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A review of soft consensus models in a fuzzy environment

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ABSTRACT

In the consensus reaching processes developed in group decision making problems we need to measure the closeness among experts' opinions in order to obtain a consensus degree. As it is known, to achieve a full and unanimous consensus is often not reachable in practice. An alternative approach is to use softer consensus measures, which reflect better all possible partial agreements, guiding the consensus process until high agreement is achieved among individuals. Consensus models based on soft consensus measures have been widely used because these measures represent better the human perception of the essence of consensus. This paper presents an overview of consensus models based on soft consensus measures, showing the pioneering and prominent papers, the main existing approaches and the new trends and challenges.

1. Introduction

The term "consensus" has been used for years, even centuries, in a variety of context and areas. It generally concerns various situations in which there is a set of experts who present their testimonies which basically concern their opinions on some alternatives, topics, courses of action, etc., in question. The experts can be both individuals as such, smaller groups, larger groups, even organizations if they can be considered uniform with respect to the issues considered and testimonies [1,2].

Consensus can be meant in various ways and group decision making (GDM) contexts. First, consensus is related with the state of agreement in a group in the sense that the individuals exhibit a state of common feeling as to the values in question. Strictly speaking, consensus has been meant from this perspective as a full and unanimous agreement [3,4], though it has been deemed questionable if such a state is possible in virtually all real world situations [5]. Second, which is related to the first sense given above, consensus is meant as a way to reach consensus. This involves an evolution of the testimonies of the group towards consensus with respect to their testimonies; this evolution can be free or facilitated (moderated) by a special individual [1,6]. Third, consensus can be meant as a way in which decisions should be meant in multi-person settings [7]. Basically, consensus decision making aims at attaining the consent, not necessarily the agreement, of the participants by accommodating views of all parties involved to attain a decision that will yield what will be beneficial to the entire group, not necessarily to the particular individuals who may give consent to what will not necessarily be their first choice but because, for instance, they wish to cooperate with the group. The full consent, however, does not mean that each individual is in full agreement [1]. Therefore, consensus boils down to cooperation in contrast to most GDM setting, notably voting, gaming, etc., which boil down to a competition.

It is clear that, ideally, consensus should refer to unanimity of individuals because the option or course of action attained will be best representative for the entire group. Obviously, unanimity may be difficult to attain, in particular in large and diversified groups of individuals as is the case in real world settings, and that is why milder benchmarks (definitions) of consensus have been employed exemplified by [1,8]: unanimity minus one (U − 1), i.e., that all individuals but one support the decision, unanimity minus two (U − 2), i.e., all but two support the decision, unanimity minus three, (U − 3), etc. Moreover, some measures like 80%, 2/3, etc., can be employed, and even the so called rough consensus may be assumed which does not assume any specific rule which is to be determined later. Notice that all these milder definitions of consensus are still crisp and do not involve any imprecise (fuzzy) specification. We can well imagine here a fuzzy majority exemplified by "most", "almost all", "much more than a half" and so on [9,10]. In such a way, we could introduce the concept of soft consensus as more adequate to model the GDM problems. We will discuss such fuzzy specifications, notably the concepts of a fuzzy majority and

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soft consensus [11], later in detail because they constitute a main topic here.

Virtually, all consensus reaching processes proceed in a multistage setting, i.e., the individuals change their opinions step by step until, possibly, some consensus is reached [1,2]. Of course, this presupposes that the individuals are committed to those changes. At this junction, one can clearly see two situations. First, the process of changes of the testimonies proceeds in one way or another and can be modeled. Second, that process is moderated (facilitated) by a special person ("super-individual") called a facilitator, moderator, etc., who is responsible for running the consensus reaching session in question by persuading the individuals to change their testimonies by rational argument, persuasion, etc., and keeping the process within a period of time considered [1]. The second option of a moderator running a consensus reaching process is usually more effective and efficient. However, this paradigm has been predominant in recent times only [5,11–13].

Given the importance of obtaining an accepted solution by the whole group, the consensus has attained a great attention and it is virtually a major goal of GDM problems. The objective of this paper is to present a comprehensive presentation of the state of the art of all known consensus approaches, with an in-depth analysis of the respective problems and solutions as well as more relevant challenges. In particular, we focus on the consensus approaches in which there is a moderator, as they are more promising in practice and the most used in the literature, and based on the concept of fuzzy majority, which is more human-consistent and suitable for reflecting human perceptions of the meaning of consensus, i.e., the soft consensus.

This contribution is set out as follows. In Section 2, we describe the consensus processes based on moderator and the usual fuzzy GDM framework. In Section 3, we highlight the pioneer and most relevant contributions existing on consensus. In Section 4, the main fuzzy consensus approaches are described. The current trends and prospects in the development of consensus models are shown in Section 5. Finally, in Section 6, we present some concluding remarks.

2. Preliminaries

This section is devoted to describe the usual fuzzy GDM framework to develop consensus processes. In particular, we define the GDM problem, the usual consensus process, the formats of preferences used to express experts' opinions, and the concepts of fuzzy majority and linguistic quantifiers.

2.1. GDM problem

In a classical GDM situation [14,15], there is a set of possible alternatives, $X = \{x_1, \ldots, x_n\} (n \geqslant 2)$, and a group of experts, $E = \{e_1, \ldots, e_m\} (m \geqslant 2)$, characterized by their background and knowledge, who express their opinions about X to achieve a common solution. In a fuzzy context, the objective is to classify the alternatives from best to worst, associating with them some degrees of preference expressed in the $[0,1]$ interval.

GDM problems are roughly classified into two groups: homogeneous and heterogeneous [16,17]. A GDM problem is heterogeneous when the opinions of the experts are not equally important. On the contrary, if every opinion is treated equally, we face an homogeneous GDM problem. A way to implement experts' heterogeneity is to assign a weight to every individual. Weights are qualitative or quantitative values that can be assigned in several different ways [16]: a moderator can assign them directly, or the weights can be obtained automatically from the preference expressed by the experts (for example, the most consistent experts could received a higher weight that inconsistent one). The weights can be interpreted as a fuzzy subset, I, with a membership function, μ_I: $E \to [0,1]$, in such a way that $\mu_I(e_l) \in [0,1]$ denotes the importance degree of the expert within the group, or how relevant is the person in relation with the problem to be solved [18,19].

In GDM problems we could apply a selection process to find a solution set of alternatives according to the preferences provided by the experts [15,20], without taking into account the level of agreement between experts. The selection process is composed of two steps [20,21]: aggregation of preferences provided by the experts and exploitation of the aggregated preference obtained previously. However, this process can lead sometimes solutions that are not well accepted by some experts in the group [1,2], because they could consider that the solution achieved does not reflect their preferences, and hence, they might reject it. To overcome this problem, it is advisable that experts carry out a consensus process, where the experts discuss and negotiate in order to achieve a sufficient agreement before applying the selection process. For this reason, GDM problems are usually faced by applying a consensus process and a selection process before a final solution can be given [22,23].

2.2. Consensus process

It can be seen that there are several problems related to the consensus reaching process, and the very essence of consensus. First, a crucial point is the very meaning of consensus. As we have already mentioned, traditionally consensus is defined as a full and unanimous agreement. Several authors have introduced consensus measures assuming values in interval $[0,1]$, with 0 meaning no consensus and 1 meaning full consensus, being the rest of assessments in $(0,1)$ different partial consensus degrees [3,4]. However, it has been considered unrealistic in most realistic setting and hence milder definitions exemplified by "unanimity minus k" have been advocated [8]. Though they have served their purpose, they have not been yet considered to be fully adequate for reflecting the very essence of consensus. Due to some inherent differences, individuals rarely arrive at that unanimous agreement, and even if this was the case, the consensus activity could be too costly in real cases. So, we have to do some reconsiderations on both the essence of consensus and that of the consensus reaching process. In such a way, consensus can be viewed not necessarily as a total and unanimous agreement. It could happen that the experts are not willing to fully change their testimonies so that consensus will not be a unanimous agreement, i.e., we could not get the same testimony for all, but some set of individual testimonies which are similar enough.

The next problem of relevance is the modeling of the consensus reaching processes. Basically, we have a set of testimonies provided by the particular individuals which concern in general opinions as to the values of some quantities. Initially, these testimonies have been equated with some utilities resulting from some courses of actions, probabilities of them, and alike. In general, they have been assumed to be in matrix form [24–26]. More recently, preferences have been more and more popular as a more flexible and less formally restrictive representation [5,11].

Other problem is related with the management of human subjectivity in the consensus process. Since the process of decision making, in particular of group type, is centered on humans, coming with their inherent subjectivity, imprecision and vagueness in the articulation of opinions, the theory of fuzzy sets, introduced by Zadeh [27], has delivered new tools in this field for a long time. Fuzzy logic is a more useful tool to represent often human preferences encountered in most real cases and provides us a more general and richer representation of individual opinions than other tools as a probability theory. The advent of fuzzy logic and fuzzy preference relations has changed the field of GDM and consensus. In a fuzzy context, for a long time, it has been considered much more promising to run the consensus reaching session with the help of a special agent, called a moderator or facilitator [1,2], whose task is to help the individuals involved while changing their testimonies towards consensus, by rational argument, persuasion, etc. Clearly, he or she should be supported by some information to be provided by tools and techniques. Fuzzy logic can play here a considerable role by providing means for the representation and processing of imprecise information and testimonies [28].

As aforementioned, there are two approaches in the formulation of consensus. The first, traditional, is the one started by Coch and French [24], French [25] and Harary [26] in which the process is modeled by using matrix calculus or Markov chains to model the time evolution of changes of opinions toward consensus. The second, more promising in practice, is the one in which there is a moderator. In the following, we describe the second one as it is the most used in the literature.

A consensus process is a negotiation process developed iteratively and composed by several consensus rounds, where the experts accept to change their preferences following the advice given by a moderator. The moderator knows the agreement degree in each round of the consensus process by means of the computation of some consensus measures. The consensus process can be divided in several steps:

1. Firstly, the problem to be solved is presented to the experts, along with the different alternatives among they have to choose the best one.
2. Then, experts can discuss and share their knowledge about the problem and alternatives in order to facilitate the process of latterly expressing their opinions.
3. Experts provide their preferences about the alternatives in a particular preference representation format.
4. The moderator receives all the experts' preferences and computes some consensus measures that will allow him to identify if an enough consensus state has been reached or not.
5. If an enough consensus state has been reached, the consensus process stops and the selection process begins. Otherwise, we can apply an advice generation step where the moderator, with all the information that he/she has (all preferences expressed by experts, consensus measures and so on) can prepare some guidance and advice for experts to more easily reach consensus. Note that this step is optional and is not present in every consensus model.
6. Finally, the advice is given to the experts and the first round of consensus is finished. Again, experts must discuss their opinions and preferences in order to approach their points of view (Step 2).

2.3. Preference representation formats

Fixed a set of alternatives in a GDM problem, there exist several preference representation formats that can be used by experts to provide their preferences about that set of alternatives. The most common ones are:

- *Preference orderings*. The preferences of an expert $e_l \in E$ about a set of feasible alternatives X are described as a preference ordering $O^l = \{o^l(1), \ldots, o^l(n)\}$ where $o^l(\cdot)$ is a permutation function over the indexes set $\{1, \ldots, n\}$ for this expert [29]. Thus, an expert gives an ordered vector of alternatives from best to worst.
- *Utility values*. An expert $e_l \in E$ provides his/her preferences about a set of feasible alternatives X by means of a set of n utility values $U^l = \{u_1^l, \ldots, u_n^l\}$, $u_i^l \in [0, 1]$, the higher the value for an alternative, the better it satisfies experts' objective [30].
- *Preference relations*. In this case, expert's preferences on X are described by means of a $P^l \subset XxX$ characterized by a function $\mu_{P^l} : X \times X \to D$ where $\mu_{P^l}(x_i, x_j) = p_{ij}^l$ can be interpreted as the preference degree or intensity of the alternative x_i over x_j expressed in the information representation domain D. Preference relations are the representation format most used in GDM. Different types of preference relations can be used according to the domain used to evaluate the intensity of the preference:

1. *Fuzzy preference relations* [31,32]: If $D = [0,1]$ every value p_{ij}^l in the matrix P^l represents the preference degree or intensity of preference of the alternative x_i over x_j: $p_{ij}^l = 1/2$ indicates indifference between x_i and x_j, $p_{ij}^l = 1$ indicates that x_i is absolutely preferred to x_j, and $p_{ij} > 1/2$ indicates that x_i is preferred to x_k. It is usual to assume the additive reciprocity property $p_{ij}^l + p_{ji}^l = 1 \ \forall i, j$.
2. *Multiplicative preference relations* [33]: If $D = [1/9, 9]$ and then every value p_{ij}^l in the matrix P^l represents a ratio of the preference intensity of the alternative x_i to that of x_j, i.e., it is interpreted as x_i is p_{ij}^l times good as x_j: $p_{ij}^l = 1$ indicates indifference between x_i and x_j, $p_{ij}^l = 9$ indicates that x_i is unanimously preferred to x_j, and $p_{ij}^l \in \{2, 3, \ldots, 8\}$ indicates intermediate evaluations. It is usual to assume the multiplicative reciprocity property $p_{ij}^l \cdot p_{ji}^l = 1 \ \forall i, j$ too.
3. *Linguistic preference relations* [12,13,34]: If $D = S$, where S is a linguistic term set $S = \{s_0, \ldots, s_g\}$ with odd cardinality $(g + 1)$, $s_{g/2}$ being a neutral label (meaning "equally preferred") and the rest of labels distributed homogeneously around it, then every value p_{ij}^l in the matrix P^l represents the linguistic preference degree or linguistic intensity of preference of the alternative x_i over x_j.

2.4. Fuzzy majority and fuzzy linguistic quantifiers

The majority is traditionally defined as a threshold number of individuals. Fuzzy majority could be defined as a soft concept of majority which is expressed by a fuzzy linguistic quantifier exemplified by "most", "almost all", "much more than a half", etc., which can be formally handled by, first of all, a calculus of linguistically quantified propositions, notably due to Zadeh [35], and also by using Yager's [36] OWA (Ordered Weighted Average) operators or other aggregation operators, which provide a much needed generality and flexibility [37–39].

The concept of a fuzzy majority equated naturally with a fuzzy linguistic quantifier was introduced into GDM by Kacprzyk [9,10,31,40,41], but one should bear in mind that Nurmi's seminal paper [42] on novel definitions of GDM solutions under fuzzy preferences and crisp (but valued) majorities is here a point of departure. In both these approaches, new definitions of consensus winners, popular solution concepts, have been proposed. The fuzzy majority has then been the key point for the new definitions of soft consensus proposed by Kacprzyk and Fedrizzi [5,11,43,44]. Applying fuzzy majority we can define the consensus degree in a more flexible way and reflect the large spectrum of possible partial agreements existing in a group of experts, and in such a way to guide easily the consensus process until wide agreement (not always total) is achieved among experts.

Quantifiers can be used to represent the amount of items satisfying a given predicate. Classic logic use only two quantifiers, "there exists" (related with "or" connective) and "for all" (related with "and" connective). However, the natural language has a greater variety of quantifiers, e.g., "about 5", "most", "at least half", "all", "as many as possible". Zadeh defined the concept of fuzzy linguistic quantifier [35] to deal with that variety of quantifiers existing in the human discourse. Fuzzy linguistic quantifiers are used to include the fuzzy majority in the computation of consensus measures and for deriving new solution concepts in GDM [5,11,43,44].

Usually, the semantics of a fuzzy linguistic quantifier is represented by using fuzzy subsets. In the natural discourse we identify both absolute quantifiers and relative quantifiers. Former represent absolute amounts as "about 2" or "more than 5". The semantics of an absolute quantifier is defined by means of a fuzzy set Q characterized by a membership function $Q(r) \in [0,1]$, $r \in R^{+}$, being $Q(r)$ the degree in which the amount r is compatible with the quantifier represented by Q. Latter represent proportion type statements as "most", "at least half". Similarly, the semantics of a relative quantifiers is defined by a fuzzy set Q characterized by a membership function $Q(r) \in [0,1]$, $r \in [0,1]$, such that $Q(r)$ indicates the degree in which the proportion r is compatible with the meaning of the quantifier it represents.

3. Pioneer and prominent contributions

In this section we provide an historical perspective on the consensus approaches in decision making. To do so, we revise the pioneer and prominent contributions in the field.

The first mathematical approaches of consensus reaching processes started with the pioneering works by French and his collaborators in the late 1940s and early 1950s.

L. Coch, J.R.P. French. *Overcoming resistance to change*. Human Relations 1(4) (1948) 512–532.
J.R.P. French. *A formal theory of social power*. PsychologicalReview 63(3) (1956) 181–194.

Basically, they employed matrix calculus to model the time evolution and reaching of the consensus process. Particularly, on the one hand, Coch and French describe the impact of involving people in changes that affect them. They conduct experiments on the effect of involving employees in changing work procedures in a manufacturing organization. High-involvement groups, in which employees were involved from the beginning, not only outperformed the no-participation groups but also increased productivity, while the no-participation groups productivity dropped and grievances and quits increased. And on the other hand, French introduces a simple model of how a network of interpersonal influence enters into the process of opinion formation. He exploits the patterns of interpersonal relations and agreements that can explain the influence process in groups of agents. These pioneer contributions are considered the beginning of participative management in decision making.

Drawing on the algebra of a Markov chain process, the consensus theory is developed in a more general form by Harary, De Groot and French in the following pioneer contributions, respectively:

F. Harary. *On the measurement of structural balance.* Behavioral Science 4 (1959) 316–323.
M.M. De Groot. *Reaching consensus*. Journal of the American Statistical Association 69(345) (1974) 118–121.

S. French. *Consensus of opinion*. European Journal of Operational Research 7(4) (1981) 332–340.

These initial formulations describes the formation of group consensus, but do not provide an adequate account of settled patterns of disagreement. But in anycase, we can affirm that the first mathematical modeling of consensus reaching process started with these works. We should point out that De Groot's work has shown a high impact in the decision making community being considered a highly cited paper.

Later, many models of consensus reaching (formation) have been proposed, notably in the realm of so called rational consensus. In this sense, a prominent contribution is proposed by Lehrer and Wagner in the following book:

K. Lehrer, C. Wagner. *Rational Consensus in Science and Society*. Dordrecht, Reidel, 1994.

In this case, the essence of rational consensus is that a consensus reaching procedure is not just a pooling or aggregation but changes of testimonies occur and are rationally motivated.

In a different perspective, Ragade proposes to conceptualize consensus within the fuzzy environment in the following pioneer contribution:

R. Ragade. *Fuzzy sets in communication systems and consensus formation systems*. Journal of Cybernetics 6 (1976) 21–38.

He examines some applications of fuzzy set theory in the area of communications and information systems. We should point out that this is the first work which addresses the problem of consensus reaching modeling in a fuzzy environment.

In the classical consensus approaches aforementioned, the notion of consensus has conventionally been understood in terms of strict and unanimous agreement. However, the human perception of consensus is typically "softer", and people are generally willing to accept that consensus has been reached when most actors agree on the preferences associated to the most relevant alternatives. A milestone was here a special issue of the influential Synthese journal,

B. Loewer. *Special issue on consensus*. Synthese 62(1) (1985) 1–122.

Among many papers therein, the most prominent one for our purpose is that by Loewer and Laddaga.

B. Loewer, R. Laddaga. *Destroying the consensus*. Synthese 62(1) (1985) 79–96.

who have clearly made the first approach for a soft concept of consensus saying that:

> ...It can correctly be said that there is a *consensus* among biologists that Darwinian natural selection is an important cause of evolution though there is currently *no consensus* concerning Gould's hypothesis of speciation. This means that there is a *widespread agreement* among biologists concerning the first matter but *disagreement* concerning the second ...

A crisp majority as, e.g., more than 75% would not evidently reflect the very essence of the above given quotation. This statement suggests that a fuzzy majority is appropriate, and that it makes sense to speak about a consensus degree, or proximity to the "ideal" consensus.

According to Loewer and Laddaga, Kacprzyk and Fedrizzi introduce the concept of a fuzzy majority using Zadeh's fuzzy linguistic quantifier to define soft consensus measures in the following prominent contributions:

J. Kacprzyk, M. Fedrizzi. *Soft consensus measure for monitoring real consensus reaching processes under fuzzy preferences*. Control and Cybernetics 15(3–4) (1986) 309–323.
J. Kacprzyk, M. Fedrizzi. *A 'soft' measure of consensus in the setting of partial (fuzzy) preferences.* European Journal of Operational Research 34(3) (1988) 316–325.
J. Kacprzyk, M. Fedrizzi. *A 'human-consistent' degree of consensus based on fuzzy logic with linguistic quantifiers*. Mathematical Social Sciences 18(3) (1989) 275–290.

Then, the classical operational definition of consensus is expressed by a linguistically quantified proposition as:

$$\begin{aligned}&\text{“}Most\ (Q1)\ of\ the\ important\ (B)\ individuals\ agree\\&as\ to\ almost\ all\ (Q2)\ relevant\ (I)\ alternatives\text{”}\end{aligned} \tag{1}$$

where: $Q1$ and $Q2$ are fuzzy linguistic quantifiers [35], e.g.,"most" and "almost all", and B and I stand for fuzzy sets denoting the importance/relevance of the individuals and alternatives.

These Kacprzyk and Fedrizzi's works constitute the basis of many soft consensus models proposed later. In the following we present some of the most prominent contributions:

- Herrera, Herrera-Viedma and Verdegay define the first soft consensus model in GDM problems with fuzzy linguistic preferences:

  F. Herrera, E. Herrera-Viedma, J. L. Verdegay, *A model of consensus in group decision making under linguistic assessments*. Fuzzy Sets and Systems 78(1) (1996) 73–87.

This is a prominent soft consensus contribution which has shown a high impact in the fuzzy decision making community being considered a highly cited paper according to the scientific database Essential Science Indicators. Authors present a new consensus model for GDM problems based on fuzzy linguistic preference relations defined in an ordinal fuzzy linguistic approach [45,46]. As main novelty authors define two types of soft consensus measures, consensus degrees and proximity measures. Both measures are computed in the three representation levels of a preference relation: level of preference, level of alternative, and level of preference relation. The consensus degrees indicate how far the set of experts is from the maximum consensus, and the proximity measures indicate how far each individual is from current consensus labels over the preferences. In such a way, this proposal provides moderator a complete consensus instrument to control the consensus reaching process.

- Later, assuming also a fuzzy linguistic context the same authors present the first consensus model which is guided by both consensus and consistency measures,

  F. Herrera, E. Herrera-Viedma, J. L. Verdegay. *A rational consensus model in group decision making using linguistic assessments*. Fuzzy Sets and Systems 88(1) (1997) 31–49.

This new consensus model also provides moderator consistency measures to guide the decision process. Then, we can achieve more consistent solutions, i.e., to avoid the effects of the inconsistencies existing in the experts' opinions.

- Other prominent contribution in soft consensus was proposed by Herrera-Viedma, Herrera, and Chiclana,

  E. Herrera-Viedma, F. Herrera, F. Chiclana. *A consensus model for multiperson decision making with different preference structures*. IEEE Transactions on Systems Man and Cybernetics-Part A: Systems and Humans 32(3) (2002) 394–402.

They define a new consensus model to deal with decision situations in which the experts can use different representation formats to express their preferences. This contribution contains two main novelties. Firstly, consensus measures are computed by comparison between experts' solutions and not between experts' preferences (as it would happen in previous consensus approaches). In such a way, authors overcome the problem of consensus computing when we use different preference formats in GDM problems. And secondly, using these measures, a feedback mechanism based on rules to aid experts change their testimonies is defined. In such a way, consensus reaching process could be guided automatically, without moderator, avoiding the possible subjectivity that moderator could introduce in the consensus reaching process. We should point out that this consensus contribution is a highly cited paper according to the Essential Science Indicators too.

- Herrera-Viedma, Martinez, Mata and Chiclana deal with the consensus problem under fuzzy multi-granular linguistic preferences, i.e., by assuming that experts could use different linguistic domains to express their opinions.

  E. Herrera-Viedma, L. Martinez, F. Mata, F. Chiclana. *A consensus sup- port system model for group decision-making problems with multigranular linguistic preference relations*. IEEE Transactions on Fuzzy Systems 13(5) (2005) 644–658.

The main novelty of this contribution is to present an automatic control system to guide the consensus process that substitutes the moderator's actions. This consensus model uses the consensus degrees to decide when the consensus process should finish and the proximity measures to define a recommendation system that recommends experts about the preferences that they should change in the next consensus rounds. Currently, this contribution is also considered a highly cited paper according to the Essential Science Indicators.

- Finally, other seminal consensus contribution is proposed by Herrera-Viedma, Alonso, Chiclana, and Herrera in

  E. Herrera-Viedma, S. Alonso, F. Chiclana, and F. Herrera. *A consensus model for group decision making with incomplete fuzzy preference relations*. IEEE Transactions on Fuzzy Systems 15(5) (2007) 863–877.

The main of this soft consensus approach is to provide tools to support the consensus processes in presence of incomplete information or missing values in GDM problems. In such a way, the authors define the first soft consensus model without moderador which is controlled automatically by means of three kind of measures: consensus measures, consistency measures and incompleteness measures too. Similarly, this contribution is considered a highly cited paper in Essential Science Indicators.

4. Consensus approaches in GDM

In the literature, we can find different consensus approaches according to different criteria as reference domain, concept of coincidence, generation method of recommendations, and guiding measures. According to the reference domain used to compute the soft consensus measures, we find some approaches based on the expert set and others on the alternative set. According to the coincidence concept used to compute the soft consensus measures, we find some consensus approaches based on strict coincidence among preferences, consensus approaches based on soft coincidence among preferences, and consensus approaches based on coincidence among solutions. According to the generation method of recommendations we find approaches in which the moderator is who guides the consensus reaching process and generates the recommendations to the experts to increase the consensus level in the next round of consensus, and on the other hand, we also find other approaches in which the process is guided automatically, without moderator's participation. And finally, according to the guiding measures we can find consensus approaches guided only by soft consensus measures and consensus approaches guided using also others kind of measures, as for example, consistency measures. In the following, we describe all these different approaches.

4.1. Consensus approaches according to reference domain used to compute the soft consensus measures

As aforementioned, according to the reference domain used to obtain the consensus measures, two different consensus approaches can be found. On the one hand, consensus measures focused on the expert set have been proposed in [5,11,23,44,47]. In these contributions consensus degrees are obtained in three steps: (i) for each pair of individuals, a degree of agreement as to their opinions between all the pairs of options are computed, (ii) these agreement degrees are combined to obtain a degree of agreement of each pair of individuals as to their preferences between $Q1$ pairs of options, and finally, and (iii) these agreement degrees are combined to obtain a agreement degree of $Q2$ pairs of individuals as to their preferences between $Q1$ pairs of options, which is the consensus degree of the group of experts.

On the other hand, consensus measures focused on the alternative set have been proposed in [12,13,21,48–50]. In these consensus approaches, the consensus measures are computed by considering the three levels of a preference relation: (i) level of preference, which indicates the consensus degree existing among all the m preference values attributed by the m experts to a specific preference, (ii) level of alternative, which allows us to measure the consensus existing over all the alternative pairs where a given alternative is present, and (iii) level of preference relation, which evaluates the social consensus, that is, the current consensus existing among all the experts about all the preferences. With these measures we can know the consensus situation in each representation level, and in such a way to identify which experts are close to the consensus solutions, or which alternatives present more problems to reach consensus.

Comparing both approaches the latter seems better to design consensus processes that allow us to guide the experts to change their preferences during the discussion process.

4.2. Consensus approaches according to the coincidence method used to compute the soft consensus measures

In any consensus process, it is primordial to establish consensus measures which can be used to evaluate the current consensus stage. On the one hand, some authors use soft consensus measures valued in $[0,1]$, where a value close to 1 indicates a high level of consensus and a value close to 0 indicates a low level of consensus [22,31,43,48,51]. On the other hand, some authors have proposed soft consensus measures based on linguistic labels [12,13,52] to evaluate the level of consensus instead of using numerical values in $[0,1]$. Anyway, to obtain the level of consensus achieved in each discussion round, the similarity among the opinions provided by the experts on the alternatives is measured. Then, we use the coincidence concept to compute soft consensus measures [52] according to three different approaches [53]:

1. *Strict coincidence among preferences.* The coincidence is obtained by means of similarity measured among expert preferences. We find two possible assessments: value 1 meaning a total coincidence and value 0 meaning non-existent coincidence. See in [13,43] how this coincidence concept is used to define soft consensus measures.
2. *Soft coincidence among preferences.* Again, the coincidence is obtained by means of similarity measured among expert preferences. However, we further consider different partial coincidence degrees. So, in this case we assume a gradual conception of the coincidence that we can assess in $[0,1]$. See in [5,11,37,43,48–50,54] some examples of use of this coincidence approach. We should point out that this coincidence approach is very extended in GDM.
3. *Coincidence among solutions.* The coincidence is obtained by means of similarity measured among the individual solutions obtained from the experts' preferences. In this case, the coincidence is also a gradual concept assessed in $[0,1]$ [30,55]. We work on the positions of the alternatives observed in the individual solutions and the collective solution. We should point out that this coincidence approach provides us a more realistic consensus measure among experts.

We should point out that the second and third method are the most useful approaches to design consensus processes that allow us to advice the experts during the consensus reaching process. The second is specially applied in contexts of GDM under preference relations and the third one in decision situations under different formats of preference representation.

4.3. Consensus approaches according to the generation method of recommendations supplied to the experts

In the consensus reaching processes the generation of recommendations to be sent to the experts allows us to improve the consensus state. From this point of view, the first consensus approaches proposed in the literature [5,11–13,22,43,54] can be considered as basic approaches based on a moderator who provided the recommendations to the experts. The moderator's goal in each round is to address the consensus reaching process towards success by achieving the maximum possible agreement degree and reducing the number of experts outside of the consensus. However, the moderator can introduce some subjectivity in the process. To overcome this problem, making more effective and efficient the decision making processes, new consensus approaches have been proposed by substituting the moderator figure or providing moderator with better analysis tools.

In [30,48,50,56]. some consensus approaches incorporating a feedback mechanism substituting the moderator's actions have been developed. In these approaches, proximity measures are calculated to evaluate the proximity between the individual experts' preferences and the collective one. These proximity measures are used to identify the preference values provided by the experts that are contributing less to reach a high consensus state. And then, the feedback mechanism gives advice to those experts to find out the changes they need to make in their opinions to obtain a solution with better consensus degree.

On the other hand, consensus approaches have been proposed using a novel data mining tool [57], the so called action rules [58], to stimulate and support the discussion in the group. The purpose of an action rule is to show how a subset of flexible attributes should be changed to obtain an expected change of the decision attribute for a subset of objects characterized by some values of the subset of stable attributes. According to it, these action rules are used to indicate and suggest to the moderator with which experts and with respect to which options it may be expedient to deal.

We should point out that the current consensus trends are committed to develop automated feedback mechanisms that replace the moderator, especially when consensus processes are developed in crowded social environments, such as Web 2.0 [59,60].

4.4. Consensus approaches according to the kind of measures used to guide the consensus reaching process

Using preference relations, the experts provide their preferences by focusing only on two elements once at a time. This allows by reducing uncertainty and hesitation while leading to the higher of consistency. However, the definition of a preference relation does not imply any kind of consistency property, and, due to the complexity of most GDM problems, experts' preferences can be inconsistent [61–63]. Fortunately, the lack of consistency can be quantified and monitored [13,64,65], and it has been used as a parameter to validate the final solution obtained after consensus reaching process [13,16,62,63]. Therefore, some consensus approaches that combine both consistency and consensus measures to guide the consensus processes have been proposed in [13,48,56]. Usually, in these approaches a consensus/consistency level is calculated as a weighted aggregation of the consistency level and the consensus degree, and it is used as a control parameter to decide if the consensus reaching process must finish and the selection process can be applied. The use of the consistency measures in the consensus approaches avoids misleading solutions, which cannot be detected by the consensus approaches using only consensus degrees [5,12,22,43,50,54].

We should point out that the incorporation of other additional criteria in the consensus process, as consistency measures, contributes to enrich the consensus reaching processes and to achieve more adequate solutions in the GDM problems.

5. Current trends in the development of consensus models

In this section, we present some current trends in the field of consensus models together with some open questions and prospects about them. We identify four current trends:

1. Adaptive consensus models.
2. Trust based consensus models.
3. Dynamic and changeable consensus models.
4. Consensus models based on agent theory.

5.1. Adaptive consensus models

The automatic consensus models existing in the literature act in a similar way during all consensus rounds although the conditions of GDM problem change [30,48,50,56]. However, it seems appropriate to distinguish different situations: if the agreement degree is "high" we should implement recommendation strategies that result in little change in the preferences of experts in order to achieve a consensus acceptable status in a few rounds. If the agreement degree is "low", we should implement recommendation strategies that result in many changes in the preferences of experts to achieve a consensus acceptable status, and maybe many rounds might be necessary to achieve that acceptable consensus. Therefore, we should implement recommendation strategies that adjust the number of changes required depending on the degree of consensus among experts in each round. Following this idea, an adaptive consensus model has been proposed by Mata et al. in [66]. This model adapts the number of changes required to the experts in each round of consensus, so that as the degree of consensus increases the number of changes required decreases. Then three levels of consensus are set: very low, low and medium. With the consensus level is very low or low we search for the furthest preferences on all experts. If the consensus level is medium, we search for the preferences on the furthest experts.

There are still some open questions about adaptive consensus models:

- It would be desirable to extend adaptive consensus models to decision making contexts managing different formats of preference representation.
- To study a mechanism to guarantee the convergence of the adaptive consensus models is still a challenge.

5.2. Trust based consensus models

We can also find that in practical decision making situations the group of experts could vary over time. For example, we could find new and important experts to solve the decision process or we could identify experts unsuitable for the decision process or we could require to simplify a subgroup of experts in order to facilite the achievement of the solution. This happens in decision processes developed in online community that share a common interest. In such decision contexts we find a great number of experts participating in the decision process, it is usual to establish the communication among experts by means of opinion polls and forums, and there not exist automatic methods to guide the consensus and it is difficult to achieve acceptable consensus among the experts. To address these situations, a new consensus model has been proposed in [60]. This consensus approach manages the decision frameworks with large number of experts. So, before to develop the consensus process the large group of experts is simplified into a subgroup of experts or spokespersons by means of a clustering algorithm. This is done by maintaining the diversity on the opinions of the whole group. The experts that are discarded provide trust degrees on experts that compose that subgroup of experts, and in suc a way a trust network is built [67]. Then, consensus process is applied with that subgroup of selected experts. During the consensus process discarded experts can change their trust evaluations on the subgroup of selected experts. When the consensus process finishes the final opinions given by that subgroup of selected experts is used to obtain the final solution.

The use of trust degrees in consensus processes is still in an early stage of development and several future challenges have still to be solved:

- Development of automatic process to compute the trust degrees in decision making.
- To introduce in the consensus models advanced trust management models that have been used satisfactorily in other frameworks, as recommender systems [68].
- Using the trust criteria as other possibility to guide the consensus process.

5.3. Dynamic and changeable consensus models

In the real decision process we could find that the set of alternatives could be dynamic, i.e., that some alternatives might disappear and new ones appear through the decision making time. This could happen because or the availability of some of alternatives changes while experts are discussing and making the decision, or experts evaluate the alternatives poorly or we find better alternatives to solve our decision problem. However, classical consensus models assume static sets of alternatives. To solve these situations, new consensus approaches have been proposed in [69,70]. These approaches provide a tool to deal with dynamic decision frameworks by allowing to change the alternatives that compose the set of solution alternatives. Therefore this consensus model introduces a procedure to remove old bad alternatives, and other to insert new ones. Those alternatives that are not available or present low evaluations are replaced by new and good alternatives. Experts provide their preferences on the replacements of alternatives and the system acts consequently. On the other hand, when new alternatives emerge, the system evaluates their goodness and looks for those worst alternatives existing in the set of alternatives to initiate the replacement process.

With respect to the dynamic and changeable aspects to be considered in consensus models, some questions are still open:

- Development of consensus models by considering the variable time in decision making problems as it was proposed by the Saaty in [71].
- To study all elements of a GDM problem that could vary throughout decision making problem, as for example, the importance of the alternatives or experts.

5.4. Consensus models based on agent theory

In [72] a new consensus proposal based on agent theory is defined. It introduces two main novelties: (i) a new tool to visualize the alternatives based on ontologies that is more suitable for the discussion, (ii) and advanced format to represent the individual preferences. An alternative is represented as a pair s: (T_s, A_s) where T_s is a hierarchy of components and A_s is a set of values of the attributes. In previous models, the discussion in the group was not explicitly represented in the model. Here the discussion phase among agents is represented by a ontology and the arguments used during the discussion are represented in that discussion ontology. Therefore, the proposal uses both ontologies to support the representation of individual preferences, argumentations and it is possible to measure the consensus degree on several levels of abstraction. For example, it might be the case that there is no consensus on a particular travel to buy, both the there is consensus that international travels are desired. In such a way, this approach provides experts or agents with tools and rich information about the issues and opinions and attitudes of other agents, that could contribute positively in the development of the consensus process.

In this kind of consensus models it would be interesting to study some of the following challenges:

- To extend these consensus models to Web 2.0 contexts as it was recently done in [60].
- Similarly, to extend these consensus models to Web 3.0 contexts which are based on the development of ontologies to represent the knowledge.
- Develop visualization tools that allow to understand better the performance of the consensus model based on agent theory.

6. Concluding remarks

In this paper, we have comprehensively analyzed consensus approaches based on soft consensus measures in which the consensus reaching process is guided by a moderator. To do so, we have introduced some basic concepts to understand the topic, we have highlighted both the pioneer and most relevant contributions on consensus models, we have described several approaches of consensus in GDM according to different criteria, and we have shown the current trends in the development of fuzzy consensus models and prospects on the topic have been presented too.

We observe that the soft consensus approaches have been a very productive topic in the last years, and it will probably continue being a hot topic in the future. Additionally to the prospects presented in Section 5 we should point out other two important challenges that should be addressed in forthcoming contributions too:

- As it has been shown, consensus approaches incorporating a feedback mechanism substituting the moderator's actions have been widely adopted by researchers. Although they show some good results giving advice to the experts to increase the consensus degree and avoiding the subjectivity that the moderator can introduce, there are still some aspects to be addressed. For example, in heterogeneous situations, the consensus approaches have taking into account the importance of the experts when aggregating the experts' opinions to obtain the collective preference [12,13,73], but not when advising to the experts how to change their preferences to increase the consensus level. Then, we think that it is important to developed new feedback mechanisms considering the experts' importance when advising experts to change their preferences, adjusting the amount of advice required by each expert depending on his/her own knowledge level about the problem. It seems reasonable that experts with lower importance or knowledge level will need more advice than those experts that previously have at their disposal a large amount of information to make good decisions. Therefore, the consensus approaches should generate the recommendations in a different way depending on the expert's knowledge level in order to increase the agreement in the next consensus round.
- It is important that the experts accept the advice given by either the moderator or the feedback mechanism to increase the consensus level in the next consensus round. However, some experts can decide not to accept the recommendations and, therefore, the consensus could not be achieved. Therefore, it is important to persuade the experts to follow the advice. To do so, some psychology concepts (or principles or persuasion) in the consensus reaching process can be used. According to Cialdini [74], the different tactics that people employ to influence other fall within six basic categories: (i) social proof, people will do things that they see other people are doing, (ii) authority, people will tend to obey authority figures, even if they are asked to perform objectionable acts, (iii) linking, people are easily persuaded by other people that they like, (iv) scarcity, infrequent items or resources will generate demand, (v) consistency, if people commit, verbally or in writing, to and idea or goal, they are more likely to honor that commitment, and (vi) reciprocation, people tend to return a favor. Each of these categories is governed by a fundamental psychological principle that directs human behavior and gives the tactics power of persuasion [74]. Therefore, these principles of persuasion or weapons of influence can be used as a support for the consensus process as they address the use communication in order to change attitudes, beliefs or the behavior of others in a voluntary manner avoiding the use of coercion.

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