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GDM-R: A new framework in R to support fuzzy group decision making processes

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ABSTRACT

With the incorporation of web 2.0 frameworks the complexity of decision making situations has exponentially increased, involving in many cases a huge number of decision makers, and many different alternatives. In the literature we can find a great variety of methodologies to assist multi-person decision making. However these classical approaches are not suitable to deal with such complexity since there are no tools able to carry out automatically the decision making processes, providing graphical information about its evolution.

The main objective of this contribution is to present an open source framework fully developed in R to carry out consensus guided decision making processes using fuzzy preference relations and providing mechanism to deal with missing information. The system includes tools to visualize the evolution of the decision making process and presents various operation modes, including a test operation one which automatically creates a customized decision scenario to validate, test and compare among various decision making approaches.

1. Introduction

It has been traditionally assumed that knowledge is a sparse “commodity” in the sense that some specific individuals of the society own it, not everybody. Furthermore, it is divided in the sense that not all individuals of the “chosen ones” mentioned above have knowledge on some topic of interest or relevance to the same degree. Hence, a subgroup of individuals (experts) should be chosen to most efficiently and effectively employ that knowledge [33], and their opinions should be considered to arrive at a consensus solution accepted by the group as a whole [10]. Group decision making (GDM) consists of multiple decision makers, with different knowledge and points of view, interacting to choose the best option among all the available ones [14,32].

GDM processes have attracted research attention in the last ten years and therefore a wide range of different methodologies have been proposed [28,36]. However, new paradigms and ways of making decisions, such as web 2.0 frameworks,

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social networks and e-democracy, have made the complexity of decision making processes to increase, involving a huge number of decision makers [7]. These new scenarios require automatic tools not only to combine the information in the best possible way but also to better analyze the whole context, providing a rapid and complete insight about the current state of the process. In this direction, some efforts have already been made [6,37–40], however, these approaches present various deficiencies: (i) they do not make available graphical visualizations or output measures displaying the evolution of the process, and (ii) they do not offer the possibility of creating a data set to test and compare the performance of different approaches. (iii) They are developed as closed systems and, hence, they are not aimed to be upgraded or extended by other researchers,

In this paper, we present a new framework to carry out GDM processes, both in classical and current scenarios, whose main features are:

- The proposed system provides support for both real GDM situations and simulation environments. Being useful not only to assist decision making processes, but also to compare and validate already existing approaches and to develop new ones.
- The system provides powerful visualizations tools to quickly verify the state of the decision process. Among its various representations it depicts experts' preferences 3D maps to quickly detect those experts who are far from the consensus solution and are more reluctant to change their mind and also to detect those ones who provide more contradictory or inconsistent opinions. The system also allows the user to visually check the evolution of the global consensus and consistency among the various round of consensus.
- In many GDM situations, especially those involving a large number of alternatives to choose from conflicting and dynamic sources of information, some of the decision makers could not efficiently express their opinions over all the available options, and sometimes it is necessary to deal with incomplete information [46], being necessary to try to estimate the missing information since it could be very valuable for the decision making process. In such a way, the system is able to deal with this uncertainty.
- It carries out a number of consensus round to obtain a solution accepted by all the decision makers [13,28,36] and provides the best alternative using well known decision making algorithms [11,24].
- It is an open source framework implemented in R [1], following a modular architecture which easily enables the extension of the tool by other researchers.

The rest of the paper is set out as follows: In Section 2 we carry out a review of the main concepts of GDM, including consensus and mechanism to estimate missing preference relations. Section 3 reviews and analyzes the existing tools available carry out GDM. In Section 4, we present our new R framework to support decision making. A practical example is included in Section 5 to illustrate how the proposed system works, its usefulness and all its capabilities and visualizations tools. Finally Section 6 closures this work pointing out future research lines and summarizing the main novelties and features of the proposed framework.

2. Background

In order to make this paper as self-contained as possible, in this section we briefly introduce the main concepts used along the paper. First, we describe a classical GDM situation and, second, we focus on their different steps. Finally a brief description of R, the software environment and programming language, used to develop the proposed framework is presented.

2.1. GDM problems

A classical GDM problem may be defined as a decision situation where [21]: (i) there exists a group of two or more decision makers, $E = \{e_1, \ldots, e_m\}$ $(m \geq 2)$, (ii) there is a problem to solve in which a solution must be chosen among a set of possible alternatives, $X = \{x_1, \ldots, x_n\}$ $(n \geq 2)$, and (iii) the decision makers try to achieve a common solution. In a fuzzy context, the objective is to classify the alternatives from best to worst, associating with them some degrees of preference expressed in the [0, 1] interval.

There are various preference representation formats which can be used by decision makers to provide their testimonies [30]. Among them, preference relations are one of the commonly used because decision makers have much more freedom when expressing their opinions and they can gain in expressivity. In particular, the fuzzy preference relations are the most used in the literature [32,35,45].

Definition 1. A fuzzy preference relation P^h on a set of alternatives X, given by a decision maker e_h, is a fuzzy set on the Cartesian product $X \times X$, i.e., it is characterized by a membership function $\mu_P : X \times X \longrightarrow [0, 1]$.

A fuzzy preference relation P^h may be represented by the $n \times n$ matrix $P^h = (p^h_{ik})$, being $p^h_{ik} = \mu_{P^h}(x_i, x_k)$ $(\forall i, k \in \{1, \ldots, n\})$ interpreted as the degree or intensity of preference of alternative x_i over x_k: $p^h_{ik} = 1/2$ indicates indifference between x_i and x_k $(x_i \sim x_k)$; $p^h_{ik} = 1$ indicates that x_i is absolutely preferred to x_k; $p^h_{ik} > 1/2$ indicates that x_i is preferred to x_k $(x_i \succ x_k)$. Obviously, we have that $p^h_{ii} = 1/2$ $\forall i \in \{1, \ldots, n\}$ $(x_i \sim x_i)$.

In what follows, we are going to describe two important aspects which have to be taken into account when dealing with fuzzy preference relations in GDM problems.

2.1.1. Consistency

Due to the complexity of most GDM problems, decision makers' preferences may not satisfy formal properties that fuzzy preference relations are required to verify. Actually, the preference values can be contradictory. In [31], it was presented some properties that need to be satisfied by fuzzy preference relations to make a rational choice. Consistency is one of them, which is crucial for avoiding misleading solutions [5,12,19].

Consistency can be interpreted as a measure of the self-contradiction expressed in the preference relation and is related to the concept of transitivity [15]. A preference relation is considered consistent when the pairwise comparisons among every three alternatives satisfy a particular transitivity property. For fuzzy preference relations, there exist many properties or conditions that have been suggested as rational conditions to be verified by a consistent relation [16]. Here, we take advantage of the additive transitivity property. As it is shown in [29], additive transitivity for fuzzy preference relations can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [42]. The mathematical formulation of the additive transitivity was given by [45]:

$$(p^h_{ij} - 0.5) + (p^h_{jk} - 0.5) = p^h_{ik} - 0.5, \quad \forall i, j, k \in \{1, \ldots, n\} \tag{1}$$

Additive transitivity implies additive reciprocity. Indeed, because $p^h_{ii} = 0.5,\ \forall i$, if we make $k = i$ in Eq. (1), then we have: $p^h_{ij} + p^h_{ji} = 1,\ \forall i, j \in \{1, \ldots, n\}$. Eq. (1) can be rewritten as follows:

$$p^h_{ik} = p^h_{ij} + p^h_{jk} - 0.5, \quad \forall i, j, k \in \{1, \ldots, n\} \tag{2}$$

A fuzzy preference relation is considered to be "additively consistent" when for every three options encountered in the problem, say $x_i, x_j, x_k \in X$, their associated preference degrees, $p^h_{ij}, p^h_{jk}, p^h_{ik}$, fulfill Eq. (2).

Given a fuzzy preference relation, Eq. (2) can be used to calculate an estimated value of a preference degree using other preference degrees. Indeed, using an intermediate alternative x_j, the estimated value of p^h_{ik} $(i \neq k)$ can be obtained in three different ways (see [29]).

2.1.2. Incomplete information

Missing information is a problem that we have to consider as decision makers are not always able to provide preferences degrees between every pair of possible alternatives. It might be due to a number factor such us time pressure, lack of knowledge or data, or limited expertise related to the problem domain [46]. In order to model these situations, the following definitions express the concept of an incomplete fuzzy preference relation [29].

Definition 2. A function $f : X \longrightarrow Y$ is *partial* when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a *total* function.

Definition 3. A fuzzy preference relation P on a set of alternatives X with a *partial* membership function is an incomplete fuzzy preference relation.

According to it, the completeness level C^h for the preference relation P^h given by decision maker e_h is computed as:

$$C^h = \frac{\#EV^h}{n \cdot (n-1)} \tag{3}$$

where $\#EV^h$ is the number of preference values provided by the decision maker e_h. When $C^h = 1$ then the fuzzy preference relation is complete (all values are known).

2.2. GDM steps

The solution for a GDM problem is derived either from the individual preferences provided by the decision makers, without constructing a social opinion, or by computing first a social opinion and then using it to find a solution [32]. Here, we focus on the second one, since we are interested in obtain a solution accepted by the whole group of decision makers (see Fig. 1). In the following, we describe in more details these steps and, in the next section, we will explain how they have been implemented in the proposed framework.

2.2.1. Aggregation step

In order to obtain a collective fuzzy preference relation, the aggregation step of a GDM problem consists in combining all the preferences given by the decision makers into only one preference structure that summarizes or reflects the properties contained in all the individual preferences. This aggregation can be carried out by means of particular aggregation operators that are usually defined for this purpose [50]. Among them, the Ordered Weighted Averaging (OWA) operator proposed by Yager [48] and the Induced Ordered Weighted Averaging (IOWA) operator [49] are the most widely used.

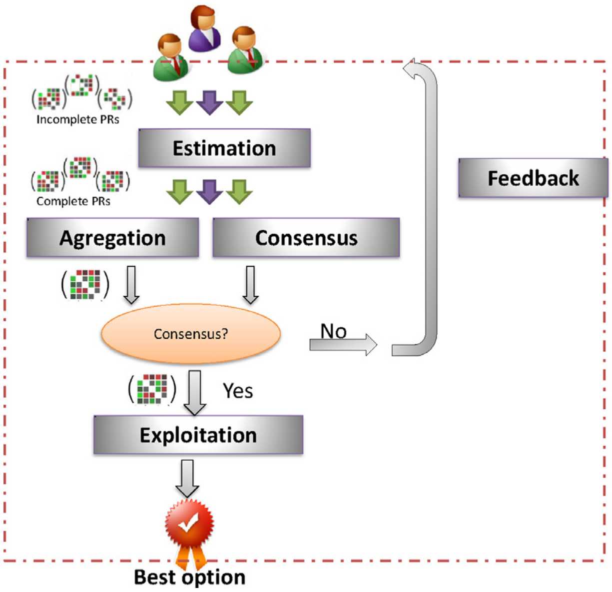


Fig. 1. Steps of a GDM process.

2.2.2. Exploitation step

In order to identify the solution set of alternatives, the exploitation step uses the information produced in the aggregation step. Here, some mechanism must be applied to obtain a partial order of the alternatives and thus select the best one(s). There are several ways to do this. A usual one is to associate a certain utility value to each alternative, based on the aggregated information, producing a natural order of the alternatives. To do so, two quantifier-guided choice degrees of alternatives can be used: a dominance and a non-dominance degree [29].

2.2.3. Consensus

The steps above, aggregation and exploitation, compose the selection process for reaching a solution of any GDM problem [21,41] without requiring any consensus among the decision makers. However, this could led to situations in which some decision makers do not agree with the final solution, since they may consider that their opinions have not been taken into account [10,43]. To avoid these situations, it is preferable to include mechanisms, which are widely known as consensus processes [28], to check the agreement among the decision makers before obtaining a solution. A consensus process is a negotiation process composed by several consensus rounds, where the decision makers agree to change their testimonies following the advice given by a moderator, which knows the agreement degree in each round of the consensus process by means of the computation of some consensus measures [13]. If an enough consensus state has been reached, the consensus process stops and the above selection process begins. Otherwise, a feedback step is applied, where the moderator, with all the available information (all preferences given by the decision makers, consensus measures and so on), can prepare some advice for the decision makers to more easily reach consensus.

2.3. R software environment

R [1] is a free and open source software environment for statistical computing and graphics, which includes a free implementation of the high-level language S [8] originally created and distributed by Bell Labs. R runs on all major operating systems, i.e., Windows, GNU/Linux, and Mac OS X and it can be considered as an alternative to traditional statistical packages such as SPSS, SAS, and Stata. R main advantages are that it allows for the user to freely distribute, study, change, and improve the software under the Free Software Foundation's GNU General Public License and that performs a wide variety of basic to advanced statistical and graphical techniques. These advantages over other statistical software encourage the growing use of R in many well-known research groups and universities, and there is an extensive research community developing frameworks in this language.

R is a functional programming language whose main data structure is the data-frame, which consist in a matrix that supports different types of values and whose rows and columns can be accessed by both index and name.

Additionally R provides very powerful tools to carry out graphical representations. Among these tools, we highlight the ones used in this framework. The lattice library [44] has been used to represent statistical plots such as barplots and scattered plots. The library scatterplot3D [34] has been used to represent 3D plots. And the library rgl3 [4] has been used to represent 3D interactive graphics.

For all the reasons explained above (multi-platform, open-source, extensive use in the research community and powerful visualizations tools) we have selected R as the only language to develop our framework. Moreover as far as we know there is no tools developed in this widely used language to support fuzzy group decision making processes.

3. Related work

In this section, we review the existing computerized tools to assist GDM processes pointing out their main weaknesses.

- In [6], it is presented a web based consensus support system dealing with different types of incomplete preference relations. It is developed to work with web environments and, to that aim, it is fully implemented using a LAMP stack (GNU/Linux operating system, Apache web server, MySQL database server and PHP programming language). This system implements the iterative decision making process proposed in [29], along with the consensus reaching process proposed in [27].
- In [39], it is presented a prototype of a decision support system (DSS) designed for dynamic mobile systems. It carries out an iterative consensus process, offering the possibility to express preferences in various representation formats such as preferences orderings, utility functions, fuzzy preference relations and multiplicative preference relations. It also presents a new approach for dealing with dynamics alternatives, that is, it is able to include new alternatives during the decision process, or to remove the old ones. It is implemented using a "client/server" architecture, being the client a mobile device sending the preferences to the server and receiving the results, whereas the server carries out the data aggregation and computation. The technologies used comprise Java and Java MIDlets for the client software, PHP for the server functions, and MySQL for the database management.
- In [40], it is presented as ontology based consensus approach and its web implementation as a tool to select wines. It is aimed to deal with a large set of alternatives by defining a fuzzy ontology which selects an smaller sub-set of the most likely ones which fulfill the decision makers' preferences, reducing the complexity of the decision process.
- In [38], a graphical monitoring tool based on Self-Organizing Maps (SOMs) is proposed. It provides a 2-D graphical interface showing the temporal evolution of the decision makers' preferences. This system is aimed to ease the analysis of information in GDM processes involving a large number of decision makers. It also provides important information such as the detection of agreement/disagreement positions within the group, the evolution of decision makers' preferences, or the level of closeness among decision makers' opinions. This tool uses both JAVA to generate the data sets from the preference relations and Matlab to compute the SOMs and obtaining the graphical representations.
- In [37], it is presented a multiagent approach of a consensus system to deal with GDM processes involving a large number of decision makers. It aims to overcome the problem of the human intervention, presenting a semisupervised operation mode in which there is no need to use a human moderator in the different consensus rounds.

From this review, the main weaknesses identified in the above tools are summarized as follows:

1. The majority of the already available tools are developed as closed systems and therefore they are not aimed to be upgraded or extended by other researchers, since in most of the cases they do not provide the source code or they are based in proprietary software.
2. They are extremely dependent of the user interface and so they cannot be adapted to work in other environments such as smart phones.
3. The available DSSs do not provide any type of graphical visualizations or output measures illustrating the evolution of the consensus process.
4. There are not many methodologies or tools that easily creates a dataset to test and compare the performance of various applications. Some initial efforts in this direction have been presented in [9,18] and [36]. In [9] it is conducted a comparative study of seven different methods for reconstructing incomplete fuzzy preference relations in terms of the consistency of the resulting complete preference relation; In [18] it has been carried out a statistical comparative study to find out the differences in group consensus that different distance measures could lead to. In [36] it has been carried out a review of various consensus methodologies and a framework in Java to compare them has been proposed.

As we will explain in the next section, the proposed framework aims to overcome the main weaknesses that these tools present as well as encompassing their main strengths in just one open source GDM framework.

4. A new framework in R to support GDM processes in a fuzzy environment

In this section, we present a new open source framework fully developed in R to automatically support GDM processes. The proposed system named as GDM-R, has been designed following a Model-View-Controller architectural pattern [22]. Therefore, the logic is completely separated from the data storage requirements and from the user interface. This design eases its adaptation to different interfaces, such us web or mobile environments, since it works totally independently from the user interface. The framework is built from various processing independent modules so it can be easily upgraded and extended just by making changes in a particular module or adding new ones. The last version of the developed R package is available in [3] and the documentation with a detailed description of all the available functions can be downloaded in [2].

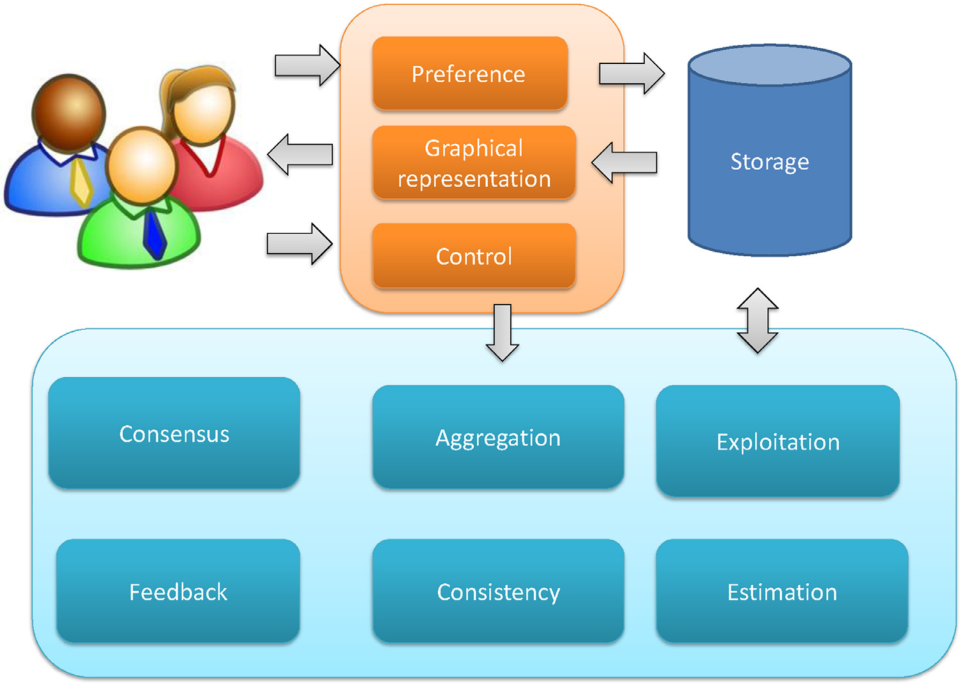


Fig. 2. Architecture of the developed framework and the main interactions between modules.

The developed framework tries to fill the gap that the other systems leave. To that aim, it includes powerful visualization tools, and enables various working modes. To do so, the system is composed of the following modules: (i) control module, (ii) preference module, (iii) estimation module, (iv) consistency module, (v) aggregation module, (vi) consensus module, (vii) feedback module, (viii) exploitation module, (ix) storage module, and (x) graphical representation module. The framework's architecture is depicted in Fig. 2 which shows the interaction among all the modules.

In the following sections, we describe the different modules, focusing on their characteristics and in relevant implementation details.

4.1. Control module

The framework presents a centralized architecture where the control module manages the whole system, and works as its starting point. This module carries out the following four main functionalities:

1. *Configuration parameters setting*. The system offers an user interface in which the user can set the main parameters of the GDM process (the meaning of most of this parameters will be explained in detail in the following sections):
   - Working mode: test, normal.
   - Number of decision makers.
   - Path where the decision makers' preferences are located. This path could point to the computer file system or could be an URL.
   - Consensus threshold: γ parameter.
   - Max number of consensus round: *MaxIter*.
   - Feedback mechanism: automatic, semisupervised, non-supervised.
   - Type of distance used in the consensus process: Euclidean, Manhattan, Jaccard, Cosine, Dice.
   - Weighting vector for the aggregation.
   - Exploitation type: dominance or non-dominance degree.
   - Consistency vs. Consensus: δ parameter.
2. *Communication with other modules*. In order to make the system fully upgradeable and extensible, the control module coordinates and initializes all the other modules.
3. *Control of the consensus rounds and the feedback mechanism*. This module also checks if enough consensus degree has been reached. Otherwise, it starts the feedback mechanism and, if necessary, asks to the decision makers to provide new fuzzy preference relations following some advice.
4. *Access to the data storage module*. This module has fully access to all the data frames in the system.

4.2. Preference module

This module is in charge of obtaining and adapting the decision makers' fuzzy preference relations. To that aim, we can distinguish two operation ways: test and normal modes.

- *Normal mode*. In this operation mode, the decision makers have to provide their complete or incomplete fuzzy preference relations by means of a CSV file, one file per decision makers, receiving the system one or more paths where the fuzzy preference relations are located. These paths can point to a file in the computer's file system where the program is running or to an URL where the files are located. In the last case, all the files will be automatically downloaded.
- *Test mode*. In this case a data set with the fuzzy preferences relations is automatically generated and the user only has to set the number of decision makers and the number of alternatives. The system can build both consistent and non-consistent fuzzy preference relations. Moreover, to test the quality of the available completion algorithms, incomplete fuzzy preference relations can be also generated.

Once all the fuzzy preference relations have been provided by the decision makers or generated by the system, they are included in an unique R data frame to be used in the next steps of the GDM process. Each row in the generated data frame corresponds to the preferences of one decision maker, not including the diagonal elements of the fuzzy preference relation. It is important to note that the system can work with any number of decision makers and any number of alternatives.

4.3. Estimation module

Prior to any other computation, the system needs to make sure that all the provided fuzzy preference relations are complete. To do so, this module carries out the iterative completion approach proposed in [29], which is based on the additive consistency property. This process is as follows: given an unknown preference value p_{ik}^h $(i \neq k)$, the iterative procedure starts by using intermediate alternatives, x_j, to create indirect chains of known preference values, (p_{ij}^h, p_{jk}^h), that will be used to derive, using the additive consistency property, the local consistency based estimated values:

$$ep_{ik}^{hj} = p_{ij}^h + p_{jk}^h - 0.5 \tag{4}$$

By averaging all the local consistency based estimated values, the overall consistency based estimated value is obtained:

$$ep_{ik}^h = \sum_{j=1, j\neq i,k}^{n} \frac{ep_{ik}^{hj}}{n-2} \tag{5}$$

In each iteration, the algorithm checks the set of pairs of alternatives for which the fuzzy preference values are unknown and can be estimated using known ones. It stops when this set is empty. Notice that the cases when an incomplete fuzzy preference relation cannot be successfully completed are reduced to those cases when no preference values involving a particular alternative are known, which means that a whole row or column of the fuzzy preference relation is completely missing. Finally, it is important to note that, although the approach proposed in [29] is used, any other algorithm of incomplete information [46] can be easily added to the framework.

4.4. Consistency module

This module calculates the self-contradiction level for each decision maker taking as a input his/her fuzzy preference relation. To that aim, it implements the consistency level based on the additive consistency proposed in [29], which defines the consistency level as the error between the provided preference relation and its estimated one.

The error between a preference value of a fuzzy preference relation P^h and its estimated one, computed in Eq. (5) is:

$$\varepsilon p_{ik}^h = |ep_{ik}^h - p_{ik}^h| \tag{6}$$

This definitions can be extended to calculate the consistency degree at three different levels, namely, pair of alternatives, alternatives and relation:

- Given a fuzzy preference relation P^h, the consistency level associated to the preference value p_{ik}^h is defined as:

  $$cl_{ik}^h = 1 - \varepsilon p_{ik}^h \tag{7}$$

  The lower the value of cl_{ik}^h, the higher the value of εp_{ik}^h and the more inconsistent is p_{ik}^h with respect to the rest of information.
- Given a fuzzy preference relation P^h, the consistency level associated to a particular alternative x_i is defined as:

  $$cl_i^h = \frac{\sum_{\substack{k=1 \\ i\neq k}}^{n} (cl_{ik}^h + cl_{ki}^h)}{2(n-1)} \tag{8}$$

  The lower the value of cl_i^h, the more inconsistent these preference values are.

- The consistency level of a fuzzy preference relation P^h is defined as follows:

$$cl^h = \frac{\sum_{i=1}^{n} cl_i^h}{n} \tag{9}$$

When $cl^h = 1$, the preference relation P^h is fully consistent. Otherwise, the lower cl^h the more inconsistent is P^h.

Finally, in a GDM problem, the global consistency measure is computed as follows:

$$CL = \frac{\sum_{h=1}^{m} cl^h}{m} \tag{10}$$

When $CL = 1$, all the decision makers are completely consistent. The lower CL is, the more inconsistent the group of decision makers is.

4.5. Aggregation module

This module is in charge of fusing all the fuzzy preference relations, $\{P^1, \dots, P^m\}$, given by the decision makers into a group one, P^c. To so, this module receives the data frame with all the preferences and stores the aggregated matrix in a separated data frame. The aggregation can be done in various ways:

- *Using an OWA operator* [48]. The OWA operator, ϕ_Q, carries out the aggregation as follows:

$$p_{ik}^c = \phi_Q\left(p_{ik}^1, \dots, p_{ik}^m\right) = \sum_{j=1}^{m} w_j \cdot p_{ik}^{\sigma(j)}, \tag{11}$$

  where σ is a permutation function such that $p_{ik}^{\sigma(j)} \geq p_{ik}^{\sigma(j+1)}$, $\forall k = 1, \dots, m-1$; Q is a fuzzy linguistic quantifier [51] that represents the concept of fuzzy majority [32] and it is used to calculate the weighting vector of ϕ_Q, $W = (w_1, \dots, w_n)$ such that, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following expression [48]:

$$w_j = Q(j/n) - Q((j-1)/n), \ \ \forall j \in \{1, \dots, n\} \tag{12}$$

- *Using an IOWA operator* [49]. Yager and Filev defined the IOWA operator as an extension of the OWA operator to allow a different reordering of the values to be aggregated. In this sense, in [29], an additive consistency based IOWA operator (AC-IOWA) was presented, where the ordering is induced depending on each decision maker's consistency, from the most to the least consistent one. In this case, the system automatically sets the decision makers' weights according to the consistency of the opinions provided by them in each round of consensus. To compute it, the system includes an implementation of the AC-IOWA operator presented in [29], although any other IOWA operator could be added [17].
- *Using a weighting vector set by the user*. This weighting vector can be used to set the decision makers' degree of importance in the aggregation. This way, the user can set the importance of each decision maker's opinion. This is especially useful in situations where the information handled by the decision makers is not equally relevant [17]. For example, when a group of medical experts expresses their opinions on the possible illness that a patient presents, its diagnostics must not be considered with equal relevance, given that, there will be more experienced medical experts, and, hence, all the opinions shall not be equally reliable.

4.6. Consensus module

This module receives as an input the data frame with all the decision makers' fuzzy preference relations and the aggregated matrix, and it calculates at each step of the process both the consensus degree, which measure the current level of agreement among all the decision makers, and the proximity measures, which quantify how far is each decision maker from the group opinion.

In order to reach agreement achieved among all the decision makers, the system computes coincidence existing among them [26]. To do so, the system, as most of the consensus approaches proposed in the literature, determines consensus degrees given at three different levels of a fuzzy preference relation [13,25]: pairs of alternatives, alternatives, and relation.

1. For each pair of decision makers (e_h, e_l) $(h = 1, \dots, m-1,\ l = h+1, \dots, m)$ a similarity matrix, $SM^{hl} = (sm_{ik}^{hl})$, is defined as:

$$sm_{ik}^{hl} = 1 - d(p_{ik}^h, p_{ik}^l) \tag{13}$$

   where d: $[0, 1] \times [0, 1] \to [0, 1]$ is a distance function [20]. The closer sm_{ik}^{hl} to 1, the more similar p_{ik}^h and p_{ik}^l.
2. A consensus matrix, $CM = (cm_{ik})$, is calculated by aggregating all the $(m-1) \times (m-2)$ similarity matrices by means of an aggregation function, ϕ:

$$cm_{ik} = \phi(sm_{ik}^{hl}), \ \ h = 1, \dots, m-1, \ \ l = h+1, \dots, m \tag{14}$$

3. Once the consensus matrix has been computed, the consensus degrees are obtained at three different levels:

(a) *Consensus degree on pairs of alternatives*, cp_{ik}. It is defined to measure the consensus degree among all the decision makers on the pair of alternatives (x_i, x_k). This is expressed by the element of the collective similarity matrix *CM*:

$$cp_{ik} = cm_{ik} \tag{15}$$

(b) *Consensus degree on alternatives*, ca_i. It is defined to measure the consensus degree among all the decision makers on the alternative x_i, and it is obtained by aggregating the consensus degrees of all the pair of alternatives involving it:

$$ca_i = \phi(cp_{ik}),\ \ k = 1, \ldots, n \wedge k \neq i \tag{16}$$

(c) *Consensus degree on the relation*, *CR*. It expresses the global consensus degree among all the decision makers' opinions, and it is obtained by aggregating all the consensus degrees at the level of alternatives:

$$CR = \phi(ca_i),\ \ i = 1, \ldots, n \tag{17}$$

It is clear that in any decision making process a high level of both consensus and consistency are necessary. To that aim, the system computes a *consensus/consistency level* (*CCL*) that needs to surpass a minimum threshold $\gamma \in [0, 1]$ set as a control parameter in order to continue with the exploitation phase:

$$CCL = (1 - \delta) \cdot CL + \delta \cdot CR \tag{18}$$

The parameter $\delta \in [0, 1]$ is set by the user depending on how important is consistency and consensus in the final solution. If this minimum threshold γ is not surpassed, the consensus-feedback process will keep running until the minimum threshold γ is surpassed or the maximum number of consensus rounds *maxIter* have been reached. This maximum number of iterations is incorporated in order to avoid that the consensus process does not converge after several rounds of discussion.

The proximity measures for each decision maker are calculated based on the collective preference relation, P^c:

1. The proximity measure of a decision maker e_h on the pair of alternatives (x_i, x_k) to the group one, denoted as pp^h_{ik}, is calculated as:

$$pp^h_{ik} = 1 - d(p^h_{ik}, p^c_{ik}) \tag{19}$$

2. The proximity measure of a decision maker e_h on alternative x_i to the group one, denoted as pa^h_i, is calculated as:

$$pa^h_i = \phi(pp^h_{ik}),\ \ k = 1, \ldots, n \wedge k \neq i \tag{20}$$

3. The proximity measure of a decision maker e_h on his/her preference relation to the group one, denoted as pr^h, is calculated as:

$$pr^h = \phi(pa^h_i),\ \ i = 1, \ldots, n \tag{21}$$

These proximity measures will be used by the feedback module to identify those decision maker who are far from the collective solution and give them some recommendations about how they should change their preferences to reach an acceptable level of consensus.

The developed module enables to calculate the distance among the decision makers' preferences following various distance functions: Manhattan, Euclidean, Dice, Cosine, and Jarccard distance [18,20]. That way the user can set the most suitable distance depending on the characteristics of the GDM process, such as the number of decision makers or the maximum number of possible rounds. For example, in [18], it is proved that the Manhattan and the Euclidean distances increase consensus level as the number of decision makers increases and help the consensus process to converge faster than the other ones. On the other hand, the Cosine and the Dice distances results in a fairly similar consensus levels regardless of the number of decision makers, whereas the Jaccard distance function contributes the least to the speed of convergence of the consensus processes. In addition, although the arithmetic mean is used by the system an aggregation function, ϕ, different aggregation operators could be used depending on the nature of the GDM problem to solve [18].

The results of the computation of both the consensus degrees and the proximity measures in the different consensus rounds are stored in a data frame, one data frame per consensus/proximity level. That way, the system keeps track of all the intermediate results generated during all the rounds of the process.

4.7. Feedback module

The aim of the feedback mechanism is to provide advice to the decision makers using consensus/consistency criteria to easily reach the desired consensus level while keeping a high consistency level in the decision makers' fuzzy preference relations. To do so, this module carries out two main tasks: (i) identification of the preference values, and (ii) generation of advice.

A three step process is carried out to identify the decision makers, the alternatives and, finally, the particular preference values, that contribute less to the consensus/consistency level.

- To identify the fuzzy preference relations that need to be modified, the system first identifies the decision makers whose consensus/consistency level of the fuzzy preference relation is lower than the threshold value γ:

$$EXPCH = \{h \mid (1-\delta)\cdot cl^h + \delta \cdot pr^h < \gamma\} \tag{22}$$

- Then, the system selects among those decision makers' alternatives with a consensus/consistency level lower than the threshold value γ:

$$ALT = \{(h,i) \mid h \in EXPCH \ \wedge \ (1-\delta)\cdot cl_i^h + \delta \cdot pa_i^h < \gamma\} \tag{23}$$

- Finally, the fuzzy preference values to be modified are those with an associated consensus/consistency level lower than the threshold value γ:

$$APS = \{(h,i,k) \mid (h,i) \in ALT \ \wedge (1-\delta)\cdot cl_{ik}^h + \delta \cdot pp_{ik}^h < \gamma\} \tag{24}$$

Once the decision makers' preferences, which need to be modified in order to increase the consensus/consistency, have been detected, the system has to carry out those changes. To that aim, this module is able to work following three different operation modes depending on the degree of human intervention [37]:

- *Automatic mode*. In this mode, the system automatically changes the decision makers' fuzzy preference relations according to the recommendations, rp_{ik}^h, which are generated using the following equation:

$$rp_{ik}^h = (1-\delta)\cdot cp_{ik}^h + \delta \cdot p_{ik}^c \tag{25}$$

- *Semi-supervised operation mode*. In this case, the system carries out an aggregation of the original values of the decision makers' fuzzy preference relations and the recommended one. The weight value to carry out this combination can be set in the control module. There can be set one value for each decision maker.
- *Fully-supervised operation mode*. In this case, the system provides easy to follow rules. The system saves the recommendations for each decision maker in one text file per decision maker. That way each decision maker can access confidentially to the system's recommendations.

4.8. Exploitation module

This module receives as an input the matrix in which the opinions of all the decision makers have been aggregated, and provides a global ranking of the alternatives. The global ranking can be calculated following one of the following two choice degrees [24]: the quantifier guided dominance degree (*QGDD*) and the quantifier guided non-dominance degree (*QGNDD*).

- For the alternative x_i, the system computes the quantifier guided dominance degree, $QGDD_i$, which quantifies the dominance that alternative the x_i has over all the others in a fuzzy majority sense as follows:

$$QGDD_i = \phi_Q(p_{i1}^c, p_{i2}^c, \ldots, p_{i(i-1)}^c, p_{i(i+1)}^c, \ldots, p_{in}^c) \tag{26}$$

- For the alternative x_i, the system computes the quantifier guided non-dominance degree, $QGNDD_i$, which gives the degree in which the alternative x_i is not dominated by a fuzzy majority of the remaining alternatives. It is defined as follows:

$$QGNDD_i = \phi_Q(1-p_{1i}^s, 1-p_{2i}^s, \ldots, 1-p_{(i-1)i}^s, 1-p_{(i+1)i}^s, \ldots, 1-p_{ni}^s), \tag{27}$$

where $p_{ki}^s = max\{p_{ki}^c - p_{ik}^c, 0\}$ represents the degree in which x_i is strictly dominated by x_k.

4.9. Storage module

Various storage data structures have been developed to easily store and manage all the information produced by the system. To that aim, we have taken advantage of the R built-in data structure, the data frame. As it has been mentioned previously an R data frame consist in a 2D matrix whose elements can be of any type. Various data frames have been implemented for the storage module. That way, each one of the modules above stores their information in its corresponding data frame. Therefore, it is pretty easy to retrieve the necessary information for each module or to include new modules that use this information. The main data frames implemented are the following:

- *CurrentPreferences*. In each row of this data frame are stored each decision maker's fuzzy preference relation. The size of this data frame is $m \times n(n-1)$.
- *PreferenceList*. This is an array of data frames, which store the *CurrentPreferences* in each round of the consensus process.
- *ConsensusLevel1*. In each row of this data frame are stored the computation of the consensus at the preferences level.
- *ConsensusLevel2*. In each row of this data frame are stored the computation of the consensus at the alternative level.
- *GlobalConsensusPerRound*. This data frame has only one column and stores in each row the global consensus level obtained in each round.
- *ProximityLevel1*. In each row of this data frame are stored the computation of the proximity between the decision maker's fuzzy preference relation and the aggregated matrix at the preferences level.
- *ProximityLevel2*. In each row of this data frame are stored the computation of the proximity at the alternative level.

- *GlobalProximityDecisionMakers*. This data frame stores in each row the global proximity between each decision maker and the aggregated matrix.
- *GlobalProximityPerRound*. This data frame stores in each column the decision makers' average proximity for each consensus round.
- *GlobalConsistencyPerRound*. This data frame stores in each column the decision makers' average consistency in each round.

4.10. Graphical representation module

One of the main novelties that the developed system presents with respect to the existing ones is the possibility of getting a quick insight in the GDM process by means of diverse graphical representations. All these representations make the system a graphical monitoring tool to support decision makers by providing them with easily understandable visual information about the current status and the evolution of the decision process. This tool eases the analysis of diverse crucial aspects that are common in these problems, among them, we can highlight:

- Monitoring the evolution of the global consensus across the whole GDM process.
- Monitoring the decision makers' consistency along the whole GDM process. This is especially important to make sure that they are keeping an acceptable consistency level in their preferences after the recommendation rounds.
- Detection of the alternatives that are posing more controversy in the GDM process.
- Detection of those decision makers or group of them, whose preferences are further from the consensus solution, or those that are more reluctant to change their point of view.
- Detection of those decision makers that are being influenced or manipulated to provide preferences far from the consensus solution.
- Providing information to the decision makers about the GDM process, and showing them how their preferences are located with respect to the consensus one.

In the following the graphics that our system includes and how they have been developed and integrated in this framework are detailed. The graphical representations that our system includes can be divided in two wide groups, depending on whether they show the evolution among the various consensus rounds, or they display information related to a single round:

- Representation of the evolution across the consensus rounds:
  - *Consistency vs. consensus evolution in the GDM process*. This representation shows the evolution of both global consistency and global consensus in each consensus round. The desirable situation is that most of the point or at least the final ones lie over the diagonal line and the points present a positive tendency. It would mean that the final solution has reached a high level of agreement and that it is consistent. This representation also enables to detect whether the consensus process is not only helping to bring the decision makers' opinions closer but also to keep or increase their consistency.
  - *Decision maker's consistency vs. decision maker's consensus in the GDM process*. This representation allows to check how decision makers' consensus and consistency evolves during the GDM process. It also enables to visually check the different decision makers profiles depending on the shape of the curve for each decision maker. Curves with a positive tendency and located over the diagonal represent the desired situation of those decision makers that are more willing to change their opinions in the interest of increasing the global consensus while keeping a highly consistency level. Curves parallel to the y-axis represents those decision makers which are reluctant to change their mind during the process, and therefore they may require special attention.
- Representation of the consensus state in a single round:
  - *Barplot of each decision maker's proximity to the aggregated solution*. This representation enables to check who are the decision makers whose opinions are closer to achieve a high degree of consensus, and who are those with highly disagree with the proposed solution.
  - *Barplot of the average consensus achieved for each alternative*. This representation allows to quickly identify which alternatives are posing more controversy in the decision process.
  - *Barplot of the average consistency achieved for each decision maker*. This representation provides a quick insight on those decision makers providing more consistent fuzzy preference relations in the decision making process.
  - *2D representation map of the decision makers' fuzzy preference relations and the consensus solution*. This representation provides a quick insight of the current state of the decision process and enables the rapid identification of sub groups of decision makers who share similar opinions. It also eases the detection of conflicts among decision makers. Moreover, it provides the decision makers with a good idea about the status of the consensus process and how far their opinions are from the consensus solution. This 2D representation is obtained after carrying out a classical 2D multidimensional scaling reduction of the decision makers' fuzzy preference relation matrix [23]. In addition, R also offers the possibility of non-metric multidimensional scaling.
  - *3D representation of the position of each decision maker with respect to the consensus solution among with their consistency*. This plot easily allows to identify those groups of decision makers that are far from the consensus solution but keep a high degree of consistency, and, therefore, need special attention. To easily visualize this plot, we have also included an interactive representation.

5. Illustrative example

In this section, we include two illustrative examples to show how the GDM-R works and its wide range of possibilities as well as its usefulness in practice. The first example illustrates how the test environment works and explains the different graphical representations available in the system. The second example includes real data taken from the experts and deals with incomplete information in the decision makers opinions as shows how the supervised operation mode works.

5.1. Example illustrating the consensus process and the graphics analytics

5.1.1. Problem definition and parameter setting

In order to show the test capabilities of the developed framework, the test module is used to generate a data set with the decision makers' preferences. The configuration parameters are set as follows: In this example a GDM situation involving 20 decision makers and 4 different alternatives is considered. The minimum consensus threshold to be achieved is 0.8, and the maximum number of consensus rounds is 4. In addition, the initial average level of consistency of the fuzzy preference relations is 0.8 and the initial average level of consensus is 0.6.

Configuration parameters

```
M=20 #Number of decision makers
N=4 #Number of alternatives
prefererecies_file=''#File with the decision makers' preferences
consensusThreshold=0.8
numberOfRounds=4
distance='euclidean'
quantifierAggregation='most'
dominance="QGDD"
quantifierExploitation='most'
feedback="automatic"
operationMode="Simulation"
initialConsensus=0.6
initialConsistency=0.8
```

5.1.2. Graphical representations for each round of consensus

In the following we show some of the most relevant representations along with their utility to increase the quality of the GDM process are explained.

First of all, a 2D map with the position of each decision maker with respect to the aggregated solution for each consensus round is depicted in Table 1. The global solution is always displayed in the center of the plot. This type of visualization allows to ease the rapid detection of those decision makers whose opinions are far from the global solution as it is the case of decision maker 1. Hence, in real case situations, some especial actions can be taken depending on the characteristics of the process, such as discarding their opinions as they can be considered as outliers. Furthermore, it is possible to recognize how in the first round, the preferences are in general pretty spread up, but after each round of recommendations the opinions of the decision makers get closer and closer verifying that the decision making process is going on the right direction. It is worth to point out that this type of maps also allow to easily recognize those decision makers reluctant to change their opinions in order to achieve a solution accepted by the whole group by keeping track of those whose position in the map does not get closer to the global solutions with the iterations. In this sense small sub-communities of decision makers that share similar opinions can be identified as well as well as and those who exert a greater influence on their sub-communities.

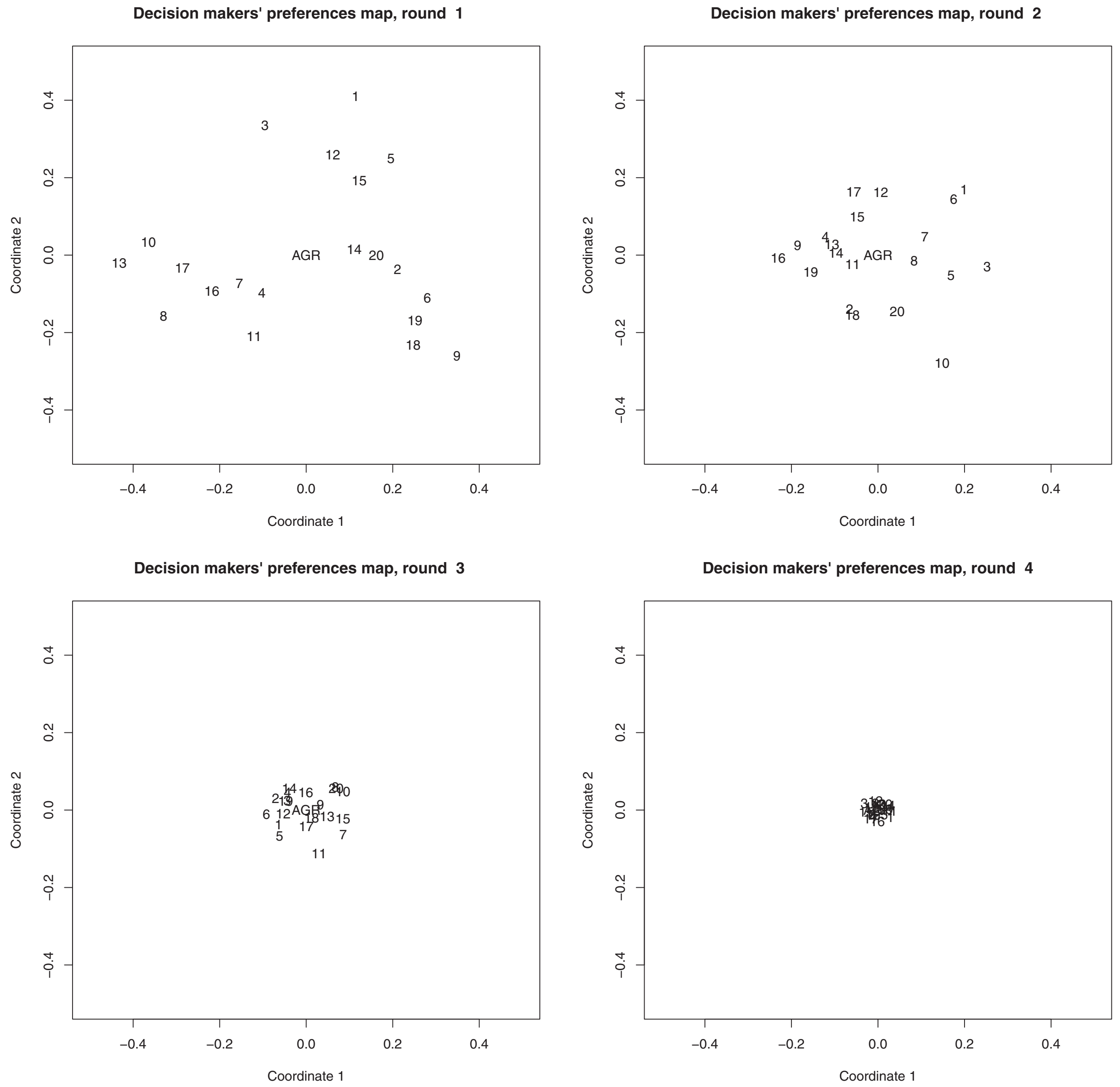
A 3D map of the decision makers preferences among with the degree of consistency for each decision maker is shown in Table 2. This type of map allow to recognize the decision makers whose preferences are more consistent and her distance to the global solution. For example, in the first iteration the decision maker number 1, presents a very high level of consistency even though his/her preferences are far from the consensus solution. Therefore, this decision maker's opinions are worth to be taken into consideration. It also allows to quickly recognize communities of decision makers who share the same points of views, and also identify those decision makers who have more influence or more persuasion power over the group. They can be recognized easily since they do not change their opinions with the time, but they attract others forming small clusters in the map that become bigger with the time. Usually, the most influential decision makers also present a high level of consistency.

In Table 3 it is presented a barplot with the decision makers average consensus and consistency degree per round, along with both lines showing the global average consensus and consistency degrees. These plots easily allow to assess the evolution of both consensus and consistency and recognize those decision makers that may present more controvert opinions, or less consistent ones, and take especial actions with those ones.

5.1.3. Results of the GDM process

The decision making process finishes when the maximum number of rounds has been overpassed or when the desired consensus degree has been achieved. In Fig. 3 it is depicted the evolution of the degree of consensus vs. the degree of

Table 1
Evolution of the decision makers' preferences among the consensus rounds.

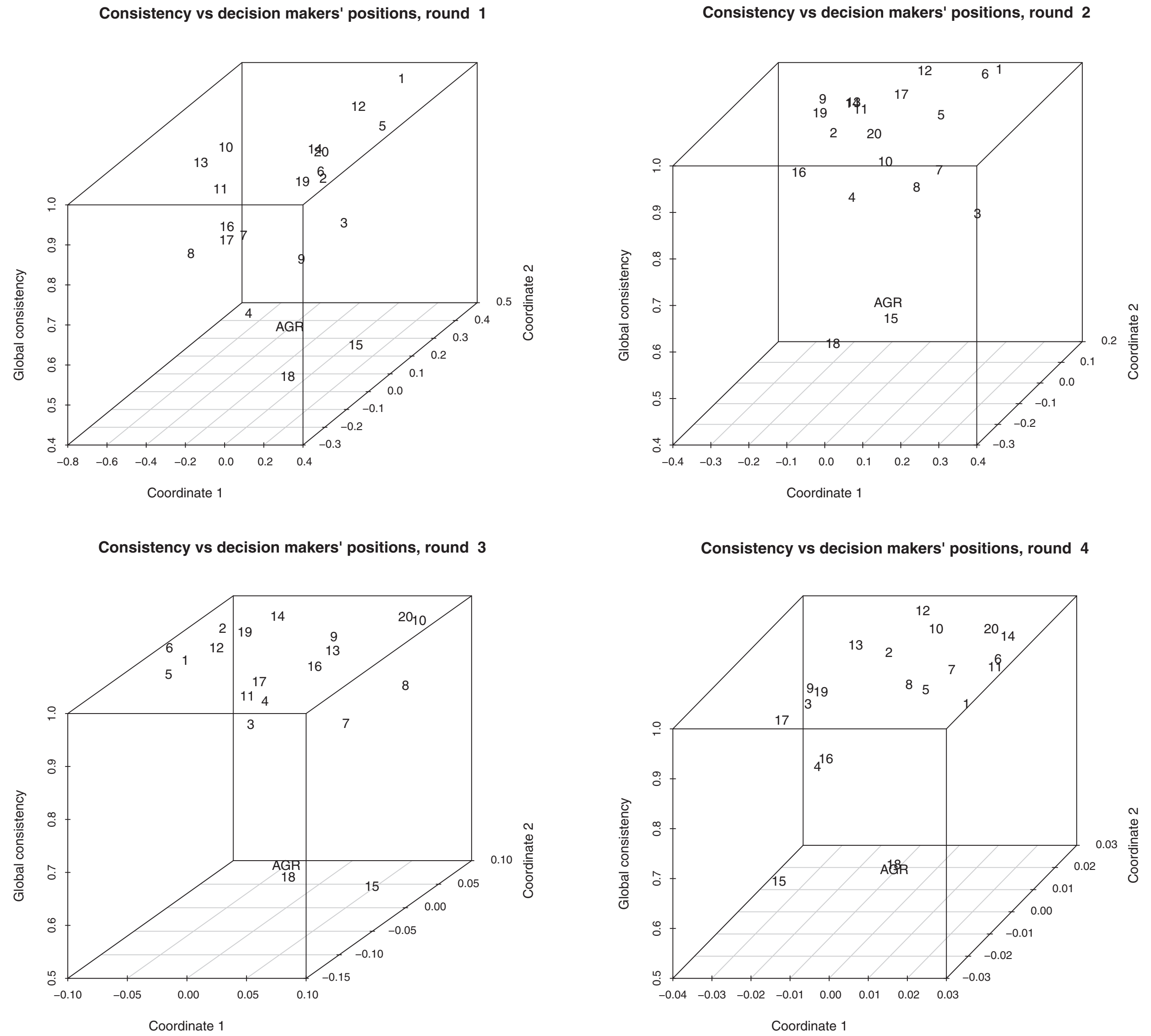


consistency for each iteration. Notice that the line slope in this plot allows to easily recognize how the decision process has gone. For example, if the line is almost parallel to the *x*-axis it means that the iterations of the decision process have only contributed to increase the global consistency. That is, in average the decision makers' opinions have become more consistent with the time, but the decision makers had not change their mind to increase the consensus. This type of line means that the decision makers are very committed to provide non contradictory solutions to the problem, but they present a non cooperative behavior towards achieving a solution accepted for the whole group.

A similar situation would happen if the line is parallel to the *y*-axis, but in this case it would mean that the consensus has improved whereas the decision makers consistency has barely changed. In this case that would mean that the decision makers are easily manipulated to change their minds, without caring about the quality of the provided solution.

The most desired solution is having a line with positive slope, like the one in Fig. 3, that means that the different rounds have contributed to positive increase both the consensus and the consistency of the decision makers. Also the average slope of this line also provide us with a general measure of how fast the consensus increase vs. the consistency, this measurement can be leverage to test the performance of different decision making approaches.

Table 2
Evolution of the decision makers preferences among the consensus rounds.



Finally, the system provides a graphical representation with the ranking of the alternatives using both the dominance and the non-dominance degrees as the one presented in Table 4. In this concrete case it was clear that the most desired alternative was the number two.

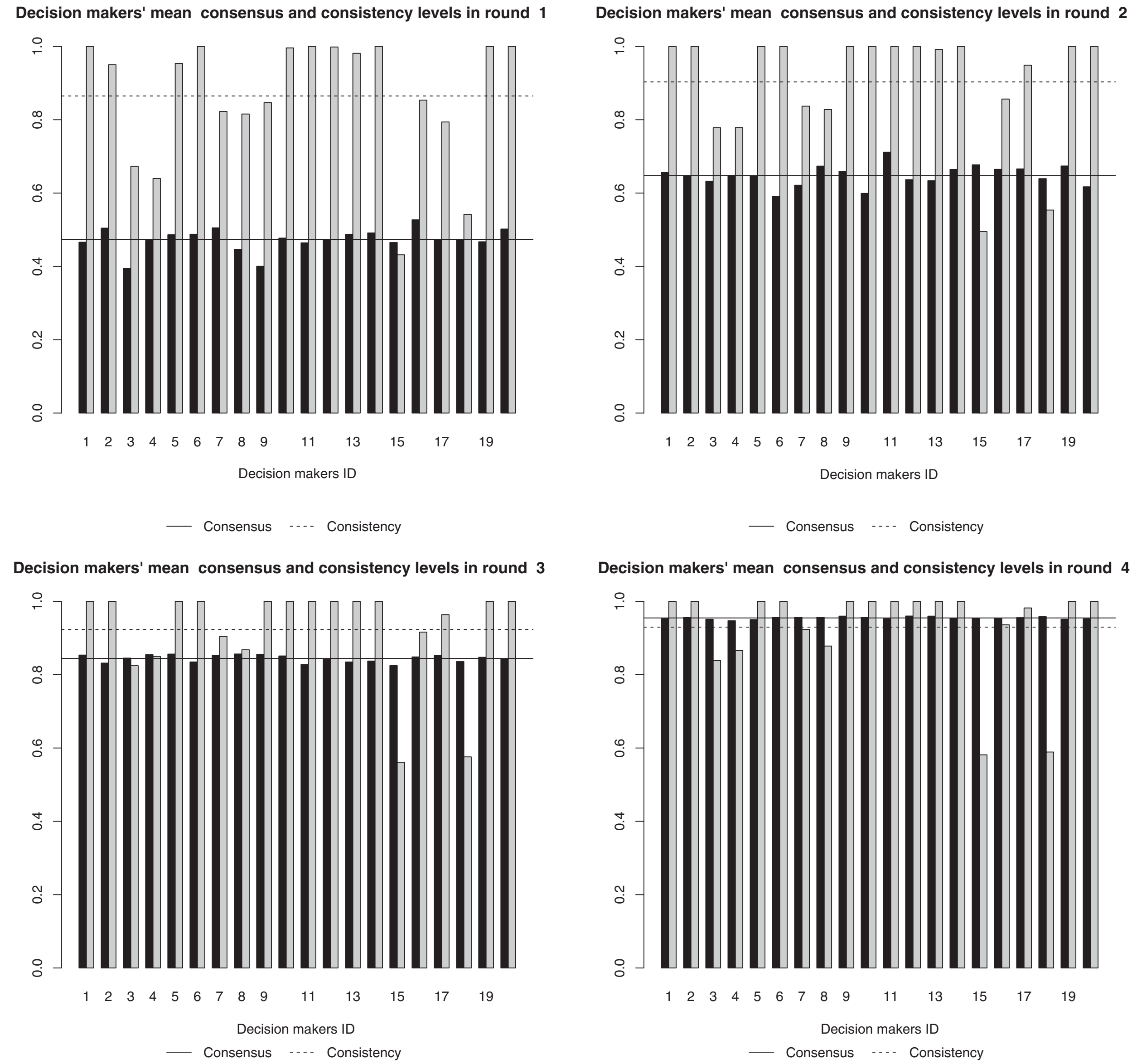
5.2. GDM process with incomplete information

In this section we analyze a real example in which 4 experts have been asked to provide their opinions to choose the best mobile phone from four different models taking into account the relationship between quality and price.

- Huawai P8
- Nexus 6
- iphone 6
- Samsung Galaxy S6

In this case the system works in supervised mode, that means that all the changes proposed by the framework needs to be accepted by the decision maker before being included in the preferences. The minimum consensus threshold to be achieved is 0.8, and the maximum number of consensus rounds is 4. The configuration parameters are as follows:

Table 3
Evolution of the decision makers' consistency and consensus in each round.



Configuration parameters

```
M=4 #Number of decision makers
N=4 #Number of alternatives
consensusThreshold=0.8
numberOfRounds=4
distance='euclidean'
quantifierAggregation='most'
dominance="QGDD"
quantifierExploitation='most'
feedback='supervised'
```

In this case we are going to focus on the interaction of decision maker e_1 with the system. The same can be applied to the rest of the experts.

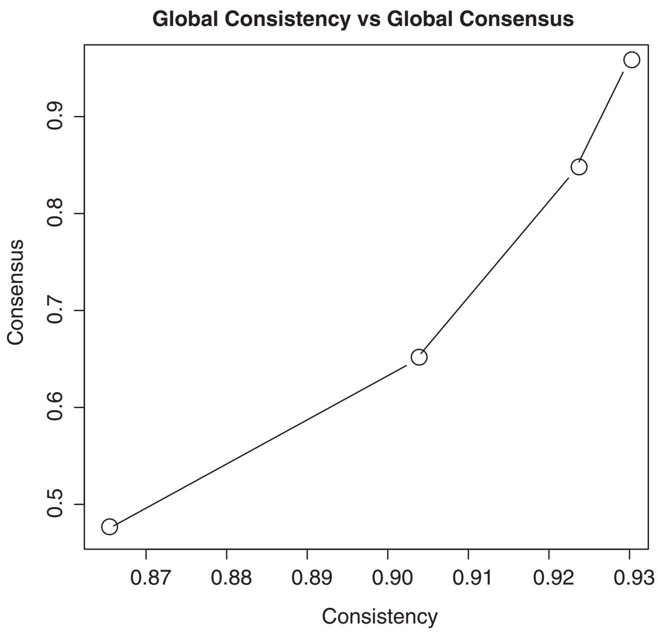


Fig. 3. Global consensus and consistency evolution along the consensus rounds.

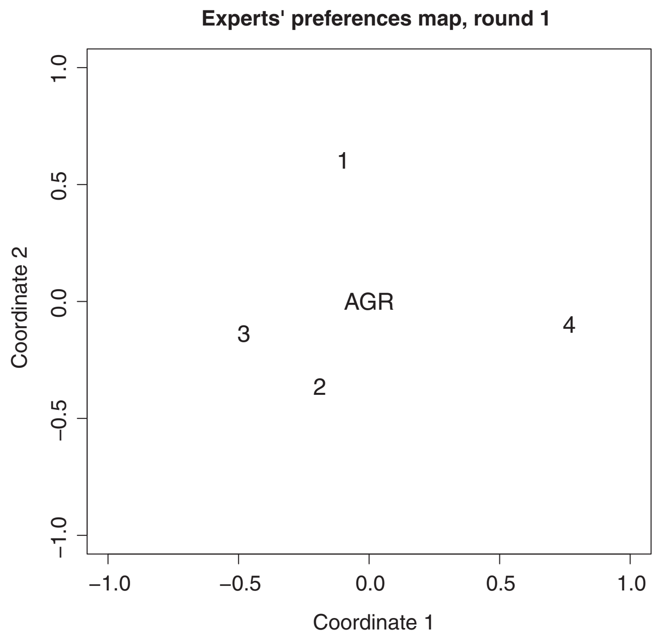
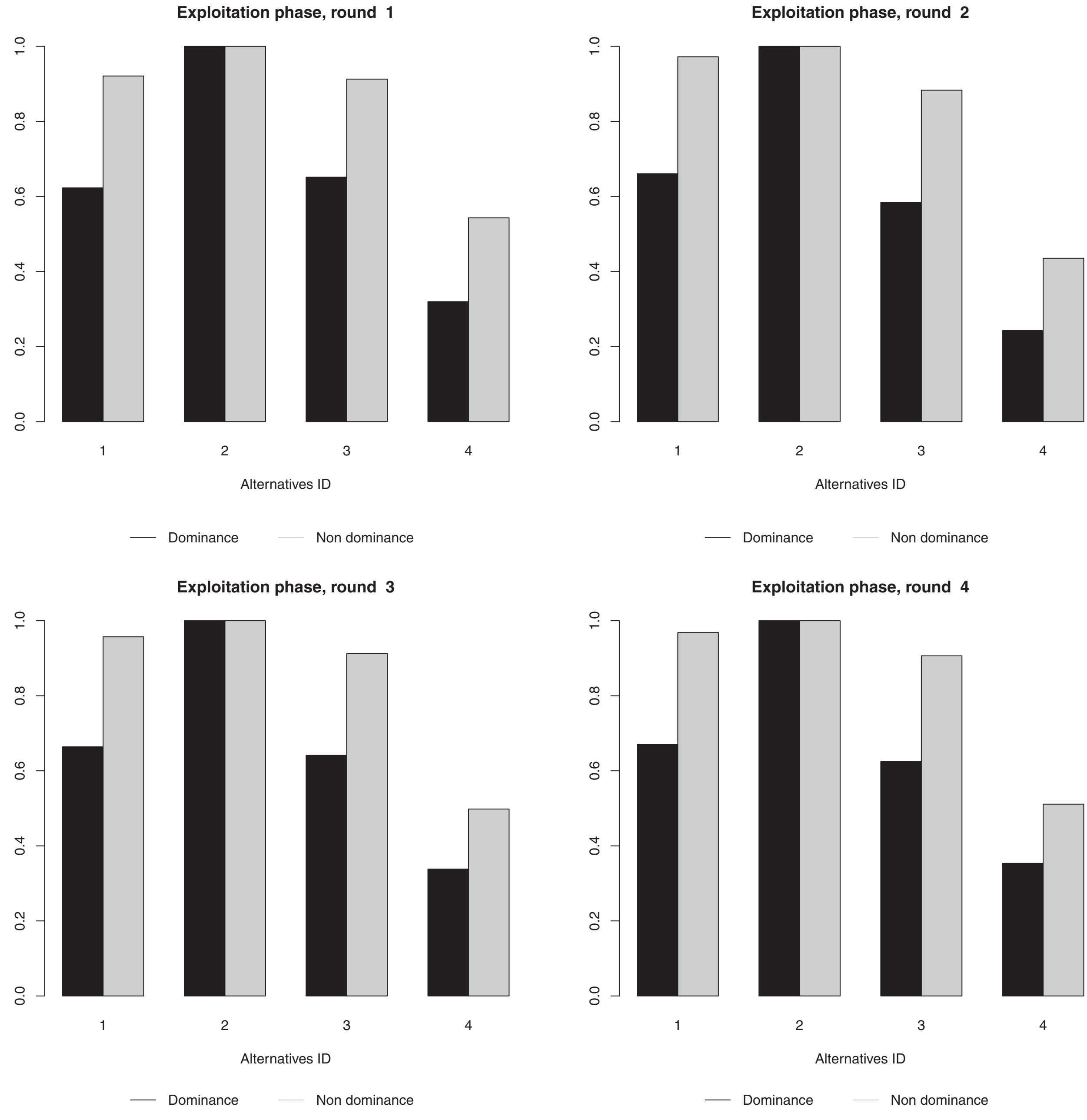


Fig. 4. Experts preferences with respect the global one after the first round of consensus.

First of all, the expert e_1 provides the following preference values: $p^1_{12} = 0.6$ and $p^1_{13} = 0.6$:

$$P^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ x & - & x & x \\ x & x & - & x \\ x & x & x & - \end{pmatrix}$$

Table 4
Dominance and non-dominace degrees in the exploitation phase.



The system, using these values estimates the values p^1_{23}, p^1_{24}, p^1_{32} , p^1_{34}, p^1_{42}, p^1_{43}, and the system presents the following matrix to the user to be approved.

$$P^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ x & - & 0.9 & 0.7 \\ x & 0.1 & - & 0.3 \\ x & 0.3 & 0.7 & - \end{pmatrix}$$

Expert e_1 considers the estimated value p^1_{32} does not reflect her real preference value and $p^1_{32} = 0.8$ is inserted instead. The system alerts that there is a high inconsistency associated to this new value, and consequently e_1 realizes that there is

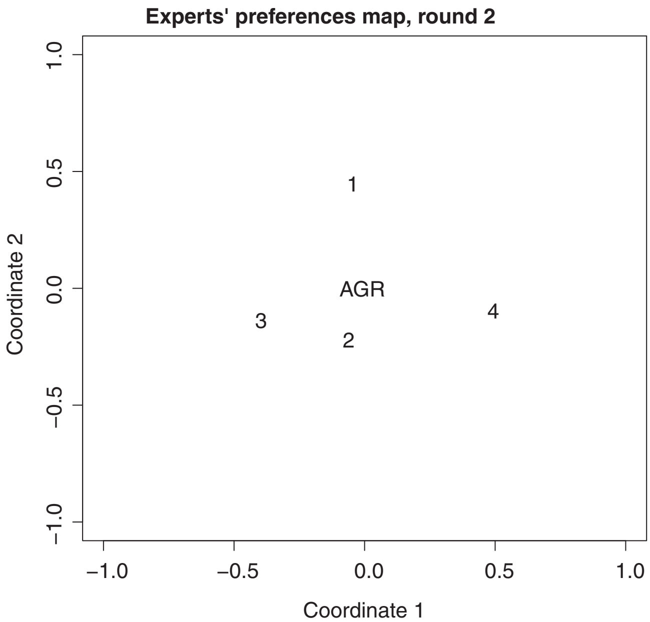


Fig. 5. Experts preferences with respect the global one after the first round of consensus.

contradiction in her preference relation ($p^1_{23} = 0.9 \Rightarrow x_1 \succ x_2$ and $p^1_{32} = 0.8 \Rightarrow x_2 \succ x_1$) and changes p^1_{32} to the value that the system initially suggested ($p^1_{32} = 0.1$).

Finally e_1 completes her preference relation accepting the values estimated by the system.

$$P^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.8 & - & 0.9 & 0.7 \\ 0.4 & 0.1 & - & 0.3 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix}$$

For the rest of the experts the system follows a similar behavior. Firstly, the experts provide the following incomplete matrices:

$$P^2 = \begin{pmatrix} - & x & 0.7 & x \\ 0.4 & - & x & 0.7 \\ 0.3 & x & - & x \\ x & 0.4 & x & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & x & 0.7 & x \\ 0.4 & - & x & 0.7 \\ 0.3 & x & - & x \\ x & 0.4 & x & - \end{pmatrix}$$

$$P^4 = \begin{pmatrix} - & x & 0.7 & x \\ 0.4 & - & x & 0.7 \\ 0.3 & x & - & x \\ x & 0.4 & x & - \end{pmatrix}$$

Which are automatically completed by the system as follows:

$$P^2 = \begin{pmatrix} 0.50 & 0.4 & 0.30 & 0.25 \\ 0.62 & 0.5 & 0.40 & 0.40 \\ 0.70 & 0.6 & 0.50 & 0.45 \\ 0.80 & 0.7 & 0.57 & 0.50 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.50 & 0.60 & 0.46 & 0.30 \\ 0.30 & 0.50 & 0.31 & 0.40 \\ 0.54 & 0.69 & 0.50 & 0.27 \\ 0.75 & 0.87 & 0.73 & 0.50 \end{pmatrix}$$

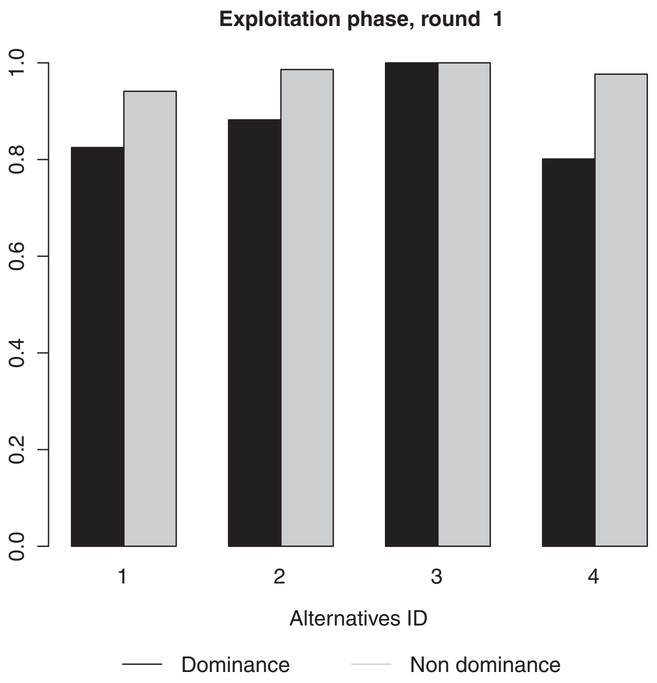


Fig. 6. Results of the GDM process.

$$P^4 = \begin{pmatrix} 0.5 & 0.4 & 0.5 & 0.7 \\ 0.6 & 0.5 & 0.6 & 0.7 \\ 0.6 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.2 & 0.3 & 0.5 \end{pmatrix}$$

In Fig. 4 a map with the experts preferences with respect to the global solution for the first iteration is depicted:

In this first iteration the consensus level reached $CR = 0.76$ and the Global Consistency $CL = 0.97$.

Since the required consensus level has not been achieved the system suggests to the experts various changes to increase the consensus level, for example for the case of expert 1 the recommendations are as follows:

```
Provide a value for (2, 1) close to 0.52
Provide a value for (2, 3) close to 0.8
Provide a value for (2, 4) close to 0.76
Provide a value for (3, 1) close to 0.44
Provide a value for (3, 2) close to 0.2
Provide a value for (3, 4) close to 0.42
Provide a value for (4, 1) close to 0.38
Provide a value for (4, 2) close to 0.35
Provide a value for (4, 3) close to 0.6
```

Since the system is working on the supervised mode the experts are the ones that decide whether they accept the proposed recommendations or not. In Fig. 5a map with the experts preferences after the feedback round is depicted.

In this second iteration the consensus level reached is $CR = 0.88$ and the Global Consistency $CL = 0.91$. Therefore enough level of agreement is achieved and no more iterations are required. Finally the result of the decision process indicates that the winner alternative is the number 3, see Fig. 6.

6. Conclusion and future work

In this contribution we have presented a critical review of the available software frameworks for computer assisted GDM, concluding that there are few available tools and the ones that have already been developed are not open source and are not able to carry out GDM processes including multiple types of preference elicitations and ways of dealing with unknown information. Moreover the majority of these tools present a non modular architecture which makes very complex for other researcher to extend or adapt to their own necessities or with test purposes.

In this contribution, we present GDM-R, a new open source framework fully implemented in R, overcoming the weaknesses of the previous software systems for GDM processes. Its main new and interesting aspects are summarized below:

- It displays various graphical representations which provide a rapid insight in the state and the evolution of the GDM process and enable to identify decision makers whose opinions are far from the group solution and those who present a non cooperative behavior in order to reach an agreement among with experts subcommunities and more influential decision makers.
- It offers a test mode which enables to set a trial scenario to try and compare the performance of different GDM approaches. It is helpful to validate and objectively compare the already existing algorithms and to develop new ones.
- The proposed framework can be easily extended to work with other types of preference relations and to include other methodologies of GDM. Therefore, other researchers can extend and customize it for comparative and test purposes.
- The developed system can be easily adapted to work in other environments such as smartphones, tablets and web, since the logic of the application is totally independent from the graphical user interface.

As future work, we point out several directions as the extension of this framework to work with different platforms such as mobile and web based environments, allowing to carry out GDM processes in environments in which the decision makers can access to the decision process from different clients. In addition, more complex approaches based on ontologies [40] and trust networks [47] will be validated and incorporated.

Acknowledgments

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