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A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets

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ABSTRACT

The complexity and impact of many real world decision making problems lead to the necessity of considering multiple points of view, building group decision making problems in which a group of experts provide their preferences to achieve a solution. In such complex problems uncertainty is often present and although the use of linguistic information has provided successful results in managing it, these are sometimes limited because the linguistic models use single-valued and predefined terms that restrict the richness of freely eliciting the preferences of the experts. Usually, experts may doubt between different linguistic terms and require richer expressions to express their knowledge more accurately. However, linguistic group decision making approaches do not provide any model to make more flexible the elicitation of linguistic preferences in such hesitant situations. In this paper is proposed a new linguistic group decision model that facilitates the elicitation of flexible and rich linguistic expressions, in particular through the use of comparative linguistic expressions, close to human beings' cognitive models for expressing linguistic preferences based on hesitant fuzzy linguistic term sets and context-free grammars. This model defines the group decision process and the necessary operators and tools to manage such linguistic expressions.

1. Introduction

Decision making is a usual process for human beings and companies in different areas such as information retrieval [4], selection [8], evaluation [9], investment [17], planning [36], etc. The complexity and importance of real world decision problems make the inclusion of multiple points of view necessary, in order to achieve a solution from the knowledge provided by a group of experts. In group decision making (GDM) complexity is often caused by the uncertainty surrounding the alternatives and the experts' knowledge. Much research has been developed on GDM problems [18,30,31], being one of the main topics, the managing and modelling of uncertainty by different forms of information, used by experts to provide their preferences, such as utility vectors [5], fuzzy preference relations [21], linguistic variables [14], interval values [15], multiplicative preference relations [6], hesitant fuzzy set [39], etc. Our research deals with GDM problems defined in linguistic contexts because the use of linguistic information by experts is quite common in problems with a high degree of uncertainty [24] and has provided reliable and successful results in different GDM problems [1,43].

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In spite of previous results obtained by linguistic approaches in decision making, different authors have pointed out some necessary improvements of such approaches [20,37]:

- Most linguistic decision approaches deal with linguistic terms defined a priori, preventing other choices to express preferences in a richer way.
- Additionally, experts are restricted to providing their preferences by use of just one term, which may not always reflect exactly what they mean.

To cope with previous issues, different proposals have been introduced in the literature. Wang and Hao [40] proposed the use of proportional linguistic 2-tuple that provides the possibility of using proportions of two consecutive linguistic terms. Ma et al. [20] developed another model to increase the flexibility of linguistic preference expressions by joining different single terms in a new synthesized term, without any fixed rule for such a conversion, which is used by a fuzzy model and with measures of consistency and determinacy. Also Tang et al. [37] introduced a linguistic model that is able to manage linguistic expressions built by logical connectives ($\neg$, $\vee$, $\wedge$, $\rightarrow$) and fuzzy relations that measure the similarity between any two linguistic terms. Previous models provide a way of expressing richer expressions than single linguistic terms but they are either far away of common language used by experts in GDM problems or are not systematically defined. Furthermore, these models have been applied to multi-criteria decision making problems in which experts express their preferences by means of preference vectors.

The use of hesitant fuzzy linguistic term sets (HFLTS) [33] improves the previous linguistic approaches [20,37,40] in their aim of achieving the previous improvements pointed out. Because it provides experts a greater flexibility in eliciting linguistic preferences through the use of context-free grammars that fix the rules to build flexible linguistic expressions to express preferences, in particular it allows the use of comparative linguistic expressions. However, the application of a linguistic modeling to GDM is not straightforward, as can be seen from previous approaches which have not been applied to GDM problems yet. Such difficulty derives from the use of linguistic preference relations to manage experts' preferences.

Therefore, in this paper we present a new linguistic GDM model to achieve these improvements. It deals with comparative linguistic expressions that are similar to those used by experts in real world decision making problems based on HFLTS and context-free grammars, which support experts' preference elicitation in uncertain group decision making situations in which they require rich expressions in order to be able to express their preferences even when they hesitate among different terms.

This novel GDM model extends the classical GDM solving process scheme [34], based on an *Aggregation* phase that combines the experts' preferences, and an *Exploitation* phase that obtains a solution set of alternatives for the GDM problem. This is achieved by adding phases that manage the gathering of comparative linguistic expressions, their transformation into linguistic intervals modeled by HFLTS, and the necessary tools to accomplish the processes of computing with words (CWW) [25,26] in a simple and accurate way.

The remainder of the paper is structured as follows: Section 2 reviews the scheme of a GDM problem and makes a brief introduction to the fuzzy linguistic approach and the 2-tuple linguistic representation model used for carrying out the processes of CWW in the linguistic decision solving process. Section 3 deals with the elicitation of linguistic expressions based on hesitant fuzzy linguistic term sets and context-free grammars. Section 4 presents a novel GDM model dealing with comparative linguistic expressions and Section 5 solves a GDM problem by using the proposed model. Finally, some conclusions are pointed out.

2. Linguistic group decision making and computing with words

The aim of this paper is to introduce a linguistic group decision making model capable of dealing with comparative linguistic expressions as preference assessments in hesitant decision situations. Before presenting this model, in this section we briefly review some basic concepts regarding GDM, linguistic information and computing with words, which are necessary to understand our proposal.

2.1. Group decision making

Group decision making is defined as a decision situation where two or more experts, who have their own knowledge and attitudes regarding the decision problem, take part and provide their preferences to reach a collective decision [18]. The need for multiple views is quite common in complex and in organizational decision situations [2,7,19,35].

A GDM solving process applies a *selection process* to achieve a collective solution that obtains the best alternative or subset of alternatives according to experts' preferences. However, sometimes the aim of GDM is not to achieve the best solution, but rather to reach a satisfactory solution for all experts involved. In the latter situation, differences among experts are settled by negotiation, namely consensus reaching processes [28,30]. This paper focuses on *selection processes* for GDM, because they are always necessary, even for obtaining satisfactory solutions after the negotiation process, whose necessity is problem dependent.

Formally, a GDM problem is defined as a decision situation in which two or more experts, $E = \{e_1, \ldots, e_m\}$ $(m \geqslant 2)$, express their preferences over a finite set of alternatives, $X = \{x_1, \ldots, x_n\}$ $(n \geqslant 2)$, to obtain a solution set of alternatives for the decision problem [16]. Usually, each expert e_k, provides her/his preferences on X by means of a preference relation P^k, μ_{P^k} $X \times X \to D$:

$$P^k = \begin{pmatrix} p_{11}^k & \cdots & p_{1n}^k \\ \vdots & \ddots & \vdots \\ p_{n1}^k & \cdots & p_{nn}^k \end{pmatrix}$$

where each assessment, $\mu_{P^k}(x_i, x_j) = p_{ij}^k$, represents the *degree of preference* of the alternative x_i over x_j according to expert e_k. The comparison of two alternatives characterized by preference relations has been focus of interest and deeply researched in the literature [10,29].

Typically, a selection process for GDM [34] consists of two main phases (see Fig. 1).

- *Aggregation phase:* this combines the experts' preference relations by using aggregation operators to obtain a collective preference matrix, which represents the preferences provided by all experts participating in the decision problem.
- *Exploitation phase:* this selects the best alternative(s) to solve the decision problem from the collective preferences obtained in the previous phase. To do so, it may use a choice function that assigns a choice degree for each alternative [11].

2.2. Linguistic information

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, and thus, the use of a linguistic approach is necessary. A common approach to model the linguistic information is the fuzzy linguistic approach [46], which uses the fuzzy sets theory [45] to model the linguistic information. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables. In order to model linguistically the information, we have to choose the appropriate linguistic descriptors for the linguistic term set and their semantics. There are different approaches to such selections [12,22]. We will use one that consists of supplying the term set directly, by considering all the terms distributed over a scale with a defined order [44]. For example, a set of seven terms S, could be:

$$S = \{neither, very\ low, low, medium, high, very\ high, absolute\}$$

The semantics of the terms are represented by fuzzy numbers defined in the interval [0, 1], described by membership functions [38].

Because of our goal is to present a GDM model in which experts use linguistic expressions to provide their preferences over the set of alternatives, the general scheme of a GDM problem shown in Fig. 1 should be extended to manage linguistic information. According to Herrera and Herrera-Viedma in [12], the solution scheme of a linguistic decision making problem should be formed by the following steps (see Fig. 2):

- *The choice of the linguistic term set with its semantics*. This establishes the linguistic descriptors that the experts will use to provide their preferences about the alternatives regarding their knowledge and experience.
- *The choice of an aggregation operator for linguistic information*. A linguistic aggregation operator is chosen to aggregate the linguistic preferences provided by experts.
- *Selection of the best alternative(s)*. This consists of selecting the best alternative or subset of alternatives. It is carried out by following the general scheme with two phases shown in Fig. 1.

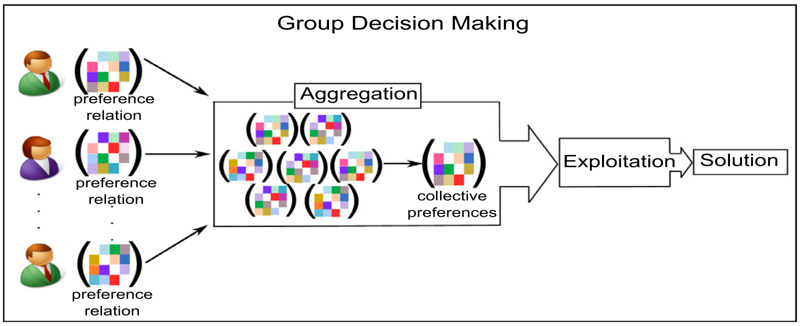

Fig. 1. General schema of a group decision making problem.

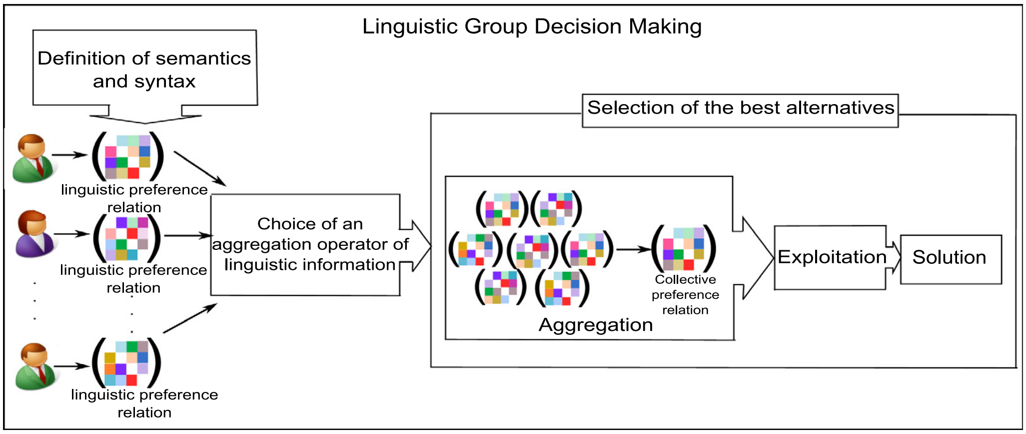

Fig. 2. Schema of a group decision making problem by using linguistic information.

Experts provide linguistic information, therefore it is necessary to apply processes of CWW in the aggregation phase by using linguistic computing models [13,40,42]. In our proposal, we will use the 2-tuple linguistic representation model introduced by Herrera and Martínez in [13] because of its precision, simplicity and interpretability in the computations with linguistic information in the aggregation phase [32].

The 2-tuple linguistic representation model extends the fuzzy linguistic approach for modeling linguistic information by introducing a new parameter called *symbolic translation*.

Definition 1 (*[13,23]*). The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between a counting of information β assessed in the interval of granularity $[0, g]$ of the term set S and the closest value in $\{0, \ldots, g\}$ which indicates the index of the closest linguistic term in S.

This concept was used to develop a linguistic representation model that represents the linguistic information by means of 2-tuples (s_i, α), $s_i \in S$, and $\alpha_i \in [-0.5, 0.5)$.

This representation model defines a set of functions to facilitate computational processes with 2-tuples [13].

Definition 2 (*[13,23]*). Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. We define the function $\Delta{:}[0, g] \to \langle S \rangle$ given by

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} i = \text{round}\,(\beta) \\ \alpha = \beta - i \end{cases} \tag{1}$$

where *round* assigns to β the integer number $i \in \{0, \ldots, g\}$ closest to β.

Remark 1. Δ is a bijective function [13] and $\Delta^{-1}{:}\langle S \rangle \to [0,g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$.

Remark 2. The conversion between a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation, $s_i \in S \Rightarrow (s_i, 0)$.

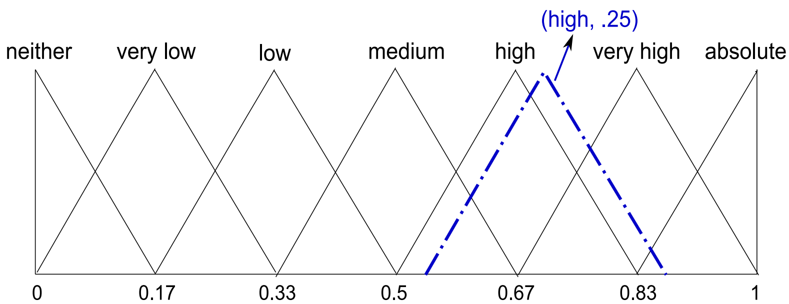

Fig. 3. A 2-tuple linguistic representation.

Let us suppose a symbolic aggregation operation over labels assessed in S = {*neither*, *very low*, *low*, *medium*, *high*, *very high*, *absolute*} that obtains as result $\beta = 4.25$, then the representation of this information by means of a 2-tuple is shown in Fig. 3.

The linguistic 2-tuple model has defined a computational model based on the functions Δ and Δ^{-1} and defines a negation operator, several aggregation functions and the comparison between two 2-tuples [13].

3. Eliciting linguistic information in hesitant situations in decision making: the use of flexible linguistic expressions

Usually experts involved in GDM problems defined in linguistic frameworks use single values to provide their preferences. For example, let's suppose a group of experts on literature $E = \{e_1, \ldots, e_m\}$ has to express its preferences on several books X = {*book_A*,*book_B*, *book_C*} in order that one of them will be published (see Fig. 4), by using the linguistic term set S, to provide the degree of preference between every two alternatives:

$$S = \{neither, \textit{very low}, low, medium, high, \textit{very high}, absolute\}$$

Expert e_1 could provide the following preference relation:

$$P^1 = \begin{pmatrix} - & high & \textit{very high} \\ medium & - & high \\ low & \textit{very low} & - \end{pmatrix}$$

with p^1_{13}, the degree of preference for *book_A* over *book_C* being *very high*. Nevertheless, when experts face decision situations in which there is a high degree of uncertainty, they often hesitate among different linguistic terms and would like to use more complex linguistic expressions, which cannot be expressed through the building of classical linguistic approaches.

Such a limitation in the expression of linguistic preferences in hesitant situations is due to the use of linguistic terms defined a priori, and because most linguistic approaches model the information by using just one linguistic term. To overcome this, different proposals [20,37,40] provide more flexible and richer expressions which can include more than one linguistic term:

- Tang and Zheng introduced in [37] a linguistic approach that allows the construction of linguistic expressions from a set of linguistic terms S, by using logical connectives ($\vee$, $\wedge$, $\neg$, $\rightarrow$), whose semantics are represented by fuzzy relations that describe the degree of similarity between linguistic terms. The set of all linguistic expressions is denoted as LE and recursively defined as:
  1. $L_i \in LE$ for $i = 1, \ldots, n$
  2. if $\theta, \phi \in LE$ then $\neg\theta, \theta \vee \phi, \theta \wedge \phi, \theta \rightarrow \phi \in LE$

Examples of linguistic expressions in LE generated from S could be:

$$\neg high \vee medium$$

$$medium \wedge high$$

- Wang and Hao proposed in [40] a linguistic modeling based on the proportions of two consecutive linguistic terms represented by 2-tuples to express linguistic expressions. A proportional 2-tuple value has a linguistic term in each 2-tuple that represents the linguistic information and a numerical value that represents its proportion in the linguistic expression.

  Examples of linguistic expressions based on proportional 2-tuples in S are:

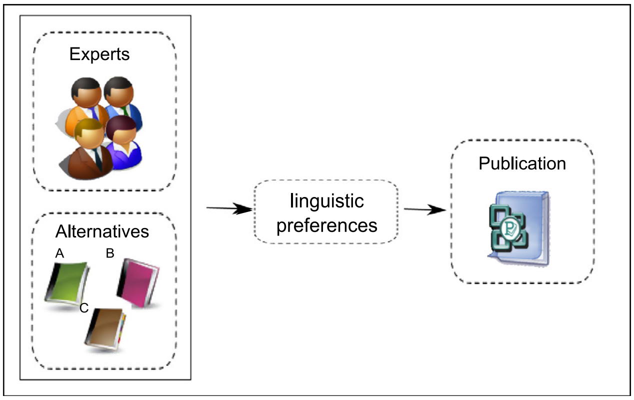

Fig. 4. Select a book to publish.

$$\{(high, 0.25), (very\ high, 0.75)\}$$

$$\{(low, 0.37), (medium, 0.63)\}$$

- Ma et al. developed in [20] a model to increase the flexibility of linguistic expressions by using multiple linguistic terms that are integrated in *synthesized comments*. There is no rule to fix the syntax of such synthesized comments obtained from multiple terms. (See Table 1)
  Because of the use of several linguistic terms in the synthesized terms, the computations on them are based on a fuzzy model and measures of determinacy and consistency of the linguistic terms.

Notwithstanding the fact that the previous approaches provide a higher flexibility with which to express linguistic expressions in hesitant decision situations, none of them is close to human beings' cognitive models or provide suitable expressions for linguistic preferences in GDM. This may be the reason none have been applied to GDM problems.

Here, we consider another possibility for generating more elaborate linguistic expressions that consists of using a context-free grammar [3]. Rodríguez et al. show in [33] how to generate comparative linguistic expressions by using a context-free grammar. Depending on the specific problem, the context-free grammar can generate different linguistic expressions. Given that our aim is to deal with hesitant situations in GDM, we have considered a similar but extended context-free grammar to that defined in [33], because it may generate comparative linguistic expressions similar to the expressions used by experts in GDM problems.

Definition 3. Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$$
$$V_T = \{lower\ than, greater\ than, at\ least, at\ most, between, and, s_0, s_1, \ldots, s_g\}$$
$$I \in V_N$$

The production rules are defined in an extended Backus Naur Form so that the brackets enclose optional elements and the symbol | indicates alternative elements [3]. For the context-free grammar, G_H, the production rules are as follows:

$$\begin{aligned}
&P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle\\
&\langle composite\ term\rangle ::== \langle unary\ relation\rangle\langle primary\ term\rangle | \langle binary\ relation\rangle\\
&\langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle\\
&\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g\\
&\langle unary\ relation\rangle ::= lower\ than|greater\ than|at\ least|at\ most\\
&\langle binary\ relation\rangle ::= \ between\\
&\langle conjunction\rangle ::= and\}
\end{aligned}$$

Remark 3. The unary relations "at least" and "at most" might be equivalent to the relations "greater or equal to" and "lower or equal to".

The expressions produced by the context-free grammar G_H, may be either single valued linguistic terms $s_i \in S$, or comparative linguistic expressions. Both types of expressions define the expression domain generated by G_H and this is noted as S_{ll}.

By using the previous grammar G_H, the expert e_1 may elicit their preferences about the books by means of comparative linguistic expressions closer to those used by human beings, such as:

$$P^1 = \begin{pmatrix} - & between\ high\ and\ very\ high & very\ high \\ at\ most\ low & - & high \\ at\ most\ low & between\ very\ low\ and\ low & - \end{pmatrix}$$

These comparative linguistic expressions generated by G_H, cannot be directly used for CWW, therefore in [33] a transformation function was proposed to transform them into HFLTS.

Table 1
Synthesized comments.

Neither	Very low	Low	Medium	high	Very high	Absolute	Comment
0	0	1	1	0	0	0	*Commonly*
0	0	0	0	0	1	1	*Excellent*

Definition 4 [33]. An HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S, where $S = \{s_0, \ldots, s_g\}$ is a linguistic term set.

For example, let S = {*neither*, *very low*, *low*, *medium*, *high*, *very high*, *absolute*} be a linguistic term set and ϑ a linguistic variable, an HFLTS could be,

$$H_S(\vartheta) = \{high, \ very \ high, absolute\}$$

The comparative linguistic expressions are transformed into HFLTS by means of the transformation function E_{G_H}. These transformations depend on of the comparative linguistic expressions generated by G_H.

Definition 5. Let E_{G_H} be a function that transforms linguistic expressions, $ll \in S_{ll}$, obtained by using G_H, into HFLTS, H_S. S is the linguistic term set used by G_H and S_{ll} is the expression domain generated by G_H:

$$E_{G_H} : S_{ll} \rightarrow H_S$$

The comparative linguistic expressions generated by G_H using the production rules are converted into HFLTS by means of the following transformations:

- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$
- $E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S \text{ and } s_j \leqslant s_i\}$
- $E_{G_H}(\text{lower than } s_i) = \{s_j | s_j \in S \text{ and } s_j < s_i\}$
- $E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S \text{ and } s_j \geqslant s_i\}$
- $E_{G_H}(\text{greater than } s_i) = \{s_j | s_j \in S \text{ and } s_j > s_i\}$
- $E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leqslant s_k \leqslant s_j\}$

Therefore, the previous comparative linguistic expressions provided by the expert e_1 over the books are transformed into HFLTS as follows:

$$P^1 = \begin{pmatrix} - & \{high, \ very \ high\} & \{very \ high\} \\ \{neither, \ very \ low, low\} & - & \{high\} \\ \{neither, \ very \ low, low\} & \{very \ low, low\} & - \end{pmatrix}$$

In order to facilitate the computations with HFLTS, the concept of an envelope of an HFLTS was introduced, the definition of which is:

Definition 6 [33]. The envelope of an HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of its upper bound (*max*) and its lower bound (*min*):

$$env(H_S) = [H_{S^-}, H_{S^+}], \quad H_{S^-} \leqslant H_{S^+}$$

where the upper bound and lower bound of H_S are defined as:

$$H_{S^+} = max(s_i) = s_j, \quad s_i \in H_S \quad \text{and } s_i \leqslant s_j \forall i$$
$$H_{S^-} = min(s_i) = s_j, \quad s_i \in H_S \quad \text{and } s_i \geqslant s_j \forall i$$

The CWW processes are carried out by using these linguistic intervals.

4. A group decision making model dealing with comparative linguistic expressions

In this section, it is presented a novel GDM model capable of dealing either with single linguistic terms or with comparative linguistic expressions, based on context-free grammars and HFLTS, which facilitate the elicitation of linguistic information in group decision problems dealing with hesitant situations. Once the model has been presented, it will be described an algorithm to solve GDM problems by applying such a model.

4.1. Hesitant linguistic group decision making model

This model considers that experts involved in the problem can use single linguistic terms or comparative linguistic expressions generated by the context-free grammar G_H, to express their preferences according to their needs.

The use of comparative linguistic expressions and HFLTS implies that the proposed model extends and modifies the linguistic solving scheme shown in Fig. 2 as follows (see Fig. 5):

1. *Definition of semantics, syntax and context-free grammar*: this phase is modified from Fig. 2 in order to include the definition of the context-free grammar G_H, to generate the comparative linguistic expressions.

2. *Transformation into linguistic intervals*: this new phase is necessary to manage the comparative linguistic expressions by HFLTS.
3. *Choice of aggregation operator*: this is similar to the scheme in Fig. 2, but the operators operate on linguistic intervals.
4. *Selection process*: this obtains the best alternative or set of alternatives according to the preferences provided by experts.

In the following subsections, previous phases are described in further detail.

4.1.1. Definition of semantics, syntax and context-free grammar

As it was revised in Section 2.2 and according to [12], the use of linguistic information in decision making implies establishing the linguistic descriptors that will be used by the experts to provide their preferences.

However, the use of comparative linguistic expressions and HFLTS makes the extension of this phase, and the inclusion of the definition of the context-free grammar G_H, which generates comparative linguistic expressions used by experts to provide their preferences in the GDM problem, necessary.

The context-free grammar G_H is problem dependent, but for GDM problems, that presented in Definition 3 provides a good basis because the comparative linguistic expressions obtained are suitable for expressing linguistic preferences in preference relations.

4.1.2. Transformation of the linguistic expressions into linguistic intervals

In this phase the linguistic expressions of the preference relations provided by experts are transformed into linguistic intervals to facilitate the processes of CWW in the *selection phase*. To do so, once the linguistic descriptors have been selected and their semantics defined, the experts taking part in the GDM problem provide their preference relations P^k, $k \in \{1, \ldots, m\}$, by using single linguistic terms or comparative linguistic expressions, $\mu_{P^k} : X \times X \to S_{ll}$:

$$P^k = \begin{pmatrix} p_{11}^k & \cdots & p_{1n}^k \\ \vdots & \ddots & \vdots \\ p_{n1}^k & \cdots & p_{nn}^k \end{pmatrix}$$

in which each assessment $p_{ij}^k \in S_{ll}$, represents the preference degree of the alternative x_i over x_j according to expert e_k. These preferences are expressed in the expression domain S_{ll}, generated by G_H, to facilitate the elicitation of linguistic expressions.

It is necessary to transform both types of terms into HFLTS for carrying out the processes of CWW in the *selection phase* (see Fig. 5). Hence, the elicited preferences are transformed into HFLTS by means of the transformation function E_{G_H}, which must be defined in this phase. Similarly to the previous phase, the transformation function introduced in Definition 5 provides an initial basis for GDM problems:

$$E_{G_H}\left(p_{ij}^k\right) = H_S\left(p_{ij}^k\right)$$

where $i,j \in \{1, \ldots, n\}$ and n is the number of alternatives.

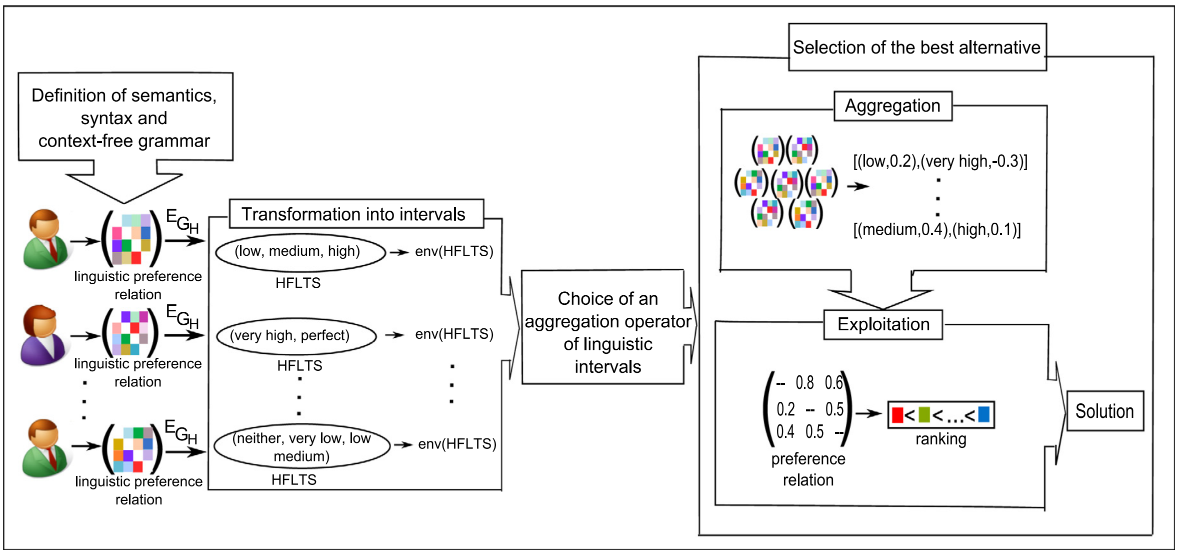

Fig. 5. Scheme of the proposed group decision making model.

To carry out the computations in the *selection phase*, according to the computing model defined for HFLTS, the envelope for each HFLTS that will be used to aggregate the preferences provided by experts is obtained:

$$env(H_S(p_{ij}^k)) = [p_{ij}^{k-}, p_{ij}^{k+}]$$

Therefore, the preference relations provided by experts are represented by linguistic intervals as follows:

$$P^k = \begin{pmatrix} [p_{11}^{k-}, p_{11}^{k+}] & \dots & [p_{1n}^{k-}, p_{1n}^{k+}] \\ \vdots & \ddots & \vdots \\ [p_{n1}^{k-}, p_{n1}^{k+}] & \dots & [p_{nn}^{k-}, p_{nn}^{k+}] \end{pmatrix}$$

4.1.3. Choice of an aggregation operator for linguistic intervals

Similarly to [12], in this case, a suitable aggregation operator will be selected to deal with linguistic intervals obtained in the previous phase.

This choice is problem dependent and usually one or more aggregation operators are necessary to obtain a solution to the GDM problem.

4.1.4. Selection of the best alternative(s)

The selection process looks for the solution set of alternatives for the GDM problem. It follows the scheme presented in Fig. 5 composed of two steps that are further detailed below.

1. *Aggregation of the preference relations represented by linguistic intervals.*

So far, the elicited preferences of experts have been modeled by linguistic intervals. In the aggregation step, such preferences will be aggregated by the operators selected previously to obtain a collective preference for each alternative.

Taking advantage of the linguistic intervals, this model interprets those preferences from two points of view in hesitant situations, with the lower value of the envelope being the pessimistic perception and the greater value the optimistic perception. A double aggregation process is carried out keeping initially both perceptions apart (see Fig. 6).

(i) *Pessimistic/optimistic collective preference relations.*
It is necessary to choose a linguistic aggregation operator φ, which will depend on the problem. Such an operator will be used to aggregate separately the right and left limits of the linguistic intervals, obtaining two collective preference relations P_C^+ and P_C^-, which represent the collective optimistic and pessimistic perceptions of the aggregated preferences, respectively. These collective preferences are represented by 2-tuple linguistic values.

$$P_C^+ = \begin{pmatrix} (s_r,\alpha)_{11}^+ & \dots & (s_r,\alpha)_{1n}^+ \\ \vdots & \ddots & \vdots \\ (s_r,\alpha)_{n1}^+ & \dots & (s_r,\alpha)_{nn}^+ \end{pmatrix} P_C^- = \begin{pmatrix} (s_r,\alpha)_{11}^- & \dots & (s_r,\alpha)_{1n}^- \\ \vdots & \ddots & \vdots \\ (s_r,\alpha)_{n1}^- & \dots & (s_r,\alpha)_{nn}^- \end{pmatrix}$$

$$(s_r,\alpha)_{ij}^+ = \Delta\left(\varphi\left(\Delta^{-1}\left(p_{ij}^{k+}\right)\right)\right) \quad \forall k \in \{1,\dots,m\} \tag{2}$$

$$(s_r,\alpha)_{ij}^- = \Delta\left(\varphi\left(\Delta^{-1}\left(p_{ij}^{k-}\right)\right)\right) \quad \forall k \in \{1,\dots,m\} \tag{3}$$

being $i,j \in \{1, \dots, n\}$ and $s_r \in S = \{s_0, \dots, s_g\}$.

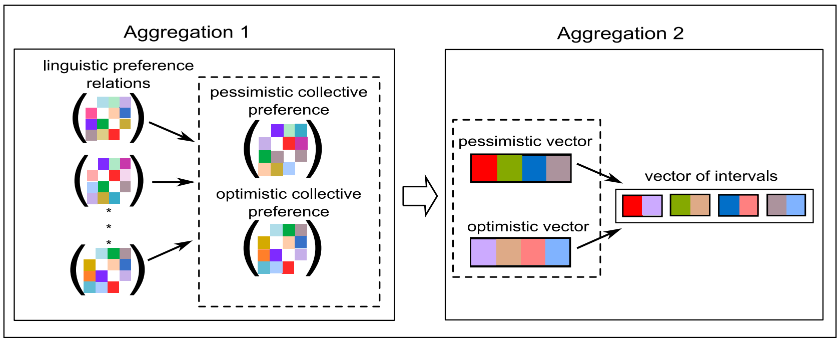

Fig. 6. Scheme of the aggregation process.

(ii) *Collective linguistic interval vector.*
A collective linguistic interval for each alternative x_i, is computed. To obtain such an interval, the collective linguistic preferences based on perceptions P_C^+ and P_C^- are aggregated by using an aggregation operator ϕ, which may or may not be the same as φ. The results obtained by these computations are the optimistic p_i^+, and pessimistic p_i^-, collective preferences for each alternative x_i:

$$p_i^+ = \Delta\left(\phi\left(\Delta^{-1}(s_r,\alpha)_{ij}^+\right)\right) \quad \forall j \in \{1,\ldots,n\} \tag{4}$$

$$p_i^- = \Delta\left(\phi\left(\Delta^{-1}(s_r,\alpha)_{ij}^-\right)\right) \quad \forall j \in \{1,\ldots,n\} \tag{5}$$

A collective linguistic interval is then built for each alternative x_i, obtaining a vector of intervals of collective preferences for the alternatives:

$$V^R = (p_1^R,\ldots,p_n^R) \tag{6}$$

where $p_i^R = [p_i^-, p_i^+]$ and $i \in \{1, \ldots, n\}$.

2. *Exploitation.*
In the exploitation step, the vector of collective linguistic intervals for the alternatives is used to obtain a ranking of alternatives and to select the best one(s) as the solution to the GDM problem. There are different approaches to ordering the alternatives [15,27,41]. We will use the approach proposed by Wang et al. in [41] because it allows a ranking of alternatives to be obtained by using intervals. Such an approach builds a preference relation between alternatives by using the following definition:

Definition 7 [41]. Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two interval utilities, the preference degree of A over B (or $A > B$) is defined as:

$$P(A > B) = \frac{max(0, a_2 - b_1) - max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)} \tag{7}$$

and the preference degree of B over A (or $B > A$) as:

$$P(B > A) = \frac{max(0, b_2 - a_1) - max(0, b_1 - a_2)}{(a_2 - a_1) + (b_2 - b_1)} \tag{8}$$

Note that $P(A < B) + P(B > A) = 1$ and $P(A > B) = P(B > A) = 0.5$ when $A = B$, i.e. $a_1 = b_1$ and $a_2 = b_2$.

Definition 8 [41]. Let P_D be a preference relation

$$P_D = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}$$

with $p_{ij} = P(p_i^R > p_j^R)$ being the preference degree of the alternative x_i over x_j, $i,j \in \{1, \ldots, n\}$ and $p_i^R = [p_i^-, p_i^+]$ and $p_j^R = \left[p_j^-, p_j^+\right]$.

Eventually, a choice function is used to generate a solution set of alternatives for the decision problem. Different choice functions have been proposed in the literature [11]. We will use a non-dominance choice degree, *NDD*, that indicates the degree to which an alternative is not dominated by the remaining ones. Its definition is given as follows:

Definition 9 [27]. Let $P_D = [p_{ij}]$ be a preference relation defined over a set of alternatives X. For the alternative x_i, its non-dominance degree, NDD_i, is obtained as:

$$NDD_i = min\{1 - p_{ji}^S,\ j = 1,\ldots,n, j \neq i\} \tag{9}$$

where $p_{ji}^S = max\{p_{ji} - p_{ij}, 0\}$ represents the degree to which x_i is strictly dominated by x_j.
And the non-dominated alternatives obtained are:

$$X^{ND} = \{x_i | x_i \in X, NDD_i = max_{x_j \in X}\{NDD_j\}\} \tag{10}$$

4.2. Algorithm of the proposed GDM model

Once the proposed GDM model has been explained in detail, we present an algorithm to solve GDM problems defined in qualitative frameworks in which experts may elicit comparative linguistic expressions applying such a model.

Let us suppose a set of alternatives, $X = \{x_1, \ldots, x_n\}$, and a set of experts, $E = \{e_1, \ldots, e_m\}$, who express their preference relations by using single linguistic terms or comparative linguistic expressions.

The algorithm is as follows:

1. Defining the semantics and syntax of the linguistic term set S.
2. Defining the context-free grammar G_H.
3. Gathering the preference relations P^k provided by experts $k \in \{1, \ldots, m\}$.
4. Transforming the preference relations into HFLTS by using the transformation function E_{G_H} (see Definition 5).
5. Obtaining for each HFLTS its envelope $\left[p_{ij}^{k-}, p_{ij}^{k+}\right]$ (see Definition 6).
6. Selecting two linguistic aggregation operators φ and ϕ. These two operators might be the same.
7. Obtaining the pessimistic and optimistic collective preference relations P_C^-, P_C^+, by using the linguistic aggregation operator φ.
8. Computing a pessimistic and optimistic collective preference for each alternative applying Eqs. (4) and (5) by using the linguistic aggregation operator ϕ.
9. Building a vector of intervals $V^R = (p_1^R, \ldots, p_n^R)$, of collective preferences for the alternatives $p_i^R = \left[p_i^-, p_i^+\right]$ by using Eq. (6).
10. Building a preference relation P_D, by using the Eqs. (7) and (8).
11. Applying a non-dominance choice degree NDD by using Eq. (9).
12. Rank the set of alternatives and select the best one(s).

To understand the proposed GDM model easily, a GDM problem will be solved following the steps of the algorithm.

5. Illustrative example

Let us suppose that a conference committee composed of 3 researchers $E = \{e_1, e_2, e_3\}$, wants to grant a best paper award in an International Conference. There are four selected papers, X = {John's paper, Mike's paper, David's paper, Frank's paper}. Because of the uncertainty among the papers, it is difficult for the researchers to use just one linguistic term to provide their preferences. To facilitate the elicitation of their preferences they can use comparative linguistic expressions close to human beings' cognitive processes.

To solve the proposed GDM problem the algorithm presented in Section 4.2 is applied.

1. *Defining the semantics and syntax of the linguistic term set S*

   In GDM problems, experts provide their assessments by means of preference relations, where an assessment represents the preference degree for one alternative over another. A linguistic term set suitable to express such assessments is:

   $$S = \{neither(n), very\ low(vl), low(l), medium(m), high(h), very\ high(vh), absolute(a)\}$$

2. *Defining the context-free grammar G_H.*

   We will use the context-free grammar introduced in Definition 3.

3. *Gathering the preferences provided by experts.*

$$P^1 = \begin{pmatrix} - & at\ most\ vl & vh & at\ most\ vl \\ at\ least\ vh & - & between\ h\ and\ vh & at\ most\ m \\ l & at\ most\ l & - & greater\ than\ h \\ at\ least\ h & greater\ than\ m & at\ most\ m & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & at\ most\ l & greater\ than\ m & lower\ than\ m \\ greater\ than\ m & - & h & vl \\ at\ most\ vl & l & - & greater\ than\ h \\ between\ h\ and\ vh & vh & between\ n\ and\ l & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & greater\ thanm & between\ h\ and\ vh & l \\ at\ most\ l & - & at\ least\ h & greater\ than\ m \\ lower\ than\ m & at\ most\ l & - & vh \\ h & at\ mostl & vl & - \end{pmatrix}$$

4. *Transforming the preference relations into HFLTS.*

$$P^1 = \begin{pmatrix} - & \{n, vl\} & \{vh\} & \{n, vl\} \\ \{vh, a\} & - & \{h, vh\} & \{n, vl, l, m\} \\ \{l\} & \{n, vl, l\} & - & \{vh, a\} \\ \{h, vh, a\} & \{h, vh, a\} & \{n, vl, l, m\} & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & \{n, vl, l\} & \{h, vh, a\} & \{n, vl, l\} \\ \{h, vh, a\} & - & \{h\} & \{vl\} \\ \{n, vl\} & \{l\} & - & \{vh, a\} \\ \{h, vh\} & \{vh\} & \{n, vl, l\} & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & \{h, vh, a\} & \{h, vh\} & \{l\} \\ \{n, vl, l\} & - & \{h, vh, a\} & \{h, vh, a\} \\ \{n, vl, l\} & \{n, vl, l\} & - & \{vh\} \\ \{h\} & \{n, vl, l\} & \{vl\} & - \end{pmatrix}$$

5. *Obtaining for each HFLTS its envelope.*

$$P^1 = \begin{pmatrix} - & [n, vl] & [vh, vh] & [n, vl] \\ [vh, a] & - & [h, vh] & [n, m] \\ [l, l] & [n, l] & - & [vh, a] \\ [h, a] & [h, a] & [n, m] & - \end{pmatrix} P^2 = \begin{pmatrix} - & [n, l] & [h, a] & [n, l] \\ [h, a] & - & [h, h] & [vl, vl] \\ [n, vl] & [l, l] & - & [vh, a] \\ [h, vh] & [vh, vh] & [n, l] & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & [h, a] & [h, vh] & [l, l] \\ [n, l] & - & [h, a] & [h, a] \\ [n, l] & [n, l] & - & [vh, vh] \\ [h, h] & [n, l] & [vl, vl] & - \end{pmatrix}$$

6. *Selecting two linguistic aggregation operators.*

Without loss of generality and for the sake of simplicity, in the aggregation phase we use the arithmetic mean aggregation operator based on 2-tuple defined as follows:

Definition 10 [13]. Let $x = \{(s_1, \alpha_1), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean $\bar{x}$ is computed as:

$$\bar{x} = \Delta\left(\frac{1}{n}\sum_{i=1}^{n}\Delta^{-1}(s_i, \alpha_i)\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^{n}\beta_i\right) \tag{11}$$

7. *Obtaining the pessimistic and optimistic collective preference relations.*

$$P_C^+ = \begin{pmatrix} - & (m, 0) & (vh, .33) & (l, -.33) \\ (vh, -.33) & - & (vh, 0) & (m, .33) \\ (l, -.33) & (l, 0) & - & (a, -.33) \\ (vh, 0) & (h, .33) & (l, 0) & - \end{pmatrix}$$

$$P_{C_{12}}^+ = \Delta\left(\frac{1}{3}(\Delta^{-1}(vl, 0) + \Delta^{-1}(l, 0) + \Delta^{-1}(a, 0))\right) = (m, 0)$$

$$P_C^- = \begin{pmatrix} - & (vl, .33) & (h, .33) & (vl, -.33) \\ (m, 0) & - & (h, 0) & (l, -.33) \\ (vl, -.33) & (vl, -.33) & - & (vh, 0) \\ (h, 0) & (m, 0) & (n, .33) & - \end{pmatrix}$$

$$P_{C_{12}}^- = \Delta\left(\frac{1}{3}(\Delta^{-1}(n, 0) + \Delta^{-1}(n, 0) + \Delta^{-1}(h, 0))\right) = (vl, .33)$$

8. *Computing a pessimistic and optimistic collective preference for each alternative* (see Table 2).

$$p_1^- = \Delta\left(\frac{1}{3}(\Delta^{-1}(vl, .33) + \Delta^{-1}(h, .33) + \Delta^{-1}(vl, -.33))\right) = (l, .11)$$

9. *Building a vector of intervals* $V^R = (p_1^R, p_2^R, p_3^R, p_4^R)$ *for the alternatives* (see Table 3).

Table 2
Pessimistic and optimistic collective preference for each alternative.

	John	*Mike*	*David*	*Frank*
Pessimistic	l, .11	m, −.11	l, .11	l, .44
Optimistic	m, .33	h, .33	m, .11	h, −.22

Table 3
Linguistic intervals for each alternative.

John	*Mike*	*David*	*Frank*
p_1^R	p_2^R	p_3^R	p_4^R
[(l, .11), (m,.33)]	[(m, −.11), (h, .33)]	[(l, .11), (m, .11)]	[(l, .44), (h, −.22)]

10. *Building a preference relation P_D.*

$$P_D = \begin{pmatrix} - & 0.17 & 0.55 & 0.35 \\ 0.83 & - & 0.91 & 0.68 \\ 0.45 & 0.09 & - & 0.29 \\ 0.65 & 0.32 & 0.71 & - \end{pmatrix}$$

where $P_{D_{12}}$ is obtained as follows,

$$P_D(p_1^R > p_2^R) = \frac{max(0, 3.33 - 2.89) - max(0, 2.11 - 4.33)}{(3.33 - 2.11) + (4.33 - 2.89)} = 0.17$$

$$p_1^R = [p_1^-, p_1^+] = [\Delta^{-1}(l, .11), \Delta^{-1}(m, .33)] = [2.11, 3.33].$$

11. *Afterwards, a non-dominance choice degree is applied.*

$$P_D^S = \begin{pmatrix} - & 0 & 0.1 & 0 \\ 0.67 & - & 0.82 & 0.36 \\ 0 & 0 & - & 0 \\ 0.3 & 0 & 0.43 & - \end{pmatrix}$$

$NDD_1 = 0.33$, $NDD_2 = 1$, $NDD_3 = 0.18$, $NDD_4 = 0.64$
where NDD_1 is computed as follows,

$$NDD_1 = min\{(1 - 0.67), (1 - 0), (1 - 0.3)\} = 0.33$$

12. *Finally, the set of alternatives is ordered*,

$$x_2 > x_4 > x_1 > x_3$$

and we select the best one as the solution to the GDM problem whose linguistic interval is,

$$x_2 = [(m, -.11), (h, .33)]$$

6. Conclusions

The need for modeling complex linguistic expressions in decision making has been pointed out by several authors, because the uncertainty involved in such problems causes experts hesitate among more than one single linguistic term to express their preferences. Some approaches were developed to meet this necessity. However, none have been applied to GDM due to the lack of matching between the expressions generated by those approaches and the expressions used by experts in GDM problems.

In this paper, a GDM model has been presented that is capable of dealing with comparative linguistic expressions based on context-free grammars and HFLTS to facilitate the elicitation of linguistic information in hesitant decision situations. Such a model carries out the processes of CWW by using a simple and accurate linguistic computing model. Eventually, an illustrative GDM problem has been solved by the proposed model in order to show its performance.

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