linguistic terms is essential to yield interpretable systems. Notice that, interpretable fuzzy partitions must represent prototypes that are meaningful for the interpreter.

Although there has been a huge effort for defining, characterizing and assessing interpretability in the last decade, there is still a lot of work to be done. Namely, the comprehensibility analysis of the system explanation is almost negligible. Understanding the system behavior from its linguistic description becomes a very hard task that involves the inference level going beyond the simple assessment of the system structure readability.

This work presents a novel methodology, firstly sketched in [14], for analyzing the fuzzy inference layer of a fuzzy rule-based system (FRBS) from the comprehensibility point of view. It is mainly based on the adaptation of recent analysis techniques from a completely different research field, that of Scientometrics [15]. We will consider the use and enrichment of existing techniques for visualizing scientific information based on social network analysis [16], [17], called scientograms or visual science maps [18], to the visual analysis of the fuzzy systems' inference process. As a consequence, our new comprehensibility analysis tool will be called fuzzy inference-grams (*fingrams* from now on).

FRBSs can be either designed from expert knowledge or automatically generated from experimental data with a specific learning technique. Anyway, the correspondence of generality and specificity in between the extracted knowledge and the available examples is not always straightforward. Moreover, this fact may become a handicap. So for, a visual representation of the FRBS inference process allows us to find out how rules cover examples and how rules are related among them, because they interact to produce the overall behavior of the system.

A first software package for generation and analysis of fingrams has been implemented. It is freely downloadable as open source software as part of the GUAJE tool[1]. All application examples presented in this paper are conducted using this software. Moreover, it includes an interactive guide

[1]http://www.softcomputing.es/guaje [19]

tutorial that allows the user to become familiar with the tool. As a result, the interested reader can use GUAJE not only to reproduce the illustrative examples presented in this paper, but also to generate and analyze her/his own fingrams.

The rest of the contribution is organized as follows. Section II presents some preliminaries including basic aspects related to interpretability assessment, a brief overview on existent methodologies for visual representation and analysis of fuzzy systems, and a short introduction to the most widely known techniques for social network analysis extending the design and analysis of visual science maps. Section III introduces the fingram generation process while Section IV presents the possibilities fingram analysis offers. Section V shows some illustrative application examples. Finally, some conclusions and future works are pointed out in Section VI.

II. PRELIMINARIES

A. Assessing Interpretability of Fuzzy Rule-based Systems

There are universal indices commonly accepted for accuracy assessment. For instance, the mean square error and the number of misclassified patterns are widely used for regression and classification problems, respectively. However, this is not the case when dealing with interpretability evaluation, where the definition of such indices remains an open hot topic.

There are lots of interpretability indices focusing on specific characteristics of FRBSs. Nevertheless, finding out a universal index for interpretability seems to be an impossible mission since the considered concept is strongly affected by subjectivity. In fact, there is a need to look for two kinds of complementary indices, objective and subjective ones. On the one hand, objective metrics are needed to make feasible fair comparisons among different fuzzy systems. On the other hand, subjective measures are demanded when looking for personalized fuzzy systems. Such systems require a flexible index to be easily adaptable to the context of each problem as well as to end-user's preferences.

Interpretability indices can be grouped according to two different criteria [20], the nature of the interpretability index (structure vs. semantics) and the elements of the fuzzy knowledge base that it considers (fuzzy partitions vs. rule base). The four derived groups are: (Q1) structure at partition level, (Q2) structure at rule base level, (Q3) semantics at partition level, and (Q4) semantics at rule base level.

	Fuzzy Partition Level	Rule Base Level
Structural-based Interpretability	**Q1** Number of membership functions Number of features/variables	**Q2** Number of rules Number of conditions
Semantic-based Interpretability	**Q3** Completeness or coverage Normalization Distinguishability Complementarity Relative measures	**Q4** Consistency of rules Rules fired at the same time Transparency of rule structure Cointension

Fig. 1.   Quadrant of interpretability indices [20].

Most well-known existing interpretability indices correspond to groups Q1 and Q2, thus they focus on readability

(in terms of complexity at structural level) of fuzzy systems. In consequence, they are objective indices since they basically count the number of elements (features/variables, membership functions, rules, premises, etc.) existing in the FRBS.

Indices included in group Q3 usually measure the degree of fulfillment of semantic constraints that should be overimposed during the design process. In [6] Oliveira proposed some semantic constraints (coverage, normalization, distinguishability, etc.) required to have interpretable fuzzy partitions from the semantical point of view. The use of strong fuzzy partitions (SFP) [21] satisfies all these semantic constraints. Nonetheless, notice that, breaking the SFP property can yield more accurate systems. Therefore, there are proposals that ensure a good interpretability at this level without considering SFP [11], [22], [23].

Finally, group Q4 is the one that contains the lowest number of works in the literature. These indices advocate for extending the analysis of readability to evaluate the comprehensibility, i.e., the implicit and explicit semantics embedded in fuzzy systems [12]. There are also some papers dealing with the consistency of fuzzy rule bases and with the number of co-fired rules, i.e., rules simultaneously fired by a given input [24]–[26].

B. Visual Description and Analysis of Fuzzy Rule Bases

There are not many papers tackling with visual analysis of the fuzzy system inference process. Probably, this is due to the well-known linguistic expressivity of fuzzy systems what gives prominence to linguistic representations. However, when dealing with complex real world problems, even when the design is made carefully to maximize interpretability, the number of rules can become huge because of the curse of dimensionality characteristic of FRBSs. In those cases, looking for a plausible linguistic explanation of the inferred output, derived from the linguistic description of the fuzzy knowledge base, is not straightforward. When many rules are fired at the same time for a given input, explaining the inferred output as an aggregation of all the involved rules can be very complicated.

Some authors [27] have searched for understandable ways of interpreting the system output in terms of describing the inferred output possibility distribution by a set of previously defined linguistic terms along with some linguistic modifiers and connectives. As an alternative, other authors have made a bet for searching visual explanations of the system output [28]–[30]. In these papers, Ishibuchi et al. established a set of design constraints with the aim of producing groups of rules with only two antecedent conditions that can be represented in a two-dimensional space. These works focus on providing a visual representation able to explain the output of fuzzy rule-based classifiers to human users. Nevertheless, considering only two antecedents per rule is a strong limitation that may penalize the accuracy of the system, especially when dealing with complex and high dimensional problems.

A complete analysis of visualization requirements for fuzzy systems is provided in [31]. That contribution gives an overview on existing methodologies to yield 2D and 3D graph-

TABLE I
CHARACTERISTICS OF VISUALIZATION METHODS FOR
MULTI-DIMENSIONAL FUZZY RULES

	[32]	[33]	[34], [35]	[36]
Represent data samples		✓	✓	✓
Represent overlapping among rules at descriptive level	✓	✓		✓
Represent rule interaction at inferential level				

ical representations of fuzzy systems. It comprises visualization of fuzzy data, fuzzy partitions, and fuzzy rules. Different alternatives are available depending on the requirements of the end-user (fuzzy designer, domain expert, etc.). Moreover, requirements may change according to the visualization tasks to perform: interactive exploration; automatic computer-supported exploration; receiving feedback from users; and capturing users' profiles and adaptation.

The most relevant works on the design of visual representations for multi-dimensional fuzzy rules are those developed by Berthold et al. [32], [33]. They make a mapping from high dimensional feature spaces onto two-dimensional spaces which maintains the pairwise distances between rules. The established mapping also displays an approximation of each rule spread and overlapping. As a result, it is possible to visualize and explore multi-dimensional FRBSs in a 2D graphical representation. Authors claim such representation yields a user friendly and interpretable exploratory analysis. However, the complexity of the analysis grows exponentially with the number of variables and rules to be displayed. In consequence, in complex and high dimensional problems, the interpretation of the resulting graph is not straightforward.

Evsukoff et al. [34], [35] propose the use of an interpretation framework that helps understanding multidimensional fuzzy rules. They assign a symbol to each rule, which is represented by a Gaussian membership function. The model interpretation is based on analysis of rule weights and on a 2D linear principal component analysis projection to visualize the model.

On a different basis, Casillas et al. [36] present the so-called "transition chromatic maps" for fuzzy rules generated from uncertain data. These maps are generated as result of a visual modeling process that represents the extracted knowledge in a more understandable way, thus helping in the postprocessing, interpretation stage of knowledge discovery in databases. They allow us to see the relations among variables by observing the chromatic evolution of the surfaces on the graph.

Table I summarizes the main characteristics of the most relevant visualization methods for multidimensional fuzzy rules previously introduced. All methods make a 2D representation of fuzzy rules. Some of them represent data and some others show the existing overlapping among rules at descriptive level, but none of them represents rule interaction at inference level. This brief review shows that there is a lack of methods depicting the interaction among rules that, however, could strongly help in the comprehension of the rule base behavior.

C. Social Network Analysis

A social network is a social structure made up of individuals called "nodes", which are connected or tied by "edges" (also called ties, links, or connections) corresponding to one or more specific types of interrelations, such as friendship, common interest, or knowledge. Social network analysis (SNA) [16], [17] views social relationships in terms of network theory regarding nodes and edges. Nodes are the individual actors within the networks, and ties are the relationships among the actors. Research in a number of academic fields has shown that social networks operate on many levels, from families up to the level of nations. They play a critical role in determining the way how problems are solved, organizations are run, and individuals succeed in achieving their goals.

Given a network, the scaling algorithms have the goal to take proximity information and to obtain structures revealing the underlying organization. They use similarities, correlations, or distances to prune a graph based on proximity among pairs of nodes. The three predominant ways proposed in the literature to perform this task are analyzed below [37].

The first option introduces a link weight threshold and it only considers the links having weights above this threshold [38]. This approach is straightforward and easy to implement. However, it does not take the intrinsic structure of the underlying network into account, so the transformed network may not preserve the essence of the original one. Furthermore, the value of the threshold could be hard to adjust for the user.

The second option extracts a minimum spanning tree (MST) from a network of N vertices [39]. This approach guarantees the number of links in the transformed network is always $N - 1$. However, that does not always reflect the subjacent relevant information.

The third option imposes constraints on paths and excludes links that do not satisfy the constraints. One of the most known methods, the Pathfinder algorithm [40], [41], is frequently used due to its mathematical properties related to the preservation of the triangular inequality. Those properties include the conservation of links, the capability of modeling symmetrical but also asymmetrical relationships, and the representation of the most *salient* relationships present in the data. The result of applying Pathfinder to a network is a pruned network called PFNET.

Once PFNETs or any other kind of pruned networks are generated, there are many different methods for their automatic visualization. Force-based or force-directed algorithms are the most widely used class of algorithms for drawing graphs in the area of information science [42], [43]. Their purpose is to locate the nodes of a graph in a two or three dimensional space so that all the edges are approximately of equal length and there are as few crossing edges as possible, trying to obtain the most aesthetically pleasing view. This family of methods has Kamada-Kawai [44] and Fruchterman-Reingold [45] as their most representative methods.

Kamada-Kawai [44] is one of the most extended methods for visualizing PFNETs. Starting from a circular position of the nodes, it generates networks with aesthetic criteria such as the maximum use of the available space, the minimum number of crossed links, the forced separation of nodes, the generation of balanced maps, etc. It assigns coordinates to the nodes trying to adjust as much as possible the distances existing among them with respect to actual network distances.

In the Fruchterman-Reingold Algorithm [45], the attraction or repulsion among nodes determines in which direction a node should move. Nodes move from an original layout step by step. The step width of node movements decreases at each iteration. Once nodes stop moving, the procedure ends.

The combination of SNA through the use of network scaling algorithms and visualization methods has proved its capability to get high quality, schematic visualizations of the resulting networks in various fields: psychology (to represent the cognitive structure of a subject [40], [41]), software development (for debugging of multi-agent systems [46]), scientometrics (for the analysis of large scientific domains [18], [47]), etc.

D. Scientogram Design and Analysis

The term *scientogram*, a particular case of social network, is coined in the specialized literature to make reference to visual science maps, i.e., visual representations of scientific domains. Vargas-Quesada, Moya-Anegón et al. [18], [47], [48] proposed a methodology to create scientograms with the aim of illustrating interactions among authors and papers through citations and co-citations. The basic idea turns up from the notion of manuscript co-citation that represents the frequency with which two documents are simultaneously cited by others. It is possible to group them by author, journal, or thematic category, for instance. Of course, depending on the kind of grouping, the information that can be extracted from the generated maps is different.

The standardized co-citation measure was originally defined by Salton and Bergmark [49]:

$$MCN(ij) = \frac{Cc(ij)}{\sqrt{c(i) \cdot c(j)}} \qquad (1)$$

where Cc means co-citation, c stands for citation, i and j represent two different entities (authors, documents, journals, categories, institutions, countries, etc.).

As an illustrative example, Fig. 2 represents the scientogram of the world production in 2002. It consists of 16 thematic areas where the volume of the nodes is shown proportional to the volume of produced documents. The links represent the main connections among these areas.

Notice that, the combination of entities co-citation, PFNETs, and Kamada-Kawai considered building this scientogram makes the most important entities in the network (i.e., those sharing more sources with the rest) tend to be placed toward the center.

Finally, concerning the analysis of scientograms, according to [18], [48], there are three main measures of centrality that yield useful information with the aim of detecting and identifying the most significant nodes in a PFNET: *Centrality Degree* (regarding the number of direct links gathering in a node), *Closeness Centrality* (measuring the shortest paths among nodes, for which the inverse of the sum of the distance of a node to all other nodes would indicate its importance), and *Intermediation Centrality* or *Betweenness* (looking at nodes
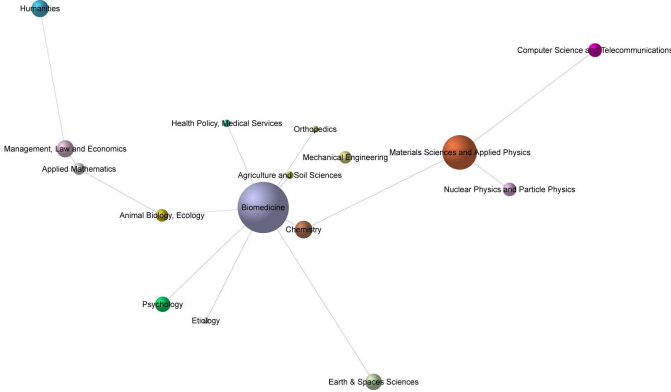
Fig. 2.   Scientogram of the thematic areas of world science, 2002.

that act as links between other nodes contained in the shortest path, for which the highest value would highlight the most central node).

III. FINGRAM DESIGN

This paper proposes a new methodology for visual representation and exploratory analysis of the fuzzy inference process in FRBSs. In such systems, various rules can be fired simultaneously by an input. Moreover, the usual behavior of FRBSs is that, given a set of problem inputs, several fuzzy rules are fired at the same time. In other words, the input space is usually covered by rules with dense overlapping among them.

In this proposal we take advantage of this characteristic of FRBSs using a set of problem instances to uncover co-fired rules. This co-firing information is used to create social networks representing fuzzy systems' inference maps, the so-called *fingrams*. In these kinds of social networks each fuzzy rule is represented by a node, and the relations among rules are represented by weighted edges whose value is computed using a specific metric. Different metrics can be used to construct a social network given a dataset of cases representing the input-output relations existing in the problem tackled, a set of fuzzy rules, and a fuzzy reasoning mechanism. As a result, fingrams show graphically the interaction among fuzzy rules at the inference level in terms of co-fired rules.

Due to the high overlapping among rules, the complete fingram is usually quite dense and difficult to analyze even for medium-size FRBSs. Fortunately, network scaling methods can be used to simplify fingrams while maintaining their most important relations.

As seen in Sec. II-C, social networks can be represented by the use of drawing methods especially designed for that purpose. Here, a specific graph representation is developed to provide the relevant information of the FRBS under study. Colors and sizes are also used to highlight distinguishing characteristics of the system, allowing the end-user to do a systematic analysis.

From a formal viewpoint, the proposed fingram definition is as follows:

Definition A *fingram* is defined by a tuple $(R, P, I, E, m, NSM, NDM)$ in which:

R is the set of fuzzy rules (nodes), denoted R_i, $1 \leq i \leq r$, with r being the number of rules.

P is the set of fuzzy partitions of input and output variables.

I is the fuzzy inference mechanism used.

E is the set of problem instances, denoted E_k, $1 \leq k \leq d$, with d being the number of instances.

m is the metric used to create M, a square weight matrix $(r \times r)$ that represents the firing interactions among fuzzy rules. The entries of that matrix are the weights associated with the links; m_{ij} is the weight of the link connecting R_i and R_j.

NSM is the considered network scaling method.

NDM is the considered network drawing method.

The remaining of the section explains in detail the procedure followed to create fingrams. The section finishes with an illustrative example.

A. Fingram generation

The generation of a fingram from a FRBS, a fuzzy inference mechanism, and a set of problem instances is made by means of the following procedure:

Procedure FINGRAM(R,P,I,E,m,NSM,NDM)

```
begin
    /* Generation of the social network defined by M using the set of fuzzy
       rules R, the set of fuzzy partitions P, the fuzzy inference mechanism
       I, the set of instances E, and the metric m.                        */
    M ← network generation (R, P, I, E, m)
    begin
        FR_i, FR_j ← get number of fired rules (R, P, I, E);
        SFR_ij ← get number of co-fired rules (R, P, I, E, m);
        M ← compute M_ij (FR_i, FR_j, SFR_ij);

    /* Scaling of the social network defined by M through the use of the
       network scaling method NSM.                                         */
    MS ← network scaling (M, NSM)
    begin
        EE ← evaluate values of edges (M, NSM);
        MS ← obtain the pruned network (M, EE);

    /* Graphical representation of the resulting pruned social network MS
       using the network drawing method NDM.                              */
    MD ← network drawing (MS, NDM)
    begin
        NI ← compute information related to nodes (MS);
        NP ← compute the network layout (MS, NDM);
        MD ← paint edges (MS, NDM, NI, NP);
```

Notice that the rest of this section is devoted to explain each of the steps of the procedure in detail.

1) Network generation: Starting from a set of fuzzy rules R, a set of fuzzy partitions P, a fuzzy inference mechanism I, a set of problem instances E, and a metric m, a social network can be built, represented by a matrix M, which shows the relations among rules.

A square matrix M $(r \times r)$ that contains all interactions inside R is computed regarding the proportion of problem instances co-firing the rules.

$$M = \begin{pmatrix} 0 & m_{12} & \dots & m_{1r} \\ m_{21} & 0 & \dots & m_{2r} \\ \dots & \dots & \dots & \dots \\ m_{r1} & m_{r2} & \dots & 0 \end{pmatrix} \qquad (2)$$

We propose the following metric, inspired by the co-citation measure of scientograms (Eq. 1):

$$m_{ij} = \begin{cases} \dfrac{SFR_{ij}}{\sqrt{FR_i \cdot FR_j}} & \text{, if } i \neq j \\ 0 & \text{, if } i = j \end{cases} \qquad (3)$$

SFR_{ij} corresponds to the number of instances for which rules R_i and R_j are fired simultaneously, while FR_i and FR_j account respectively for the total number of data pairs for which rules R_i or R_j are respectively fired, without taking care if they are fired together or not. Notice that, m_{ij} is thus normalized and the matrix M is symmetrical when using this metric.

2) Network scaling: As usual in social network design, the initial fingram is commonly quite dense and difficult to analyze even for medium-size FRBSs. So for, a network scaling method is required to simplify it while keeping the most important relations. Three options have been considered:

- Prune the network to eliminate the least informative links according to an expert. Contrary to what one may think by intuition when confronting the problem of pruning the graph, using a threshold to filter the graph is not worthy. There exist a large number of links with high weights that would imply the selection of a high threshold value, so for, producing a disconnected network. Of course, the latter does not help in the comprehension of the global system, which is our ultimate goal in this contribution.
- Use a specific scaling algorithm that preserves the most important links without producing isolated nodes, such as Pathfinder[2], previously introduced in Sec. II-C.
- Use a combination of the previously mentioned alternatives. First, links are pruned and then Pathfinder scales the resulting graph. As we will show later, this hybrid option can be used to analyze classification problems. In such case, potential inconsistencies among rules, i.e. relations among rules pointing out different classes, have to be treated carefully. So for, non-inconsistent links can be pruned, keeping just inconsistent links. Finally, as the resulting graph is still likely to be quite complicated, Pathfinder is used to simplify it.

3) Network drawing: As previously outlined in Sec. II-C, force-based algorithms are devoted to represent this kind of information in an aesthetically pleasing way. In order to visualize the pruned network in a 2D space, they assign coordinates to the nodes obtaining a graph with the most important elements placed toward the center of the image. Kamada-Kawai, through Graphviz[3] will be used in our approach because it has been proved very effective in combination with Pathfinder [18]. This solution is flexible enough to be adapted to the particularities of new scenarios we have to deal with.

Nodes are represented by circles and labeled with useful textual information (see Fig. 3):

1) The first line shows the rule identifier, R_k.
2) The second one provides the *relative coverage of that rule* (cov), i.e. the number of covered instances divided by the total number of instances. One problem instance is covered by rule R_k when the rule firing degree for that instance is greater than a predefined threshold (0.1
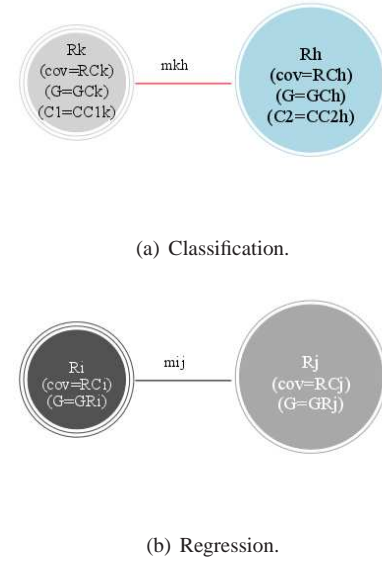

(a) Classification.

(b) Regression.

Fig. 3.   Fingram's interpretation.

in this contribution).

$$cov_{R_k} = \frac{\#\text{instances covered by } R_k}{\#\text{ instances}}$$

3) The third line shows the goodness of the rule (G), i.e. how the rule behaves with respect to the problem instances available. This goodness measure reflects how well the problem instances covered by a rule are classified or modeled. It is computed as the ratio between the differences of cumulated firing degrees produced by positive instances (properly issued) and negative ones with respect to the total cumulated firing degrees regarding all covered instances. Hence, it can take values from -1 to 1, assigning -1 to rules with low number of problem instances correctly issued and close to 1 when the rule correctly handles most problem instances.

$$G_{R_k} = \frac{\sum \text{ FDPI for } R_k - \sum \text{ FDNI for } R_k}{\sum \text{ FDCI for } R_k}$$

where FDPI stands for firing degree of positive instances; FDNI means firing degree of negative instances; and FDCI is the firing degree regarding all covered instances.

4) The fourth line of the nodes appears only in classification problems. It reflects the relative coverage of the rule output class, i.e., the number of problem instances covered by rule R_k that belong to class n divided by the total number of instances related to class n.

$$C_{R_k} = \frac{\text{instances of class } n \text{ covered by } R_k}{\#\text{ instances of class } n}$$

B. Additional fingram visualization capabilities

The proposed representation includes graphical information of special interest for FRBSs. Hence, once the fingram is pruned by Pathfinder and drawn by Kamada-Kawai, some additional visualization capabilities are incorporated which are specific for FRBS fuzzy inference analysis.

In this context, nodes represent the fuzzy rules of a FRBS, which are of the form:

$$R_x: \quad \text{IF } \textit{Input 1 is } LV_1 \text{ AND } \textit{Input 2 is } LV_2 \text{ AND } \ldots$$
$$\ldots \text{ AND } \textit{Input n is } LV_n \text{ THEN } \textit{Output is } CC$$

with (*Input i is* LV_i) being the antecedents of the fuzzy rule, and CC the output of the fuzzy rule.

The node size is established according to the number of examples covered by the rule. The higher the amount of covered examples, the bigger the node size is. For instance, Fig. 3(a) shows an example of a network with two rules (R_k and R_h) where rule R_h covers more examples than rule R_k. In addition, the border of the nodes indicates how complex the antecedents of the rules are. Single-line border indicates two premises; double-line border means three premises; and so on. Thus, the rules R_k and R_h depicted in Fig. 3(a) have three and two antecedents, respectively.

Furthermore, edges (links) among nodes represent rule co-firing information. Each link represents the relation between a pair of fuzzy rules. The higher the degree of overlapping existing over rules, the higher the edge weight and the thicker the link width in the visual representation to clearly represent this fact.

We deal with problems having either categorical or continuous outputs. Therefore we distinguish between classification and regression problems, providing particularities in their representations.

- *Classification:* Rules yielding the same class are depicted by the same color of nodes. The color of links gives useful information as well. Links between rules of the same class (output) are colored in green while potential inconsistencies (links between co-fired rules pointing out different classes) are remarked with red color (See Fig. 3(a)).
- *Regression:* The output variable[4] is ordered in its universe of discourse. This order is used to assign grey tones to nodes, from black to white. So for, the typical behavior will relate nodes with similar grayness, and related nodes showing quite different tones should be studied in detail. In this case there is no difference among links, contrary to what happens in classification problems with redundancies and inconsistencies, and they just inform about their weight (See Fig. 3(b)).

C. Illustrative example

In this section, a fuzzy rule-based classification system (FRBCS) created for the popular WINE dataset [52] is considered. The dataset is made up of 178 examples and 13 attributes (Alcohol, malic acid, ash, etc.) found in three types of wines. The FRBCS has 24 rules with three different output classes, corresponding to the three different wine kinds.

Several fingrams are built with the aim of illustrating the effect of the different network scaling methods used. The fingram plotted in Fig. 4(a), obtained without applying any network scaling technique, clearly shows the previously mentioned scaling motivations. A quite dense set of relationships

[4]We will only consider multi-input-single-output (MISO) FRBSs.

(a) Complete fingram.

(b) Fingram scaled using pruning with threshold ($\Delta = 0.6$).

(c) Fingram scaled using Pathfinder.

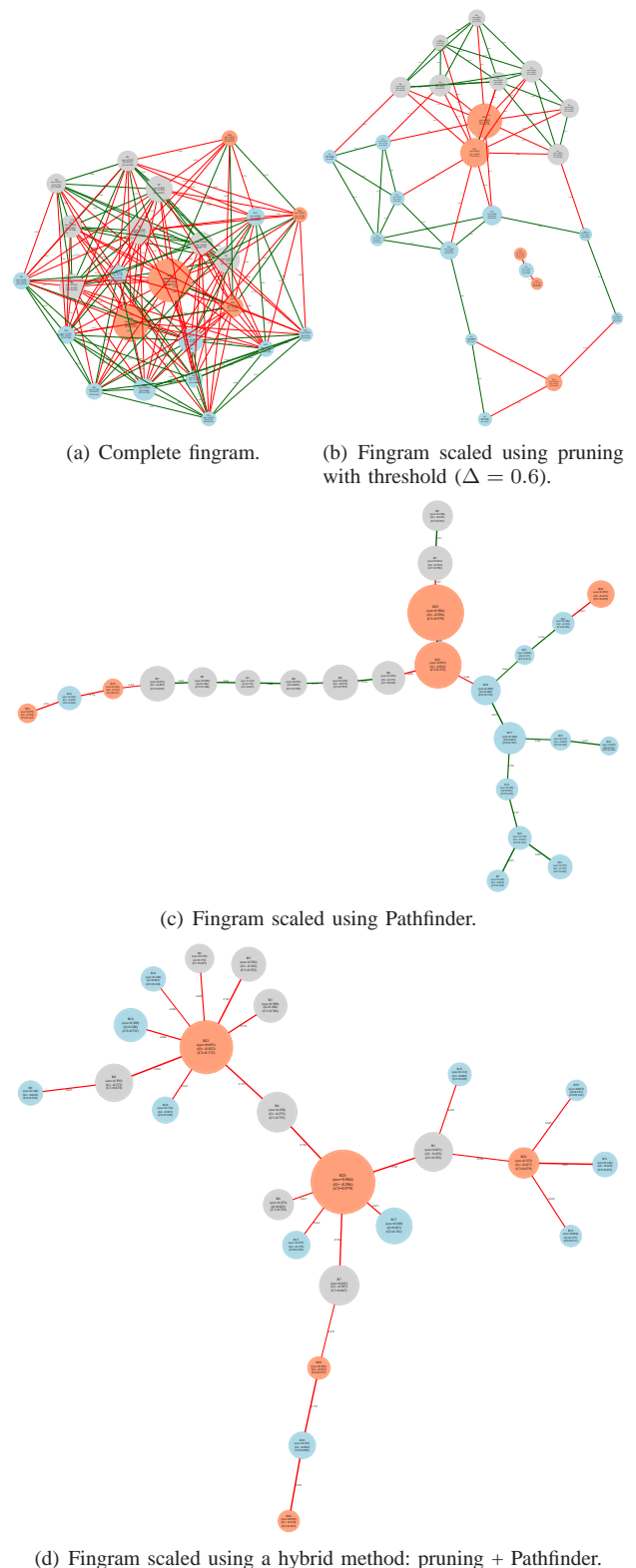(d) Fingram scaled using a hybrid method: pruning + Pathfinder.

Fig. 4. Example of fingrams: complete, pruned, scaled with Pathfinder, and with hybrid method.

among rules does not allow us to analyze easily the FRBCS behavior.

Then, the three scaling methods previously described are used to simplify the network. Fig. 4(b) shows the result of using a user-defined threshold ($\Delta = 0.6$) to prune edges.

It can be seen how the network is still quite dense, some groups of rules are isolated and the network is not visualized in an aesthetic way, thus hindering the comprehension of the whole set of rules. On the other hand, Fig. 4(c) shows the result of applying Pathfinder, whose global close-to-tree structure provides valuable information easy to interpret. As an illustration of the hybrid scaling method, Fig. 4(d) is created from the complete fingram of Fig. 4(a). There, non potential inconsistencies are pruned first (once we deal with a classification problem), while the resulting graph is simplified with Pathfinder. It can be seen how this graph only relates nodes of different color (rules with a different output class).

It is remarkable that, thanks to the combination of rule co-firing, PFNETs, and Kamada-Kawai's algorithm, information related to the inference process of the FRBSs is displayed in pretty nice scalable fingrams, as seen in Fig. 4(c). As a side effect, the most relevant fuzzy rules, i.e., those more often fired, tend to be located toward the center of the scaled fingrams, while less salient ones (in this case, rules with the lowest co-firing degrees) go to the periphery. Hence, the shape of the fingram is quite informative.

Of course, fingrams must be carefully analyzed by an expert since rules that are apparently not very relevant (like those ones in the periphery) may be essential for handling properly important cases that only happen from time to time. For instance, not common cases dealing with failures in a system controlling a nuclear reactor could be extremely important.

Moreover, it is important to highlight that our proposal is not affected by the well-known curse of dimensionality that implies the number of fuzzy rules grows exponentially with the number of inputs. Firstly, nodes directly represent fuzzy rules instead of premises, and secondly, PFNETs have been successfully applied to the analysis of large scientific domains with hundreds of co-cited entities (dual to our problem instances), allowing to relate different thematic areas (dual to our fuzzy rules in the FRBS), with the chance of also considering hierarchical representations [18]. In consequence, fingrams are able to display the interactions among a few hundreds of rules in the form of highly interpretable trees. Even when the number of rules is huge the scaled fingram can be still comfortably viewed by an expert.

For comparison purposes, Fig. 5 shows the same FRBS represented by the visualization method proposed by Berthold et al. in [33]. As it can be seen, this representation is mainly descriptive, placing rules in a 2D space through a multi-dimensional scaling. So for, the distance among rules is relevant. However, it does not provide information for rule behavior at inference level. Moreover, the Delaunay triangulation indicates direct neighbors for each rule. Unfortunately, it relates rules far away in the 2D space. Of course, that fact does not help in the comprehension of the system behavior. For example, rules R1 and R13, which do not co-fire for any problem instance (as it can be seen in Fig. 4(a)), are strongly related in Fig. 5 because of their descriptive proximity.

IV. FINGRAM EXPERT ANALYSIS

Fingrams provide an enormous potential for the representation and comprehension of the FRBS inference process.
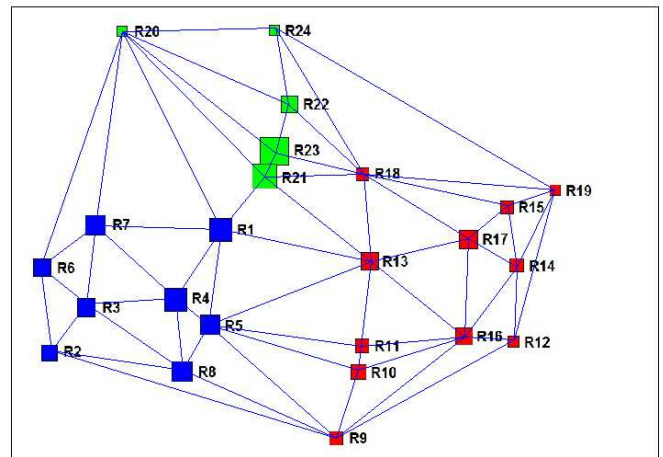

Fig. 5. Visualization of the fuzzy rule set constructed for the WINE problem using the method proposed by Berthold et al. [33]. It shows possible overlaps among rules along with rule connections in terms of closeness by Delaunay triangulation.

They relate rules jointly fired by a given input vector, making easy to uncover how the rules of a FRBS actually cover the input space. Hence, fingrams can be viewed as a powerful tool for dealing with FRBS comprehensibility analysis tasks related to quadrant Q4 (semantics at rule base level) in Fig. 1 (Sec. II-A), the least studied category in the existing fuzzy system interpretability assessment literature.

The analysis of fingrams offers many different possibilities thanks to the high amount of information this representation gives about a FRBS and its related fuzzy inference process. For instance, one can directly analyze its global structure by the exploration of the number and location of the apparent groups of rules (nodes), analyze the respective location of the rules coding for different outputs, etc. As such, we would like to highlight two exploratory tasks that provide a good base to detect and analyze particularities or anomalies in a FRBS: i) identifying the most significant rules in a FRBS from the inference viewpoint, and ii) detecting potential inconsistencies among rules in the particular case of FRBCSs.

On the one hand, it should be reminded that, because of the specific way network scaling and drawing are done, the most salient links and nodes are likely to be placed towards the center of the graphical representation. Thus, those fuzzy rules that correspond to nodes located in the periphery of the fingram, especially those which are connected with a high weight (the value of the associated link is large) to the remaining graph nodes and show a low level of coverage (cov), are good candidates to be further studied. These rules usually cover the same space than others and do not change the final output of the system, thus not affecting the accuracy of the system. This could have an interesting collateral advantage in classification problems since removing such rules is likely to increase interpretability while keeping almost the same accuracy. We will check that assumption in the Application examples section (Sec. V).

Moreover, rules that are fired more frequently (represented with bigger nodes) are usually placed in the center because

they also tend to be co-fired with more rules. Those cases where nodes covering a large number of examples are placed in the periphery must be carefully analyzed. This can be due to a fuzzy rule which covers a large part of the input space in isolation.

The usual Centrality measures that are commonly considered in the analysis of scientograms [18], [48] (see Sec. II-D) can also be successfully applied to uncover the most significant rules within a FRBS. As a first approach, we advocate for the use of the so-called *Degree of Centrality*. This means that we will point out those fuzzy rules corresponding to the nodes that concentrate the larger number of links in a fingram as the most salient ones.

On the other hand, the interaction among fuzzy rules at inference level is very difficult to be appreciated by only reading the linguistic description of FRBSs. It should be remarked that this interaction depends on the rule description but also on the fuzzy rule semantics (fuzzy partitions included in the data base) and on the inference mechanism. Even when a rule base is fully consistent at linguistic level, some possible inconsistencies may arise at inference level because of the FRBS semantics and fuzzy inference process. Such potential conflicts are difficult to detect mainly because they are partially hidden since they are typically produced by new unknown situations that were not taken into account during the learning stage (for example, data pairs not initially included when considering a data-driven FRBS derivation). Of course, such analysis is different depending on the kind of problem faced. For instance, the meaning of overlapping rules is not the same when considering either classification or regression problems.

In the former case, inconsistencies must be handled as conflicts to be solved. For instance, it may happen that several rules are jointly fired for a new given input vector and as a consequence several outputs are activated with degrees higher than zero. When two different classes are activated with very similar degrees, the situation can be labeled as an ambiguous case. Such situation is not desirable, no matter if the system is (or not) able to yield the right output class, because a slight modification in the input data may yield a wrong output. We can conclude that a FRBCS producing many ambiguous cases is not reliable and should be corrected. Fortunately, looking at fingrams we can easily uncover potential inconsistencies (when the co-fired rules yield different output classes). The larger the degree of inconsistency among fuzzy classification rules is, the higher the weight of the "inconsistent" links (co-firing degree computed by Eq. 3) will become (red edges). The interested reader is referred to [53] where a detailed explanation of some possible inconsistency problems, along with a methodology to detect and correct such inconsistencies, is presented.

Opposite, when dealing with regression problems, the well-known FRBS approximation capability is mainly based on the interpolative reasoning carried out among overlapping rules. Typically, two rules with similar premises may yield two different wrong outputs but their aggregation may result in the right inferred interpolated output. Unfortunately, these kinds of situations are quite common but very difficult to identify. Of course, from the comprehensibility point of view it would be desirable to have only one rule that directly yields the right inferred output. However, this may produce a huge number of rules what is also undesirable. Fingrams allow the expert to study and improve the system systematically as it will be shown with an example in Sec. V-C.

V. APPLICATION EXAMPLES

This section starts with an experimental setup subsection, devoted to introduce the quality indices to be considered. Then, two examples in the next two subsections display the possibilities of considering fingrams in real-world problems. The first illustrative classification example gives an idea about how to deal with the co-firing among rules, along with the inconsistencies and redundancies produced. The second example displays a small-sized but complex real-life regression application, where fingrams make easier the understanding of the rules constructed.

A. Experimental setup

We will now describe the accuracy and interpretability indices considered in this contribution.

Accuracy is computed as the percentage of misclassified instances (MC) in classification problems, and as the mean square error (MSE) in regression problems.

$$MC = \frac{1}{d}\sum_{i=1}^{d} err_i; \quad err_i = \begin{cases} 1, & \text{if } C_i \neq \hat{C}_i \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$MSE = \frac{1}{d}\sum_{i=1}^{d}(y_i - \hat{y}_i)^2 \quad (5)$$

where d means the number of problem instances, C_i the class of instance i, and $\hat{C}_i$ is the class inferred by the FRBCS given the instance i in MC. For MSE, y_i is the real output value of instance i, and $\hat{y}_i$ is the inferred output by the FRBS.

Of course, as it was pointed out in Sec. II-A, taking only one index is not enough to evaluate interpretability. Therefore, we have considered some of the interpretability indices commonly used in the literature. Probably, the most popular index is NR which stands for number of rules. As an alternative, TRL (total rule length) represents the total number of linguistic propositions into the whole rule base. Another simple index is ARL which stands for average rule length, computed as TRL divided by NR. We will also report the average number of fired rules with respect to problem instances (AFR). Notice that, a rule is counted as fired by a given data instance only in the case it is activated with a confidence firing degree greater or equal than a predefined threshold (0.1 in this contribution). In the case of classification problems we will additionally compute the average confidence firing degree of winner rules (AFD). It is measured as the average of the firing degree of the winner rule for each data sample over the whole dataset.

Moreover, the proportion of co-fired rules can also be considered to evaluate the FRBS comprehensibility. The assumption is the following: the larger the number of simultaneously fired rules for a given input vector, the smaller the comprehensibility of the FRBS.

Thus, the Co-firing Based Comprehensibility Index ($COFCI$) [54] can be used to evaluate the complexity of understanding the inference process in terms of rules co-firing information. Eq. 6 presents this index:

$$COFCI = \begin{cases} 1 - \sqrt{\frac{CI}{MaxThr}}, & \text{if } CI \leq MaxThr \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

$$CI = \sum_{i=1}^{r} \sum_{j=1}^{r} [(P_i + P_j) \cdot m_{ij}] \quad (7)$$

where r is the total number of rules in the fuzzy rule base, P_i and P_j count the number of premises (antecedent conditions) in rules R_i and R_j, while m_{ij} is the measure of co-firing (computed by Eq. 3) for the rules R_i and R_j, and $MaxThr$ is a maximum value heuristically established to get a normalized measure in the interval [0,1].

B. Generation of fingrams in a simple classification problem. Analysis of inconsistencies.

As a first example we will analyze a simple classification problem with two input variables, which can be represented in two dimensions, where the co-firing relations among rules can be easily understood. For that, the IRIS data set from UCI [52] is considered.

IRIS is perhaps the best known database to be found in the pattern recognition literature. The data set contains 3 classes of 50 instances each, so it is perfectly balanced, where each class refers to a type of iris plant. Class 1 is linearly separable from the other two; the latter are not linearly separable from each other. Notice that, only two of the four input variables of IRIS (SEPAL LENGTH and SEPAL WIDTH) have been used with the aim of allowing a 2D representation that facilitates the understanding of fingram construction.

Fig. 6 shows graphically the distribution of examples, with the selected variables SEPAL LENGTH and SEPAL WIDTH, remarking the flower class ($C1 = \bigcirc$, $C2 = +$, and $C3 = \times$). Each input is characterized by a uniform strong fuzzy partition with three linguistic terms (LOW, AVERAGE, HIGH).
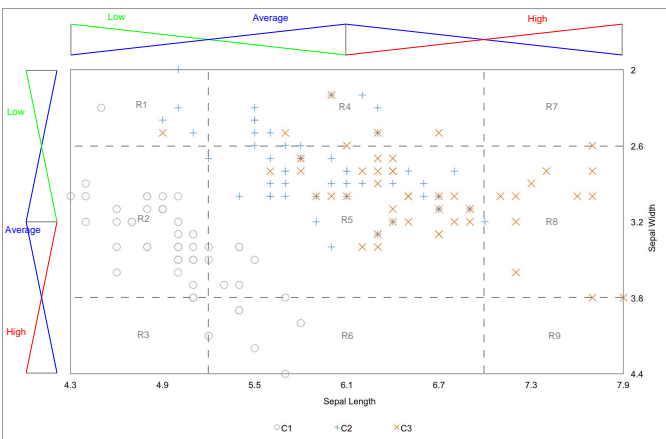

Fig. 6. Classification example: Problem instances, fuzzy partitions, and set of fuzzy rules used.

The rule base has been automatically extracted from the whole data set following the HILK fuzzy modeling methodology which is aimed at producing highly interpretable fuzzy systems [53], [55]. The rule base is generated by means of the Fast Prototyping Algorithm[5] [59]. It is made up of the following nine linguistic rules:

R_1: IF Sepal Length is   Low   AND Sepal Width is   Low   THEN Class is C2
R_2: IF Sepal Length is   Low   AND Sepal Width is Average THEN Class is C1
R_3: IF Sepal Length is   Low   AND Sepal Width is   High   THEN Class is C1
R_4: IF Sepal Length is Average AND Sepal Width is   Low   THEN Class is C2
R_5: IF Sepal Length is Average AND Sepal Width is Average THEN Class is C2
R_6: IF Sepal Length is Average AND Sepal Width is   High   THEN Class is C1
R_7: IF Sepal Length is   High   AND Sepal Width is   Low   THEN Class is C3
R_8: IF Sepal Length is   High   AND Sepal Width is Average THEN Class is C3
R_9: IF Sepal Length is   High   AND Sepal Width is   High   THEN Class is C3

It is possible to find more accurate FRBCSs for this problem in the fuzzy literature, but the objective of this example is to illustrate the creation and analysis of fingrams in classification problems.

We will detail, step by step, the different phases involved in the construction of fingrams, as they were described in Sec. III:

1) **Network generation**: With the problem instances, fuzzy partitions, and fuzzy rules previously presented (all of them illustrated in Fig. 6), we have generated a 9x9 matrix that represents the co-firing degrees. Fig. 7(a) shows that matrix with inconsistencies remarked by (*).

2) **Network scaling**: We have checked different scaling methods. First, Pathfinder is applied to the original network, obtaining a pruned matrix. Second, a hybrid scaling method is used to discover inconsistencies in the FRBCS. For that, non-inconsistent links are firstly thresholded in the original network and afterwards Pathfinder is enforced.

3) **Network drawing**: Kamada-Kawai's spring layout is selected for plotting the previously generated and scaled networks, considering the additional visualization capabilities in Sec. III-B.

The first graph, the complete non-scaled fingram (Fig. 7(b)), shows the relations among rules displayed in a perfect grid, thanks to the dimensions and partitions considered.

A simple comparison between Figs. 6 and 7 makes easy to appreciate the correspondence among the node sizes and how populated the input space regions are. For example, rule R_5 covers the central region with the largest number of instances, while rule R_9 covers the smallest amount of data samples.

In addition, the node layout perfectly reflects the relation among co-fired rules, with a central fuzzy rule (R_5) that highly overlaps with the rest, thus producing non-inconsistencies (green links) or potential inconsistencies (red links).

By carefully analyzing the dataset, a high volume of instances can be appreciated in the regions of fuzzy rules R_4 and R_5 (see Fig. 6). This can also be observed in the fingram (Fig 7), which assigns a high value (0.794) to the connection between these two rules. In addition, the highest link weight (0.897) is related to rules R_3 and R_6 as most instances they cover are located close to the border between the input space regions they handle. Notice that, a quick study of the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

11

	R1	R2	R3	R4	R5	R6	R7	R8	R9
R1	1.000	0.743(*)	0.000	0.715	0.562	0.000	0.000	0.000	0.000
R2	0.743(*)	1.000	0.597	0.531(*)	0.763(*)	0.536	0.000	0.000	0.000
R3	0.000	0.597	1.000	0.000	0.461(*)	0.897	0.000	0.000	0.000
R4	0.715	0.531(*)	0.000	1.000	0.794	0.000	0.644(*)	0.542(*)	0.000
R5	0.562	0.763(*)	0.461(*)	0.794	1.000	0.517(*)	0.517(*)	0.603(*)	0.221(*)
R6	0.000	0.536	0.897	0.000	0.517(*)	1.000	0.000	0.168(*)	0.416(*)
R7	0.000	0.000	0.000	0.644(*)	0.517(*)	0.000	1.000	0.842	0.000
R8	0.000	0.000	0.000	0.542(*)	0.603(*)	0.168(*)	0.842	1.000	0.405
R9	0.000	0.000	0.000	0.000	0.221(*)	0.416(*)	0.000	0.405	1.000

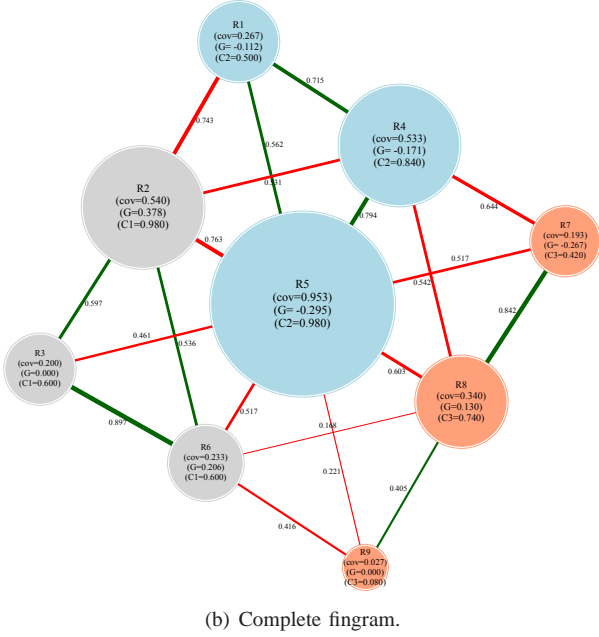(a) Co-firing matrix.

(b) Complete fingram.

Fig. 7.   Classification example: Original social network.

input space can be done, even in multi-dimensional problems, following the same sketched procedure.

The use of Pathfinder algorithm yields a pruned fingram (Fig. 8) that keeps the most salient links of the original network, what highlights those rules which are fired simultaneously a larger number of times. This fingram shows that rule R_2 is quite important due to the high interrelations with others (producing inconsistencies with rules R_1 and R_5, and non-inconsistencies with rule R_3).
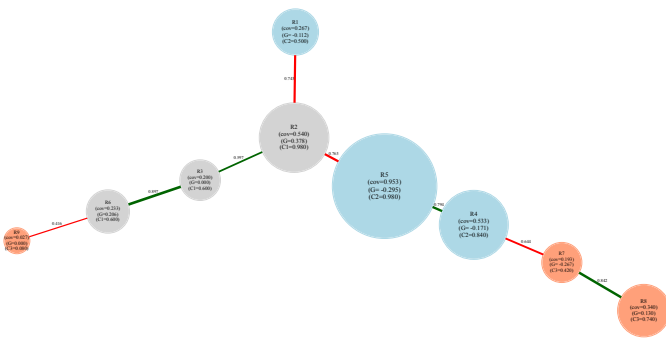

Fig. 8.   Classification example: Fingram scaled with Pathfinder.

The fingram in Fig. 9, scaled using the hybrid alternative with the aim of only keeping inconsistencies, emphasizes the main potential inconsistencies among rules, turning up those regions that do not belong clearly to a single class. Rule R_5

shows up as the main cause of conflicts. It is clear that this central rule covers most of the problem instances, and so for, it overlaps with most rules. Notice that, the input region covered by R_5 (as seen in Fig. 6) includes a large number of instances of different classes what produces these inconsistencies.
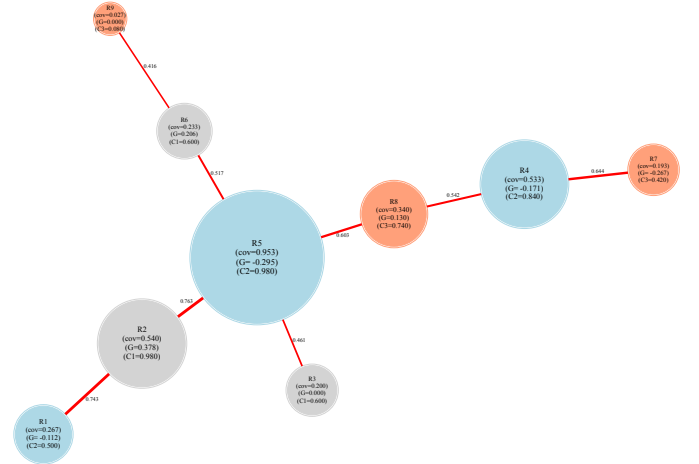

Fig. 9.   Classification example: Fingram scaled with hybrid method (Threshold + Pathfinder).

In addition, a linguistic simplification can be made from the previous FRBCS, yielding a new FRBCS with less rules but exactly the same accuracy:

R_1:	IF Sepal Length is	Low	AND Sepal Width is	Low
	THEN Class is C2			
R_{23}:	IF Sepal Length is	Low	AND Sepal Width is	NOT(Low)
	THEN Class is C1			
R_{45}:	IF Sepal Length is	Average	AND Sepal Width is	NOT(High)
	THEN Class is C2			
R_6:	IF Sepal Length is	Average	AND Sepal Width is	High
	THEN Class is C1			
R_{789}:	IF Sepal Length is	High		
	THEN Class is C3			

where R_{XY} represents the merge of original R_X and R_Y.

Fig. 10 shows the pruned fingram, created using Pathfinder, of the simplified FRBCS. As expected, it can be seen that the information associated to the new merged rules vary with respect to the original FRBCS (Fig. 7) except for rules R_1 and R_6 that keep unchanged. Nevertheless, it is remarkable how the new fingram in Fig. 10 keeps almost the same global shape of the original FRBCS (Fig. 8). The new rule R_{23} gets the central position previously taken by rule R_2 distributing the remaining rules in three branches.

It can also be appreciated that rules R_{23} and R_{45} cover all the problem instances of their output classes ($C1 = 1.000$ in R_{23} and $C2 = 1.000$ in R_{45}). So for, it is interesting to test the behavior of the system without the rest of rules of output classes C1 and C2 (R_6 and R_1, respectively). With that aim, several FRBCSs are created and tested without those rules from the simplified FRBCS.

Table II summarizes the values for the quality indices in Sec. V-A before and after the linguistic simplification, but also after the elimination of R_1 and R_6. We should again remark that we are not focused on finding out the most accurate FRBCS for the tackled problem, but on exploring the opportunities fingrams offer.

As previously mentioned, the accuracy (look at MC in

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
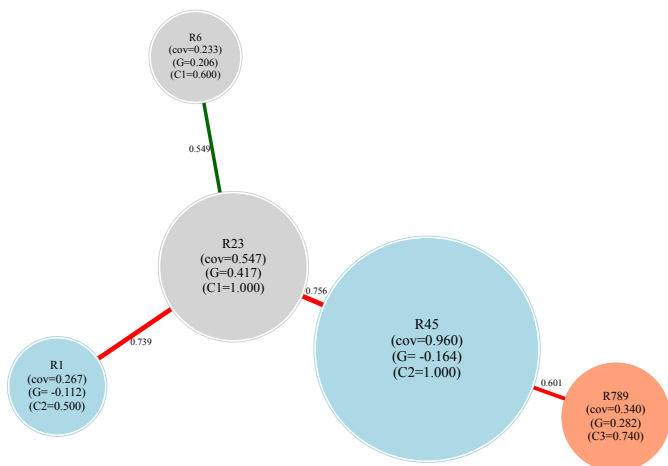
12

Fig. 10. Classification example: Fingram scaled with Pathfinder after linguistic simplification.

TABLE II
CLASSIFICATION EXAMPLE: QUALITY EVALUATION OF THE DIFFERENT FRBCSs GENERATED

Quality index	Original FRBCS	Simplified FRBCS	R_1 removal	R_6 removal	R_1 & R_6 removal
MC	0.32	0.32	**0.313**	0.34	0.333
NR	9	5	**4**	4	3
TRL	18	9	**7**	7	5
ARL	2	1.8	**1.75**	1.75	1.667
AFR	3.287	2.347	**2.08**	2.113	1.847
AFD	0.476	0.546	**0.54**	0.544	0.538
$COFCI$	0.643	0.782	**0.818**	0.815	0.86

Table II) keeps the same after applying the linguistic simplification, but the interpretability indices improve with the reduction of rules. The elimination of R_6 produces more classification errors indicating that R_6 is the winner rule for some problem instances of class C2. Only the FRBCS produced from eliminating R_1, highlighted in boldface in the table, improves both the accuracy and the interpretability of the linguistically simplified FRBCS.

C. Generation of fingrams in a small-size regression problem. Analysis of specificity and generality.

This example illustrates the use of fingrams in regression problems. An electrical network distribution problem in northern Spain [60] is analyzed. The system aims to estimate the length of the low voltage line installed in a certain village. The problem has two input variables (the *population of the village* and its *radius*) and one output variable (the *total length of the installed line*). Real data of 495 villages are available. The training set contains 396 elements and the test set includes 99 elements, randomly selected from the whole sample, taken from KEEL dataset repository[6]. Here we will use just the training set to create the fingrams thus being able to compare the accuracy results with previous works.

First of all, the problem variables are partitioned as shown in Fig. 11. The partitions of the input variables (INHABITANTS and DISTANCE) are tuned to improve the performance, while

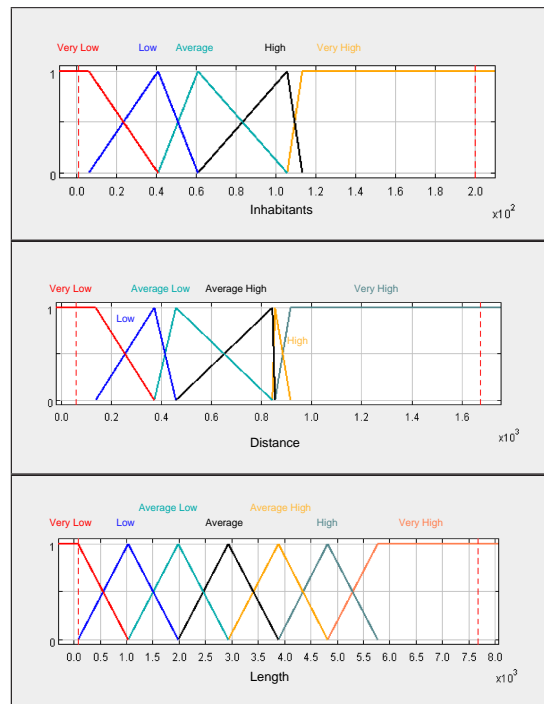[6]*http://sci2s.ugr.es/keel/datasets.php*

Fig. 11. Regression example: Fuzzy partitions for the electrical distribution problem.

the output variable is partitioned homogeneously covering the interest range, i.e. the range where problem instances are located. Using these fuzzy partitions along with FPA[7] the following set of rules is generated:

	IF		Distance is	Very Low
R_1:				
	THEN Length is	Very Low		
R_2:	IF Inhabitants is (Very Low OR Low OR Average)	AND Distance is		Low
	THEN Length is	Low		
R_3:	IF Inhabitants is	Very Low	AND Distance is	Average Low
	THEN Length is	Low		
R_4:	IF Inhabitants is	(Low OR Average)	AND Distance is	Average Low
	THEN Length is	Average Low		
R_5:	IF Inhabitants is	High	AND Distance is	Low
	THEN Length is	Average Low		
R_6:	IF Inhabitants is	(Very Low OR Low)	AND Distance is	Average High
	THEN Length is	Average		
R_7:	IF Inhabitants is	Very High	AND Distance is	Average Low
	THEN Length is	Average		
R_8:	IF Inhabitants is	Average	AND Distance is (Average High OR High)	
	THEN Length is	Average High		
R_9:	IF Inhabitants is	Very High	AND Distance is	Average High
	THEN Length is	High		
R_{10}:	IF Inhabitants is	Very High	AND Distance is	High
	THEN Length is	Very High		

This FRBS exhibits a good accuracy ($MSE = 130,046$), similar to the one obtained in [65] ($MSE = 133,763$). Anyway, we should again remind that we are not focused on finding the most accurate FRBS for the tackled problem. Our target is showing the utility of fingrams in the context of a real-world regression problem.

As explained previously in Sec. III-B, the output of each fuzzy rule will be reflected in the color of the nodes. From dark to light the node colors represent a range from low to high values. So for, the output label "VERY LOW" will be represented by the darkest node while "VERY HIGH" corresponds to the lightest one close to white. Naturally, the system will have relations among close labels and close colors, and when nodes of quite different darkness are related the expert should focus her/his attention on them.

[7]FPA can be used for classification and regression problems. Other fuzzy modeling methods can be used for regression problems, as [61]–[64].
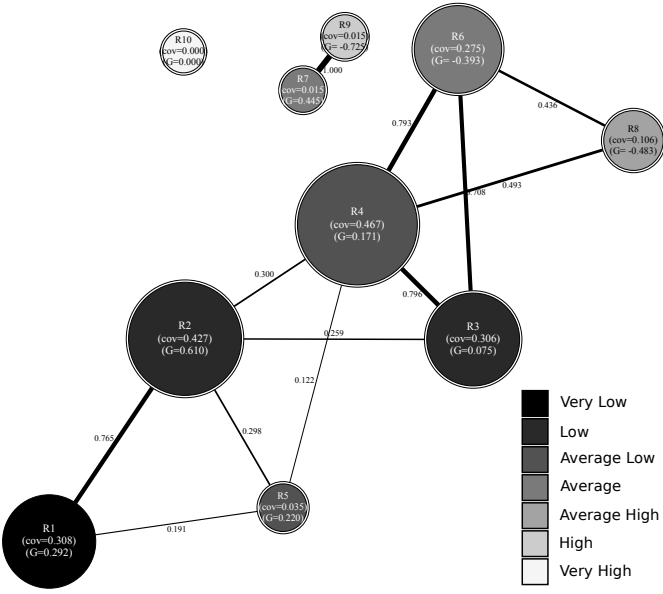
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

13

Fig. 12. Regression example: Complete fingram for the electrical distribution problem.

Fig. 13. Regression example: Fingram scaled with Pathfinder.

Fig. 12 shows the non-pruned fingram related to the inference process on the FRBS previously presented. It can be seen that the two dimensions allow the fingram to spread the nodes in a grid, relating close outputs, i.e. the evolution of darkness of the nodes is mapped smoothly. Rules R_2 and R_4 are quite general, covering almost half of the problem instances. Contrary, rules R_5, R_7, R_9 and R_{10} cover a small amount of problem instances, thus being very specific. Moreover, it is easily appreciated that rule R_{10} does not cover any example[8] ($cov = 0$), and thus it can be eliminated without any accuracy loss. In addition, all rules but R_1 have two antecedents, as it is appreciated in the single-line border of the nodes.

The fingram analysis lets us discover a special relation between rules R_7 and R_9 that appear isolated in a group, composing a kind of "fuzzy rule cluster" in a specific problem domain region. They cover some examples that no other rule covers. Moreover, they cover exactly the same examples (the related link takes value 1.0) but having different outputs. Even more, rule R_9 has a negative goodness, -0.725, so for it is a candidate to be removed, changing, if necessary, the output of R_7. An analysis of these rules must be achieved to avoid this kind of behavior. Notice that only looking R_7 and R_9 at linguistic level is not enough for detecting this kind of potential problems, but our fingram-based analysis methodology allows us to quickly identify them.

Fig. 13 shows the pruned network corresponding to the fingram scaled with Pathfinder. It emphasizes a high relation among rules R_3, R_4, and R_6. This interrelation suggests merging the three rules in a single one. To do so, a new rule, R_{346}, is constructed from R_3, R_4, and R_6 in an expert way. The antecedents of all these rules are combined and the output is taken from the middle term. This is done just as an example, and a more complex process, testing the

alternatives, could be done.

R_3:	IF Inhabitants is Very Low	AND Distance is Average Low	THEN Length is Low
R_4:	IF Inhabitants is (Low OR Average)	AND Distance is Average Low	THEN Length is Average Low
R_6:	IF Inhabitants is (Very Low OR Low)	AND Distance is Average High	THEN Length is Average
R_{346}:	**IF Inhabitants is (Very Low OR Low OR Average)**	**AND Distance is (Average Low OR Average High)**	**THEN Length is Average Low**

We will develop the proposed changes in a sequential fashion (i.e., first removing R_{10}, then removing R_9, and finally merging R_3, R_4, and R_6) and check how they affect the resulting FRBS accuracy and interpretability (as detailed in Table III).

TABLE III
REGRESSION EXAMPLE: QUALITY EVALUATION OF THE GENERATED FRBSS

Quality index	Original FRBS	R_{10} removal	R_9 removal	R_3-R_4-R_6 fusion
MSE	130,046	130,046	**125,511**	155,838
NR	10	9	**8**	6
TRL	19	17	**15**	11
ARL	1.9	1.889	**1.875**	1.83
AFR	2.463	2.463	**2.446**	1.695
$COFCI$	0.971	0.971	**0.974**	0.981

Analyzing these results we can conclude that the removal of R_{10} does not change the behavior of the system because, as mentioned, it does not cover any problem instance. Thus, MSE, AFR, and $COFCI$ remain the same while the interpretability indices related to transparency (NR, TRL, and ARL) are improved. However, deleting the rule R_9 simplifies the FRBS improving both accuracy (MSE decreases) and interpretability (all the considered interpretability indices get better values). The new fingram resulting from these two eliminations can be observed in Fig. 14. Finally, although the fusion of R_3, R_4, and R_6 reduces the accuracy of the FRBS, it could still be a good option to get a more compact and understandable FRBS (notice that, all the interpretability indices are clearly improved). Besides, a more elaborated rule

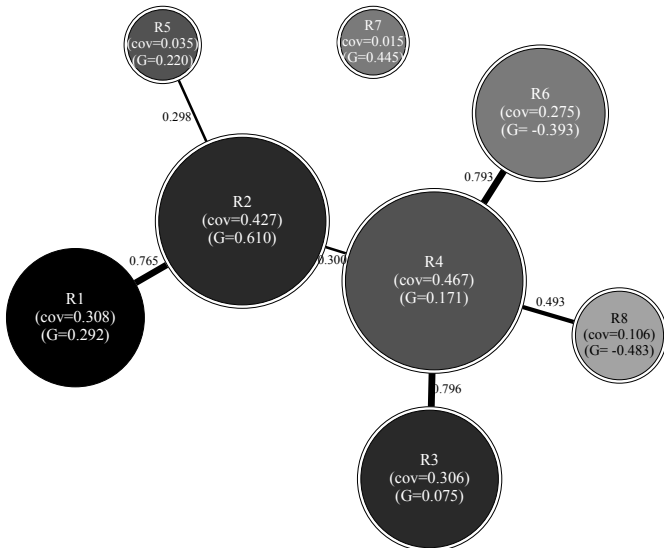fusion mechanism could be considered by the expert to reduce the accuracy loss.

Fig. 14.   Regression example: Fingram scaled of the best simplified FRBS.

VI. CONCLUSIONS AND FUTURE WORKS

This paper has introduced fingrams as a new powerful methodology for exploratory analysis of fuzzy rule bases. A brief overview of the possibilities that fingrams offer, for both design and analysis of fuzzy systems, has been illustrated through some examples. As it is a novel proposal, some of the potential uses are just outlined, opening the door to new alternatives and developments.

In the future we will extensively validate and extend the methodology. For instance, we plan to look for asymmetrical co-firing metrics able to yield additional information about consistency, generality, and/or specificity of rules.

The future of this methodology is very promising, with several applications to design or improve fuzzy systems. The human-centric simplification of a FRBS by means of the elimination or modification of rules could be done after analyzing the resulting graphs. The detection of rules that do not cover any example is very easy by just looking fingrams at first sight. Rules that have a low overlapping with others can be detected to proceed as desired, building, maybe, more general rules.

A basic simplification procedure may consist of finding and removing those non-relevant rules normally located at the periphery of the graph. Moreover, by carefully looking at fingrams we can first set a ranking of rules according to their relevance and then run a linguistic simplification procedure like the one proposed in [55].

A first software package for fingrams generation and analysis is already implemented [66] as part of the GUAJE tool, freely downloadable as open source software at *http://www.softcomputing.es/guaje*.

REFERENCES

[1] J. M. Alonso, L. Magdalena, and G. González-Rodríguez, "Looking for a good fuzzy system interpretability index: An experimental approach," *International Journal of Approximate Reasoning*, vol. 51, pp. 115–134, 2009.

[2] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning," *Parts I, II, and III. Information Sciences*, vol. 8, 8, 9, pp. 199–249, 301–357, 43–80, 1975.

[3] D. Nauck and R. Kruse, "Obtaining interpretable fuzzy classification rules from medical data," *Artificial Intelligence in Medicine*, vol. 16, pp. 149–169, 1999.

[4] L. A. Zadeh, "Outline of a new approach to the analysis of complex systems and decision processes," *IEEE Transactions on Systems Man, and Cybernetics*, vol. 3, pp. 28–44, 1973.

[5] E. H. Mamdani, "Application of fuzzy logic to approximate reasoning using linguistic synthesis," *IEEE Transactions on Computers*, vol. 26, no. 12, pp. 1182–1191, 1977.

[6] J. V. de Oliveira, "Semantic constraints for membership function optimization," *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, vol. 29, no. 1, pp. 128–138, 1999.

[7] C. Mencar and A. M. Fanelli, "Interpretability constraints for fuzzy information granulation," *Information Sciences*, vol. 178, no. 24, pp. 4585–4618, 2008.

[8] S. M. Zhou and J. Q. Gan, "Low-level interpretability and high-level interpretability: A unified view of data-driven interpretable fuzzy system modelling," *Fuzzy Sets and Systems*, vol. 159, no. 23, pp. 3091–3131, 2008.

[9] S. Guillaume, "Designing fuzzy inference systems from data: An interpretability-oriented review," *IEEE Transanctions on Fuzzy Systems*, vol. 9, no. 3, pp. 426–443, 2001.

[10] J. Casillas, O. Cordón, F. Herrera, and L. Magdalena, Eds., *Interpretability issues in fuzzy modeling*, ser. Studies in Fuzziness and Soft Computing.   Springer-Verlag, Heidelberg, 2003, vol. 128.

[11] M. J. Gacto, R. Alcalá, and F. Herrera, "Integration of an index to preserve the semantic interpretability in the multiobjective evolutionary rule selection and tuning of linguistic fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 3, pp. 515–531, 2010.

[12] C. Mencar, C. Castiello, R. Cannone, and A. M. Fanelli, "Interpretability assessment of fuzzy knowledge bases: A cointension based approach," *International Journal of Approximate Reasoning*, vol. 52, no. 4, pp. 501–518, 2011.

[13] G. A. Miller, "The magical number seven, plus or minus two: Some limits on our capacity for processing information," *The Psychological Review*, vol. 63, no. 2, pp. 81–97, 1956.

[14] J. M. Alonso, O. Cordón, A. Quirin, and L. Magdalena, "Analyzing interpretability of fuzzy rule-based systems by means of fuzzy inference-grams," in *Proceedings of World Conference on Soft Computing*, San Francisco, 2011, pp. 181–185.

[15] D. J. de Solla Price, *Little science, big science*.   Columbia University Press New York, 1986.

[16] J. Scott, *Social Network Analysis: A Handbook (2nd edition)*.   Sage Publications, 2000.

[17] S. Wasserman and K. Faust, *Social Network Analysis: Methods And Applications (Structural Analysis in the Social Sciences)*.   Cambridge University Press, 1994.

[18] B. Vargas-Quesada and F. Moya-Anegón, *Visualizing the structure of science*.   Springer-Verlag, 2007.

[19] J. M. Alonso and L. Magdalena, "Generating understandable and accurate fuzzy rule-based systems in a java environment," in *Lecture Notes in Artificial Intelligence - 9th International Workshop on Fuzzy Logic and Applications*, vol. LNAI6857.   Springer-Verlag, 2011, pp. 212–219.

[20] M. J. Gacto, R. Alcalá, and F. Herrera, "Interpretability of linguistic fuzzy rule-based systems: An overview of interpretability measures," *Information Sciences*, vol. 181, no. 20, pp. 4340–4360, 2011.

[21] E. H. Ruspini, "A new approach to clustering," *Information and Control*, vol. 15, no. 1, pp. 22–32, 1969.

[22] P. Fazendeiro, J. V. de Oliveira, and W. Pedrycz, "A multiobjective design of a patient and anaesthetist-friendly neuromuscular blockade controller," *IEEE Transactions on Biomedical Engineering*, vol. 54, no. 9, pp. 1667–1678, 2007.

[23] R. Alcalá, J. Alcalá-Fdez, and F. Herrera, "A proposal for the genetic lateral tuning of linguistic fuzzy systems and its interaction with rule selection," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 4, pp. 616–635, 2007.

[24] J. M. Alonso, L. Magdalena, and O. Cordón, "Embedding HILK in a three-objective evolutionary algorithm with the aim of modeling highly interpretable fuzzy rule-based classifiers," in *Fourth International Workshop on Genetic and Evolutionary Fuzzy Systems (GEFS)*.   IEEE, 2010, pp. 15–20.

[25] F. Cheong and R. Lai, "Constraining the optimization of a fuzzy logic controller using an enhanced genetic algorithm," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 30, no. 1, pp. 31–46, 2000.

[26] A. Márquez, F. Márquez, and A. Peregrín, "A multi-objective evolutionary algorithm with an interpretability improvement mechanism for linguistic fuzzy systems with adaptive defuzzification," in *IEEE International Conference on Fuzzy Systems*, 2010, pp. 277–283.

[27] C. Moraga, "An essay on the interpretability of Mamdani systems," in *Combining Experimentation and Theory: A Hommage to Abe Mamdani*, ser. Studies in Fuzziness and Soft Computing.   Springer, 2012, vol. 271, pp. 61–72.

[28] H. Ishibuchi, Y. Kaisho, and Y. Nojima, "Designing fuzzy rule-based classifiers that can visually explain their classification results to human users," in *3rd International Workshop on Genetic and Evolving Fuzzy Systems (GEFS)*, 2008, pp. 5–10.

[29] ——, "A visual explanation system for explaining fuzzy reasoning results by fuzzy rule-based classifiers," in *NAFIPS*, 2008, pp. 1–6.

[30] ——, "Complexity, interpretability and explanation capability of fuzzy rule-based classifiers," in *IEEE International Conference on Fuzzy Systems*, 2009, pp. 1730–1735.

[31] B. Pham and R. Brown, "Analysis of visualisation requirements for fuzzy systems," in *International conference on computer graphics and interactive techniques (GRAPHITE03)*, 2003, pp. 181–187.

[32] M. R. Berthold and L. O. Hall, "Visualizing fuzzy points in parallel coordinates," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 3, pp. 369–374, 2003.

[33] T. R. Gabriel, K. Thiel, and M. R. Berthold, "Rule visualization based on multi-dimensional scaling," in *IEEE Interational Conference on Fuzzy Systems*, 2006, pp. 66–71.

[34] A. G. Evsukoff, S. Galichet, B. S. L. P. de Lima, and N. F. F. Ebecken, "Design of interpretable fuzzy rule-based classifiers using spectral analysis with structure and parameters optimization," *Fuzzy Sets and Systems*, vol. 160, no. 7, pp. 857–881, 2009.

[35] A. G. Evsukoff, A. Branco, and S. Galichet, "Intelligent data analysis and model interpretation with spectral analysis fuzzy symbolic modeling," *International Journal of Approximate Reasoning*, vol. 52, no. 6, pp. 728–750, 2011.

[36] J. Casillas and F. Martínez-López, "Mining uncertain data with multiobjective genetic fuzzy systems to be applied in consumer behaviour modelling," *Expert Systems with Applications*, vol. 36, no. 2, pp. 1645–1659, 2009.

[37] C. Chen and S. Morris, "Visualizing evolving networks: Minimum spanning trees versus Pathfinder networks," in *IEEE Symposium on Information Visualization*, 2003, pp. 67–74.

[38] M. Zizi and M. Beaudouin-Lafon, "Accessing hyperdocuments through interactive dynamic maps," in *Proceedings of the ACM European conference on Hypermedia technology*, 1994, pp. 126–135.

[39] S. Noel, C. H. Chu, and V. Raghavan, "Visualization of document cocitation counts," in *IEEE Symposium on Information Visualisation*, 2002, pp. 691–696.

[40] D. W. Dearholt and R. W. Schvaneveldt, "Properties of Pathfinder networks," in *Pathfinder associative networks: Studies in knowledge organization*, R. Schvaneveldt, Ed.   Ablex Publishing Corporation, 1990, pp. 1–30.

[41] R. W. Schvaneveldt, F. T. Durso, and D. W. Dearholt, "Network structures in proximity data," *The psychology of learning and motivation: Advances in research and theory*, vol. 24, pp. 249–284, 1989.

[42] G. di Battista, P. Eades, R. Tamassia, and I. Tollis, *Graph drawing: Algorithms for the visualization of graphs*.   Upper Saddle River, N.J: Prentice Hall, 1998.

[43] S. G. Kobourov, "Force-directed drawing algorithms," in *Handbook of Graph Drawing and Visualization*, R. Tamassia, Ed.   CRC Press, 2012, ch. 12.

[44] T. Kamada and S. Kawai, "An algorithm for drawing general undirected graphs," *Information Processing Letters*, vol. 31, no. 1, pp. 7–15, 1989.

[45] T. M. J. Fruchterman and E. M. Reingold, "Graph drawing by force-directed placement," *Software: Practice and experience*, vol. 21, no. 11, pp. 1129–1164, 1991.

[46] E. Serrano, A. Quirin, J. Botia, and O. Cordón, "Debugging complex software systems by means of Pathfinder networks," *Information Sciences*, vol. 180, no. 5, pp. 561–583, 2010.

[47] F. Moya-Anegón, B. Vargas-Quesada, V. Herrero-Solana, Z. Chinchilla-Rodríguez, E. Corera-Álvarez, and F. J. Muñoz-Fernández, "A new technique for building maps of large scientific domains based on the cocitation of classes and categories," *Scientometrics*, vol. 61, no. 1, pp. 129–145, 2004.

[48] F. Moya-Anegón, B. Vargas-Quesada, Z. Chinchilla-Rodríguez, E. Corera-Álvarez, F. J. Muñoz-Fernández, and V. Herrero-Solana, "Visualizing the marrow of science," *Journal of the American Society for Information Science and Technology*, vol. 58, no. 14, pp. 2167–2179, 2007.

[49] G. Salton and D. Bergmark, "A citation study of computer science literature," *IEEE Transactions on Professional Communication*, vol. 22, pp. 146–158, 1979.

[50] A. Quirin, O. Cordón, J. Santamaría, B. Vargas-Quesada, and F. Moya-Anegón, "A new variant of the Pathfinder algorithm to generate large visual science maps in cubic time," *Information Processing and Management*, vol. 44, no. 4, pp. 1611–1623, 2008.

[51] E. R. Gansner and S. C. North, "An open graph visualization system and its applications to software engineering," *Software Practice and Experience*, vol. 30, no. 11, pp. 1203–1233, 2000.

[52] A. Frank and A. Asuncion, "UCI machine learning repository," 2010. [Online]. Available: http://archive.ics.uci.edu/ml

[53] J. M. Alonso, L. Magdalena, and S. Guillaume, "HILK: A new methodology for designing highly interpretable linguistic knowledge bases using the fuzzy logic formalism," *International Journal of Intelligent Systems*, vol. 23, no. 7, pp. 761–794, 2008.

[54] J. M. Alonso, D. P. Pancho, O. Cordón, A. Quirin, and L. Magdalena, "Social network analysis of co-fired fuzzy rules," in *Soft Computing: State of the Art Theory and Novel Applications*, R. R. Yager, A. M. Abbasov, M. Reformat, and S. N. Shahbazova, Eds.   Springer, 2013, pp. 113–128.

[55] J. M. Alonso and L. Magdalena, "HILK++: An interpretability-guided fuzzy modeling methodology for learning readable and comprehensible fuzzy rule-based classifiers," *Soft Computing*, vol. 15, no. 10, pp. 1959–1980, 2011.

[56] H. Bustince, M. Pagola, R. Mesiar, E. Hullermeier, and F. Herrera, "Grouping, overlap, and generalized bientropic functions for fuzzy modeling of pairwise comparisons," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 3, pp. 405–415, 2012.

[57] M. Kumar, S. Neubert, S. Behrendt, A. Rieger, M. Weippert, N. Stoll, K. Thurow, and R. Stoll, "Stress monitoring based on stochastic fuzzy analysis of heartbeat intervals," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 4, pp. 746–759, 2012.

[58] A. Lemos, W. Caminhas, and F. Gomide, "Multivariable gaussian evolving fuzzy modeling system," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 1, pp. 91–104, 2011.

[59] P. Y. Glorennec, *Algorithmes d'apprentissage pour systèmes d'inférence floue*.   Hermès, 1999.

[60] O. Cordón, F. Herrera, and L. Sánchez, "Solving electrical distribution problems using hybrid evolutionary data analysis techniques," *Applied Intelligence*, vol. 10, no. 1, pp. 5–24, 1999.

[61] Z. Deng, K.-S. Choi, F.-L. Chung, and S. Wang, "Scalable TSK fuzzy modeling for very large datasets using minimal-enclosing-ball approximation," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 2, pp. 210–226, 2011.

[62] M. H. Mashinchi, M. A. Orgun, M. Mashinchi, and W. Pedrycz, "A tabu–harmony search-based approach to fuzzy linear regression," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 3, pp. 432–448, 2011.

[63] C.-F. Juang and C.-D. Hsieh, "A fuzzy system constructed by rule generation and iterative linear SVR for antecedent and consequent

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

16

parameter optimization," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 2, pp. 372–384, 2012.

[64] R. Alcalá, M. J. Gacto, and F. Herrera, "A fast and scalable multi-objective genetic fuzzy system for linguistic fuzzy modeling in high-dimensional regression problems," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 4, pp. 666–681, 2011.

[65] O. Cordón, F. Herrera, L. Magdalena, and P. Villar, "A genetic learning process for the scaling factors, granularity and contexts of the fuzzy rule-based system data base," *Information Sciences*, vol. 136, no. 1–4, pp. 85–107, 2001.

[66] J. M. Alonso, D. P. Pancho, and L. Magdalena, "Enhancing the fuzzy modeling tool GUAJE with a new module for fingrams-based analysis of fuzzy rule bases," in *IEEE International Conference on Fuzzy Systems, World Congress on Computational Intelligence*, 2012, pp. 1082–1089.

David P. Pancho received the M.S. degree in Computer Engineering from the University of Oviedo in 2010, and the M.S. degree in Soft Computing and Intelligent Data Analysis organized from the European Centre for Soft Computing and the University of Oviedo in 2011. He obtained a grant in 2008 at the European Centre for Soft Computing to work in the Unit of "Fundamentals of Soft Computing". From 2008 to 2011 he worked in the area of Computing with Words, Actions and Perceptions. From 2011 he is Ph.D. student in Computer Science at the European Centre for Soft Computing in knowledge extraction and representation in fuzzy modeling focusing in the visual representation of this kind of knowledge. He received a diploma for the best transcript in his last year at the M.S. degree in Computer Engineering, and the Best paper award of the XV Congreso Espaol de Tecnologas y Lgica Fuzzy (Huelva, 2010).

Jose M. Alonso received the M.S. and Ph.D. degrees in Telecommunication Engineering, both from the Technical University of Madrid, Spain, in 2003 and 2007, respectively. He is currently associate researcher in the "Cognitive Computing: Computing with Perceptions" Unit at the European Centre for Soft Computing (ECSC). From Feb to Oct 2012, he was postdoc researcher under JCI-2011-09839 project with the Robesafe group in the Department of Electronics at University of Alcala, Madrid, Spain. From Nov 2007 to Jan 2012 he was postdoc researcher at ECSC. He is vice-chair of the IEEE-CIS Task Force on Fuzzy Systems Software since 2011. His research interests include fuzzy modeling for control and classification problems, assessing interpretability of fuzzy systems, knowledge extraction and representation, integration of expert and induced knowledge, advance multi-sensor fusion, WiFi localization, autonomous robotic navigation in complex environments, and development of free software tools.

Arnaud Quirin is a data mining researcher with Gradiant, Vigo, Spain. His last position was in the European Centre for Soft Computing in Mieres (Spain) from 2006 to 2012. He received his M.S. degree (2002) and his Ph.D. (2005) in Computer Science from the University Louis Pasteur of Strasbourg (France) where he was a teaching and researcher assistant until September 2006. Dr. Quirin has published more than 30 peer-reviewed scientific publications, including 3 book chapters and 8 JCR-SCI-indexed journal papers, and he is a reviewer for 6 international journals. He was involved in several national projects, private contracts and one European projects, related to the application of evolutionary algorithms to image classification. He currently participates in several R&D programs in the fields of evolutionary algorithms, multi objective graph-based mining, fuzzy multi classifier systems, and genetic fuzzy algorithms.

Oscar Cordón (SM11) is Full Professor with the University of Granada (UGR), Granada, Spain. He was the founder and leader of that Universitys Virtual Learning Center between 2001 and 2005. From 2006 to 2011 he was one of the founding researchers of the European Centre for Soft Computing, Mieres, Spain, in his role of Principal Researcher, and he is still contracted as Scientific Consultant at the Centre. He has been, for more than 15 years, an internationally recognized contributor to R&D Programs in fuzzy systems, evolutionary algorithms, and ant colony optimization. He has published around 280 peer-reviewed scientific publications including a research book on genetic fuzzy systems and 66 JCR-SCI-indexed journal papers, advised 13 Ph.D. dissertations, participated in 34 research projects and contracts (coordinating 20 of them), and co-edited 8 special issues in international journals and 3 research books, and given 12 plenary talks in national and international conferences. By October 2012, his publications had received 1956 citations, carrying an h index of 23, and being included in the 1% of most-cited researchers in the world (source: Thomson Reuters's Web of Knowledge). He also has an approved international patent on an intelligent system for forensic identification that is being commercialized in Mexico. He is Associate Editor of 11 international journals and was recognized as the IEEE TRANSACTIONS ON FUZZY SYSTEMS Outstanding Associate Editor in 2008. He is a reviewer for more than 30 international journals. Since 2004, he has taken many different representative positions with Eusflat and the IEEE Computational Intelligence Society (CIS). He is currently a member of the IEEE CIS AdCom (term 20102012). Prof. Cordón received the UGR Young Researcher Career Award in 2004. In July 2010, he received the IEEE CIS Outstanding Early Career Award in its 2011 edition, the first such award conferred. In 2011, his developments on soft computing methods in forensic identification were recognized with the IFSA Award for Outstanding Applications of Fuzzy Technology and with the granting of the EUSFLAT Best Ph.D. Thesis Award to Dr. Ibañez's PhD dissertation, co-advised by Dr. Cordón.

Luis Magdalena received the M.S. and Ph.D. degrees in Telecommunication Engineering, both from the Technical University of Madrid (UPM), Spain, in 1988 and 1994, respectively. He has been an Associate Professor in Computer Science at the Department of Applied Mathematics of the UPM since 1995, where he was previously Assistant Professor since 1990. In 1998 he Co-founded the Company Daedalus (spin-off of UPM). From 2003 to 2005 he was Deputy Director (and promoter) of the Centro de Domtica Integral (Smart House Research Center), at UPM. In 2006 he joined the European Center for Soft Computing as Director General. His research interests include Soft Computing and its application, particularly in the field of robotics and automation. He has been actively involved in research projects funded by National and European programs, as well as by private companies, being project leader in more than fifteen of those projects. He is co-author of the book "Genetic Fuzzy Systems: Evolutionary Tuning and Learning of Fuzzy Knowledge Bases" (World Scientific, 2001). He is co-editor of the books "Technologies for Constructing Intelligent Systems" (two volumes, Phisica Verlag, 2002), "Interpretability Issues in Fuzzy Modeling" (Springer, Studies in Fuzziness and Soft Computing, 2003), "Accuracy Improvements in Linguistic Fuzzy Modeling" (Springer, Studies in Fuzziness and Soft Computing, 2003), and Foundations of Reasoning under Uncertainty" (Springer, Studies in Fuzziness, 2010). He has also authored over one hundred and fifty papers in books, journals and conferences, and edited special issues of different journals, being Area Editor for Engineering applications of the "International Journal of Uncertainty, Fuzziness and Knowledge Based Systems", and member of the Editorial Board or regular reviewer for over ten international journals. From 2001 to 2005 he has been the President of the "European Society for Fuzzy Logic and Technologies", and Vice-president of the International Fuzzy Systems Association from 2007 to 2011. He is Senior member of IEEE, and member of the AdCom (2011-13) and Chair of the University Curricula Sub-Committee (2010-11) of IEEE Computational Intelligence Society. He is also member of IEEE Systems, Man and Cybernetics Society. Since 2009 he is a member of the Scientific Advisory Committee of the Spanish Research Council (CSIC). He has been member of the Program Committee of more than thirty National and International Conferences, serving as General, Program or Publicity Chair for some of them.